Correlated Persuasion

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Story

- Two firms (two senders) selling similar products target different regional markets, say Hong Kong and Singapore.
- They persuade their own customer base (receivers) by designing their advertising + marketing campaigns.
- Their products' similarity suggests positive correlation in quality.
- HK customers' purchasing decisions are influenced by both firms' advertising campaigns. (Likewise for Singaporean customers.)
  $\Rightarrow$ information spillover/ leakage

Questions

- How would the correlation in senders' qualities affect their **persuasion strategies**?
  - Compared to the benchmark independent case, more informative or less informative persuasion?
- Does the correlation **benefit or hurt the senders**? What about **receivers**?
  - The equilibrium level of **information revelation**
- What are the implications for *product design* and *transparency design*?

Model

- Two ex-ante identical senders: Sender 1 and Sender 2
- Each sender i is endowed with a proposal with binary quality $U_i \in \{l, h\}$ with $h > l$ and joint distribution:

	$U_2 = l$	$U_2 = h$
$U_1 = l$	$(1-\mu)^2 + \rho$	$\mu(1-\mu) - \rho$
$U_1 = h$	$\mu(1-\mu) - \rho$	$\mu^2 + \rho$

- $\mu \in (0, 1/2)$: average quality.
- $\rho \in [0, \bar{\rho}]$: correlation parameter, where $\bar{\rho} = \mu(1-\mu)$.

Model

- Two receivers: Receiver 1 and Receiver 2
- **Receiver** i decides whether to adopt **Sender** i**'s proposal.**
- His payoff depends only on U_i, but not U_j, $j \neq i$.
- For simplicity, receiver i adopts iff sender i's proposal quality has a *posterior (that* $U_i = h$*)* **no less than** $1/2$.
- Sender i gets a positive payoff iff Receiver i adopts her proposal.

Strategies

- Sender i persuades by costless design of signal (info structure) about U_i.
  - She has no direct control over info revelation of U_j.
- The marginal distribution over U_i **conditional only on sender i's own signal realization** m_i is generically denoted by **posterior** $p_i = \Pr(U_i = h | m_i)$.
- Wolog: sender i's strategy is a distribution over posteriors such that its **mean equals the prior**.
- Both receivers have access to the signal realizations/posteriors of **both senders**.
- Receiver i adopts Sender i's proposal iff

$$\Pr(U_i = h | p_i, p_j) \geq \frac{1}{2}.$$

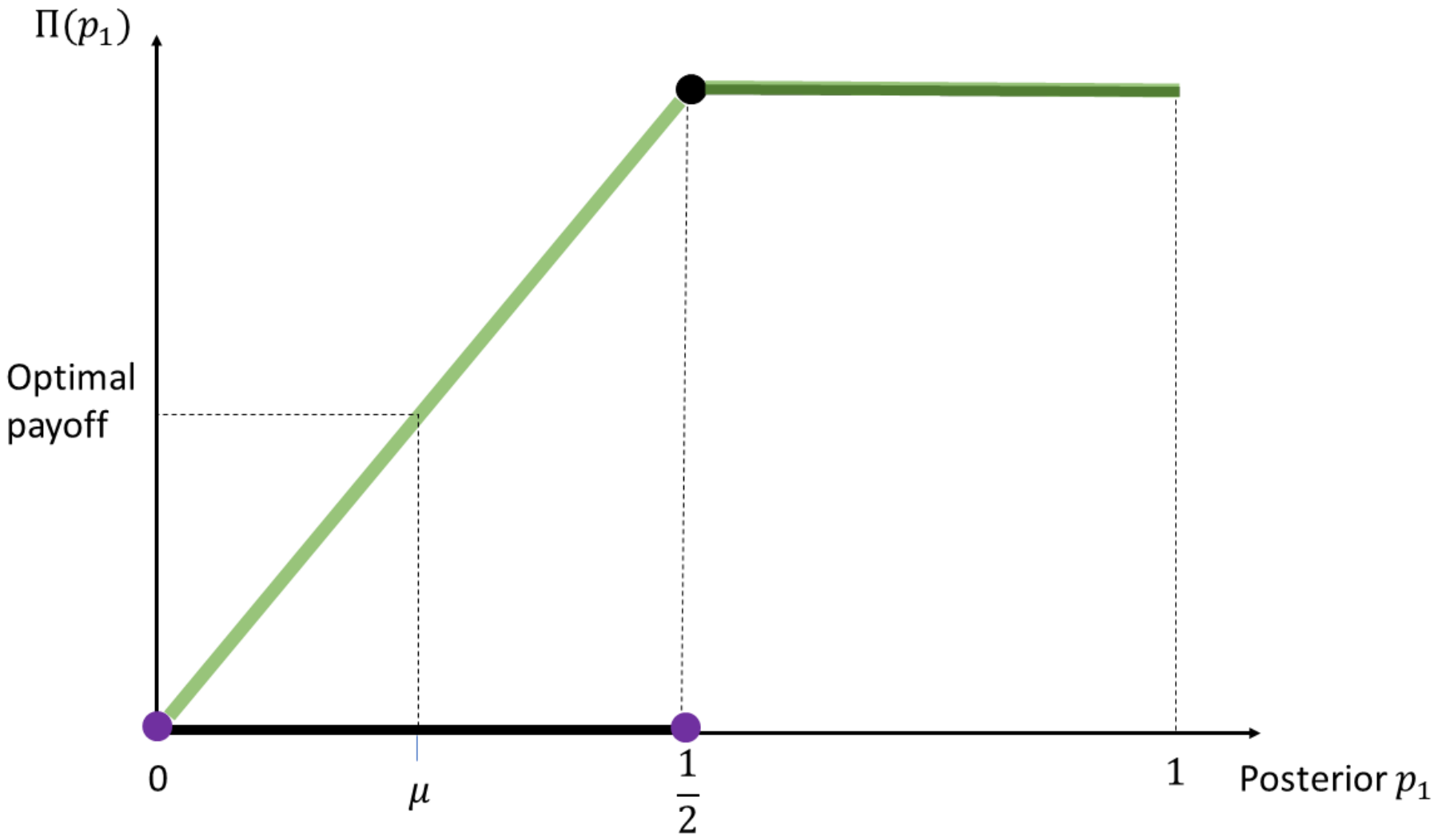
Timeline

1. Sender 1 and 2 simultaneously post their signals /info structures about their respective U_i.
2. Receiver 1 and Receiver 2 observe the signal realizations by both senders.
3. Receiver 1 adopts Sender 1's proposal iff the (combined) posterior of U_1 is no less than $1/2$.
   Receiver 2 adopts Sender 2's proposal iff the (combined) posterior of U_2 is no less than $1/2$.
4. The players collect their respective payoffs.

Equilibrium

- Focus on the **symmetric equilibria** between the senders' play.
- If the symmetric equilibria can be Pareto ranked, we select the **senders-preferred** one.

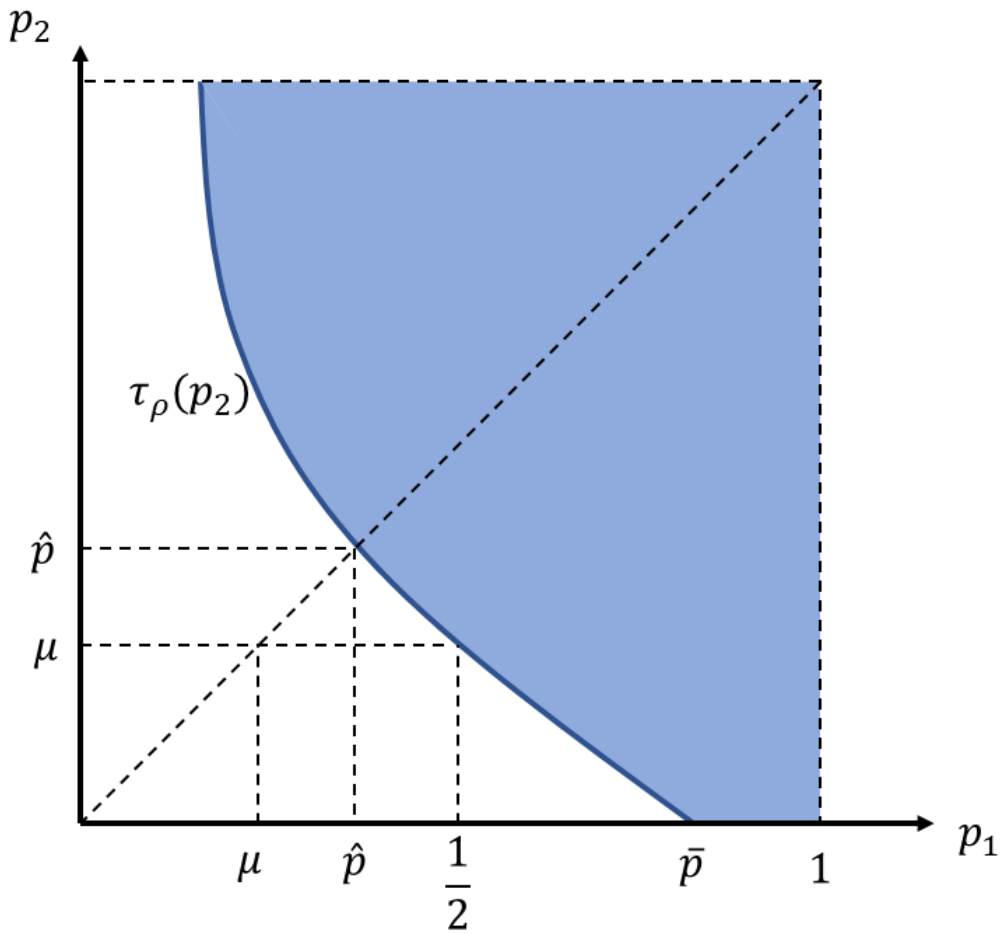
The Independent Benchmark



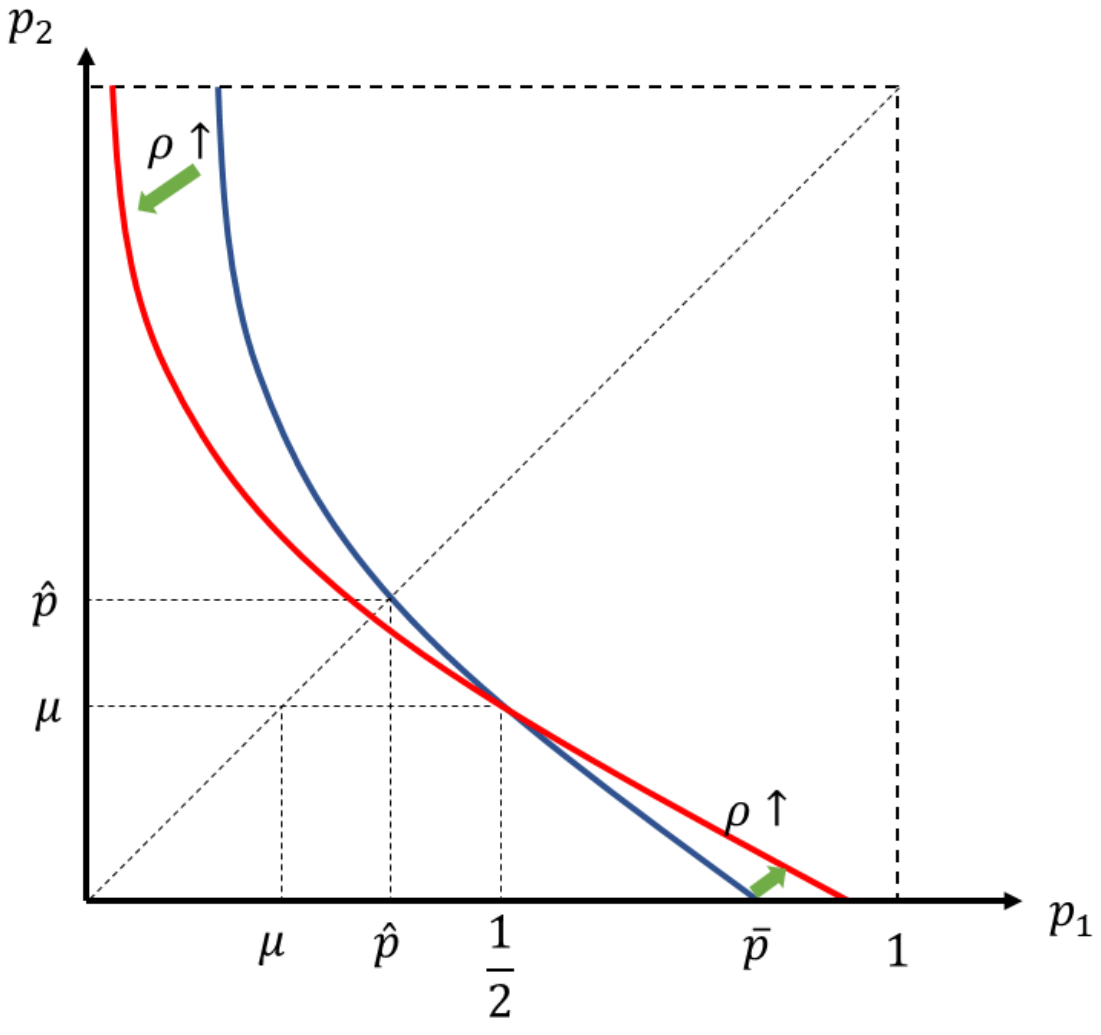
Persuasion under Correlation

Let $\rho > 0$. Sender 1 succeeds in persuasion iff

$$\Pr\left(U_1 = h | p_1, p_2\right) \geq \frac{1}{2} \Leftrightarrow p_1 \geq \tau_\rho\left(p_2\right).$$



Persuasion under Correlation: Increase in Correlation



Payoff Function

- KG11: The optimal signal can be found by constructing the concavification of payoff function in own posterior.
- If sender 2 adopts strategy σ_2, sender 1's payoff function is:

$$\Pi\left(p_1;\sigma_2\right)=\sum_{\{p_2\in supp\{\sigma_2\}:\Pr(U_i=h|p_1,p_2)\geq 1/2\}}\Pr\left(p_2|p_1,\sigma_2\right),$$

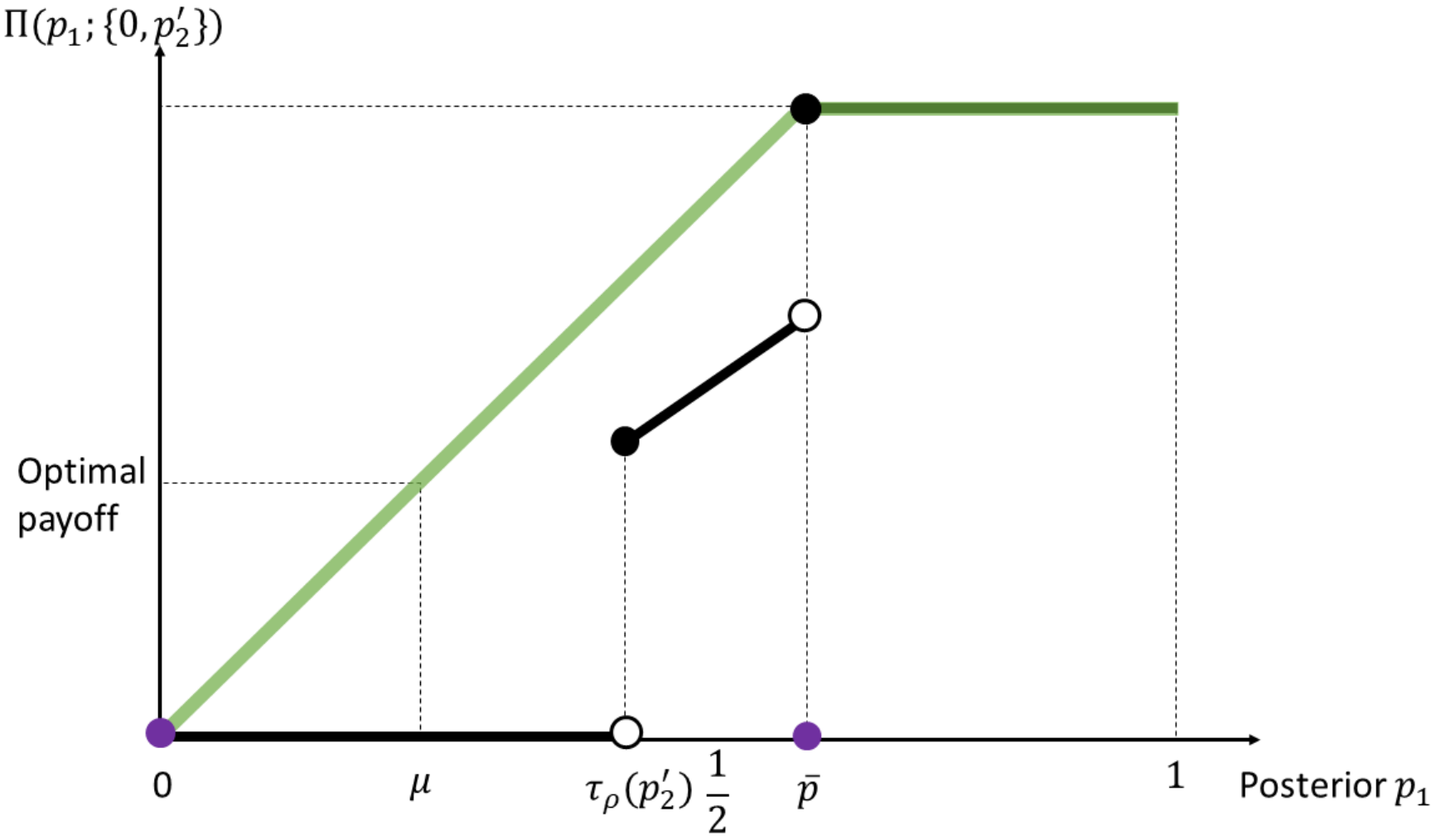
where

$$\Pr\left(p_2|p_1,\sigma_2\right)=\sigma_2\left(p_2\right)\left(1+\frac{\rho}{\mu^2\left(1-\mu\right)^2}\left(p_2-\mu\right)\left(p_1-\mu\right)\right).$$

- Fixing strategy σ_2, good news by Sender 1 implies Sender 2 is more likely to bring good news too.
- This effect is more salient if ρ is large.

Payoff Function

Say σ_2 has support $\{0, p_2'\}$. Sender 1's payoff function may look like:



Structure of Symmetric Equilibria

- Coordinated eqm: supported only on $\{0, \hat{p}\}$
- Uncoordinated eqm: supported on $\{0, \bar{p}\}$ and possibly more.

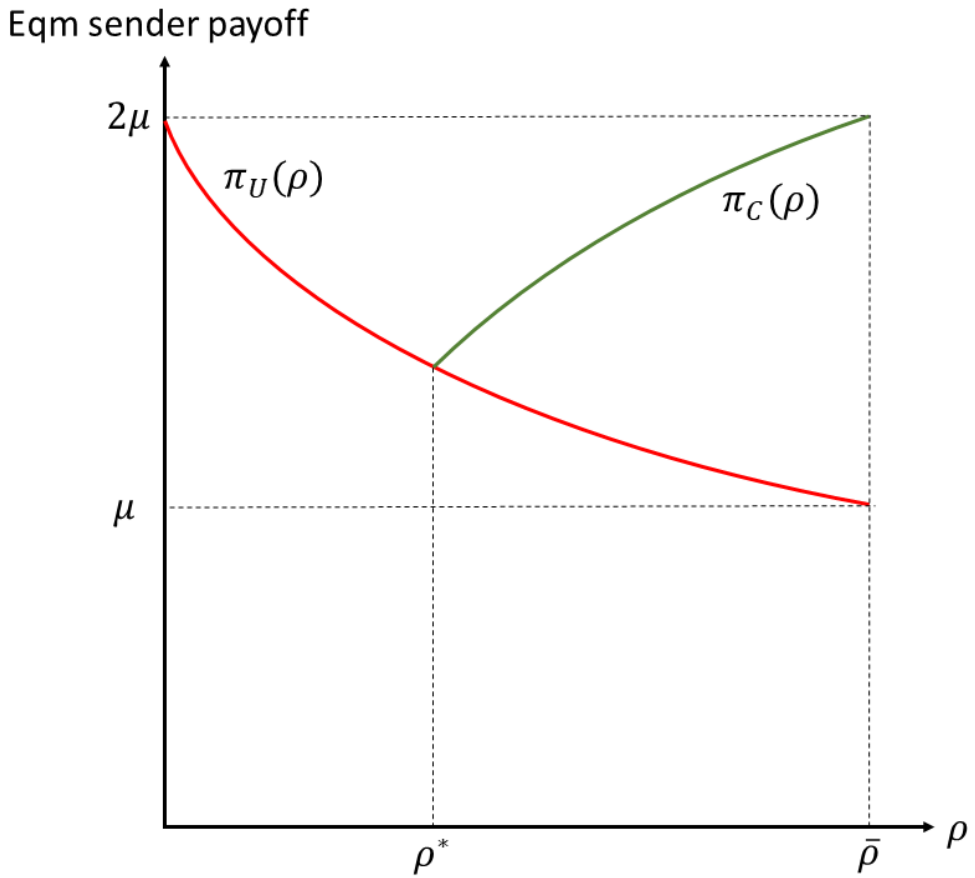
Lemma

These are the only two types of symmetric equilibria. Whereas uncoordinated equilibrium always exists, a coordinated equilibrium exists iff $\rho \geq \rho^*$ for some $\rho^* \in (0, \bar{\rho})$.

Equilibrium Payoff

Payoffs of uncoordinated and coordinated equilibrium are

$$\pi_U(\rho) = \mu\frac{1}{\bar{p}} \text{ and } \pi_C(\rho) = \mu\frac{\Pi(\hat{p};\{0,\hat{p}\})}{\hat{p}}.$$

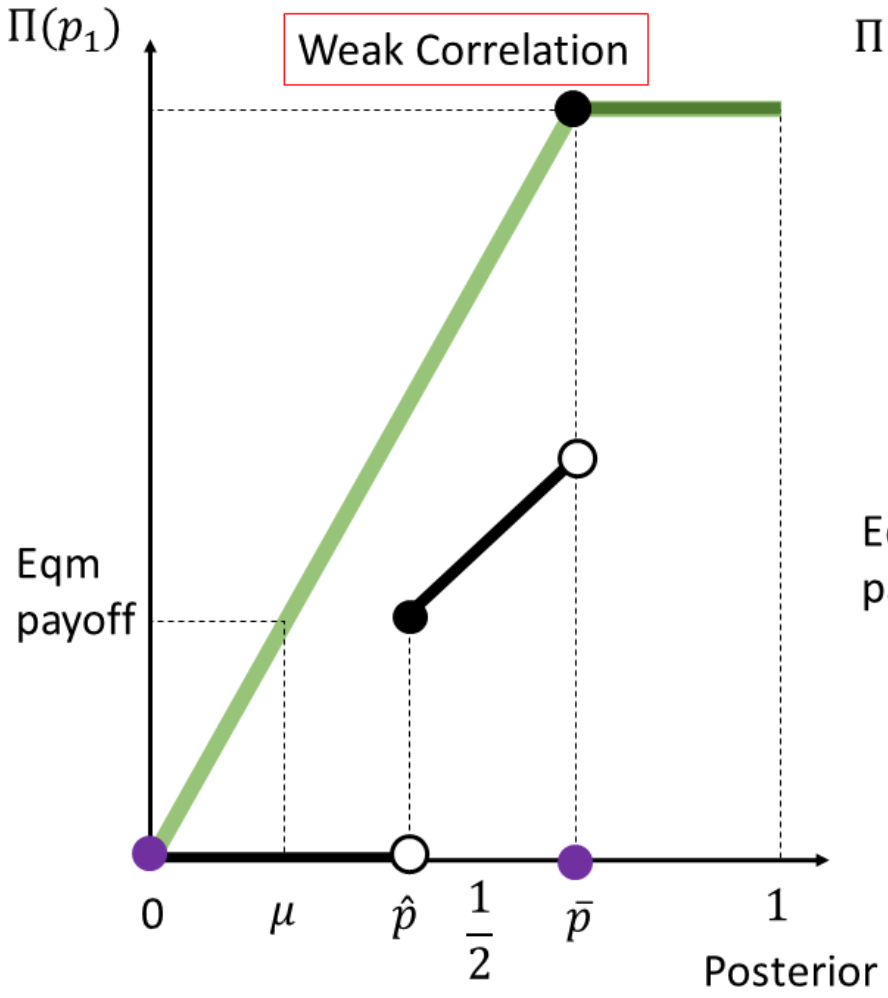


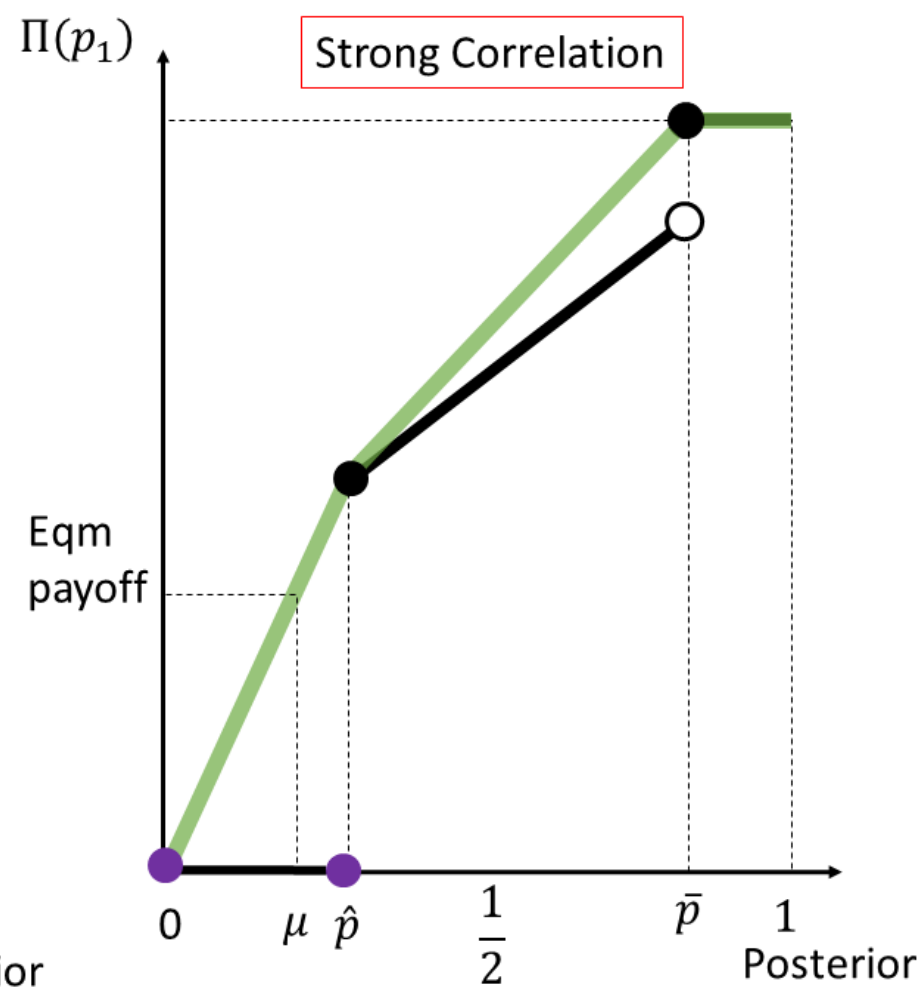
Optimal Symmetric Equilibrium

Proposition

If $\rho < \rho^*$, the optimal symmetric equilibrium is uncoordinated supported on $\{0, \bar{p}\}$.
If $\rho \geq \rho^*$, the optimal symmetric equilibrium is coordinated supported on $\{0, \hat{p}\}$.

The Effect of Correlation





The Effect of Correlation on Info Revelation

Exploit fellow sender's good news (calls for weak disclosure) or overcome his bad news (calls for strong disclosure)?

If the **correlation is low**,

- not too costly to counter his bad realization.
  - $\bar{p}$ is low
- his good signal realization is not that helpful anyway;
  - $\hat{p}$ is high
- my good signal realization doesn't mean his is likely to be good;
  - $\Pi(p_1; \sigma_2)$ is low for $p_1 \in (\mu, \bar{p})$.

$\Rightarrow$ **More informative disclosure to counter correlation**.

- go for $\{0, \bar{p}\}$.

The Effect of Correlation on Info Revelation

Exploit fellow sender's good news (calls for weak disclosure) or overcome his bad news (calls for strong disclosure)?

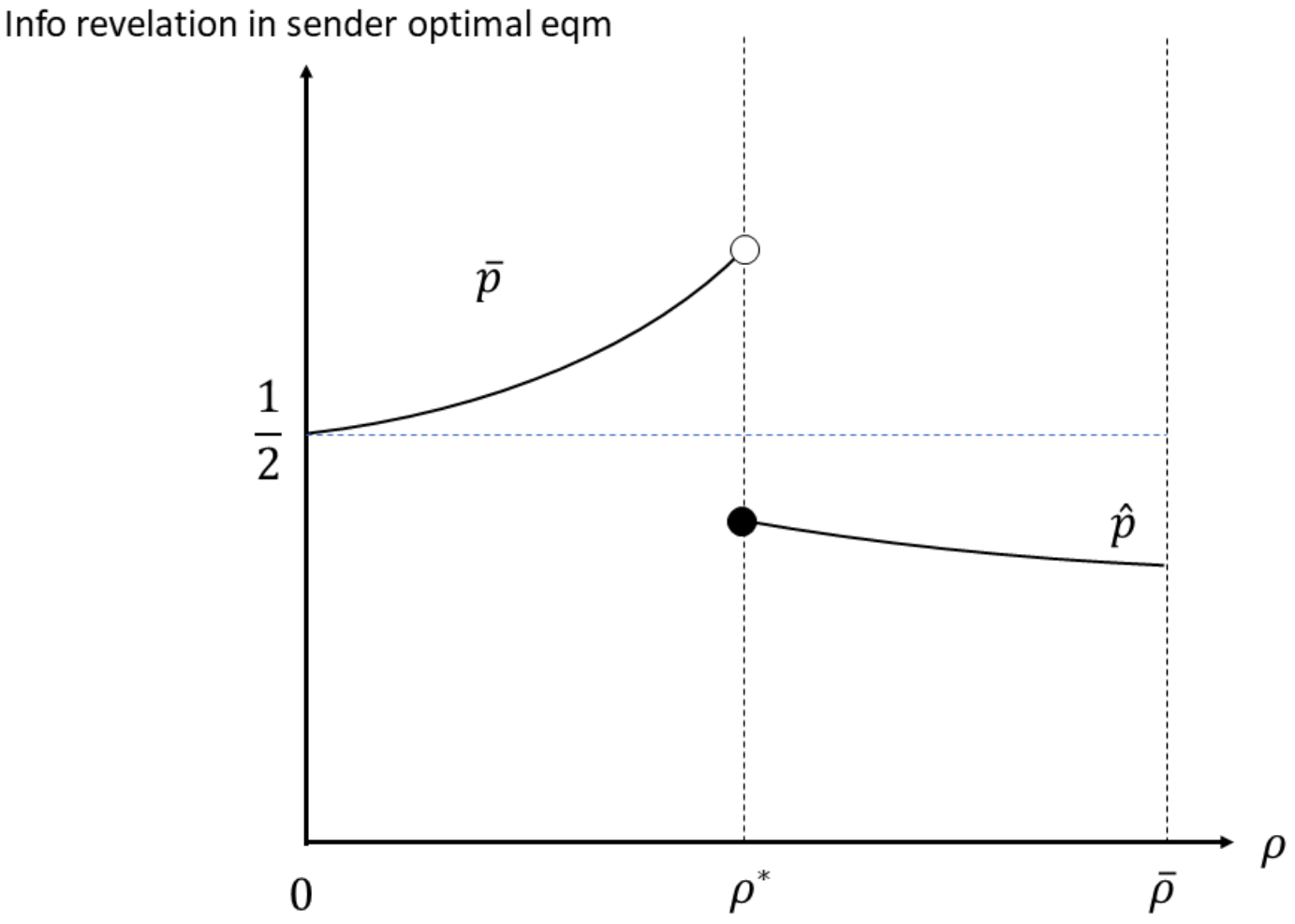
If the **correlation is high**,

- very costly to counter his bad realization.
  - $\bar{p}$ is high
- his good realization is very helpful;
  - $\hat{p}$ is low
- my good signal realization does imply his is likely to be good;
  - $\Pi(p_1; \sigma_2)$ is high for $p_1 \in (\mu, \bar{p})$.

$\Rightarrow$ **Less informative disclosure to exploit correlation**.

- go for $\{0, \hat{p}\}$.

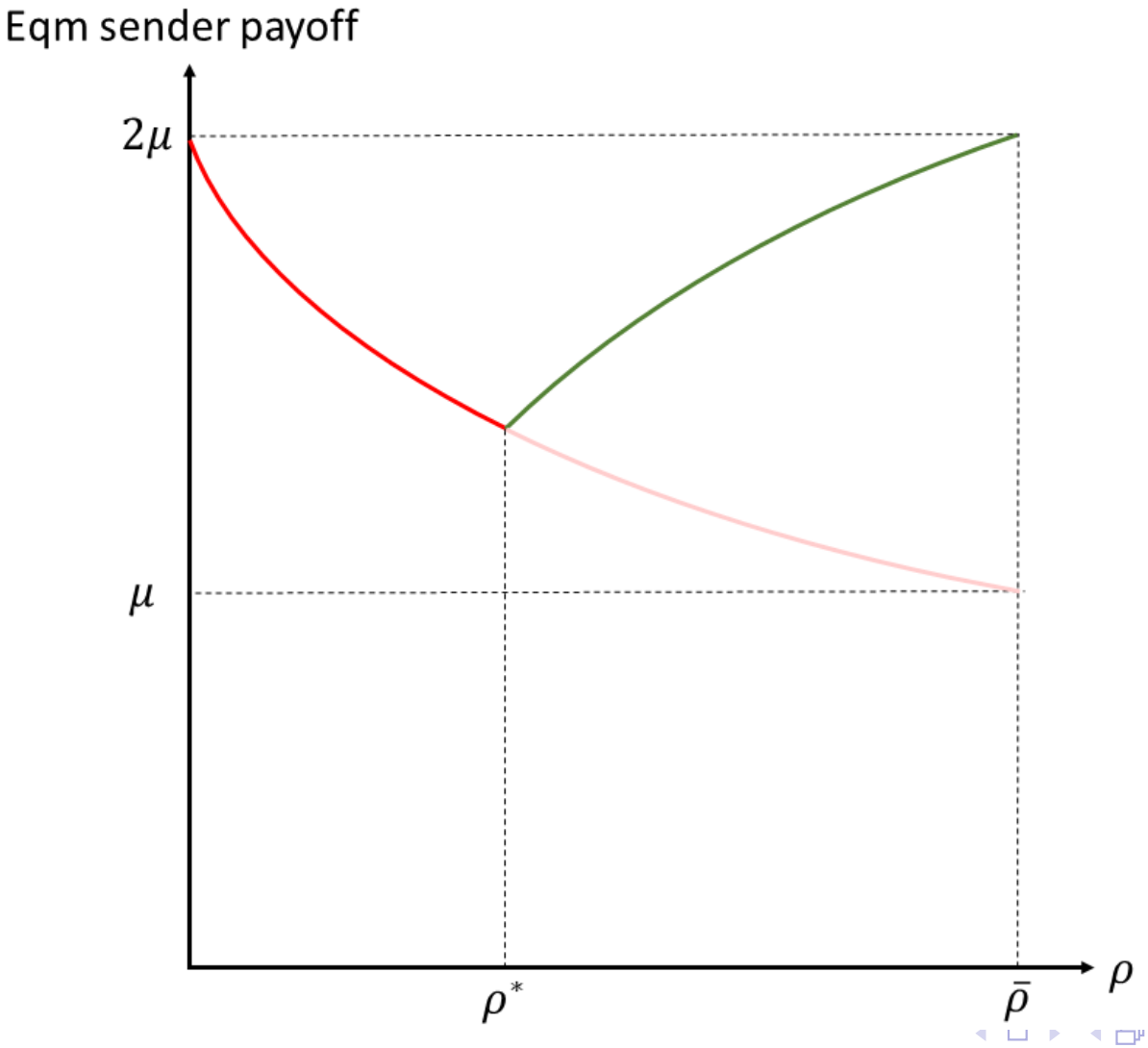
The Effect of Correlation on Info Revelation



The Effect of Correlation on Sender Payoff

- The overall effect of info spillover/leakage is a **negative externality** between the senders — a loss of control over the signal received by their target receiver.
- The eqm magnitude of negative externality is **non-monotone in the degree of correlation**.
- At $\rho < \rho^*$, senders counter correlation by more aggressive revelation, **exacerbating** the info leakage problem.
- At $\rho > \rho^*$, senders are able to coordinate with less informative revelation, **mitigating** the info leakage problem.

The Effect of Correlation on Sender Payoff

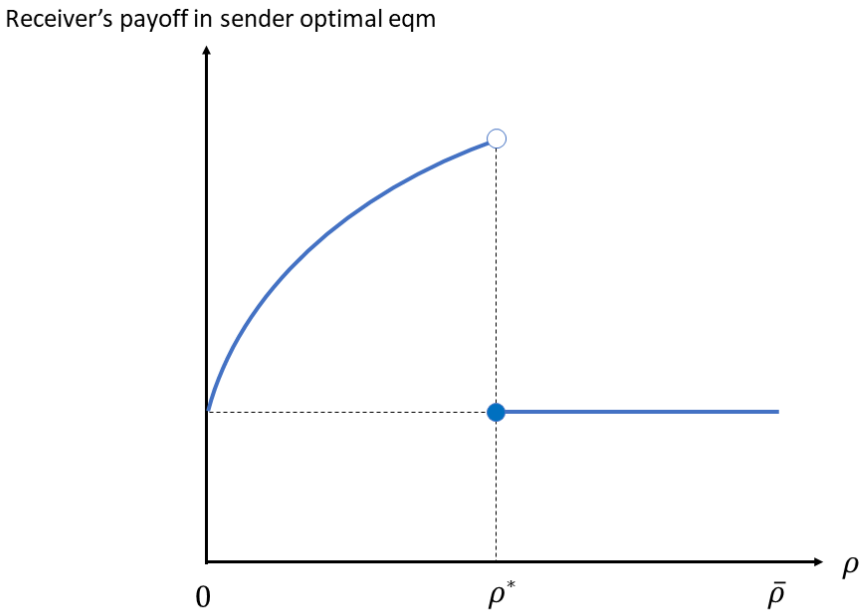


The Effect of Correlation on Receiver Payoff

Suppose receiver gets a positive payoff iff she makes the right ex-post decision.

Corollary

Relative to the independence benchmark, the receiver benefits from correlated persuasion iff $\rho < \rho^*$.



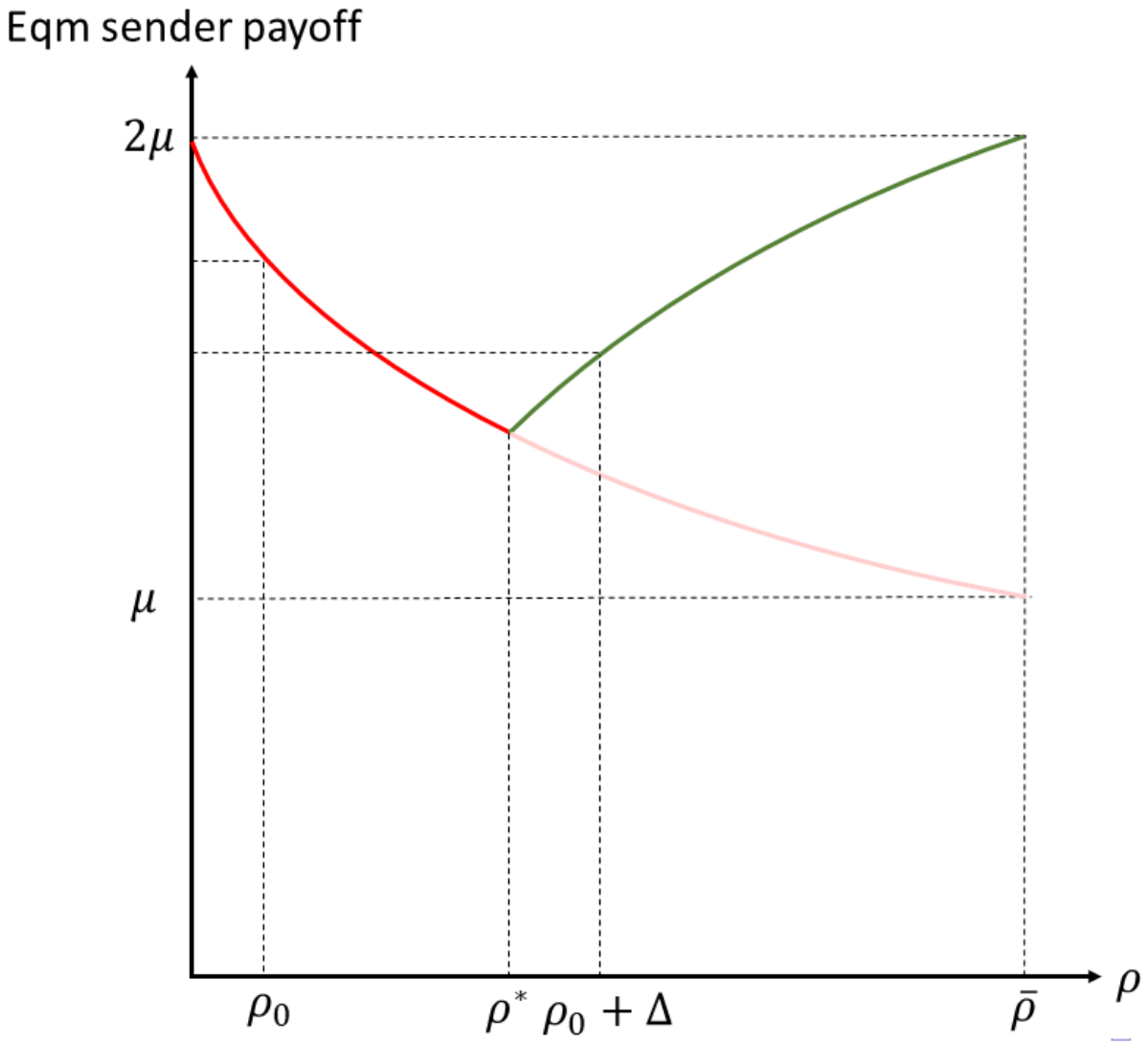
Implication for Proposal Design

- Will senders homogenize or differentiate proposal designs?
- Augment the game with an **initial stage of proposal design**.
- Sender 1 chooses between design A1 and B1, and sender 2 simultaneously chooses between design A2 and B2.
- Designs A1 and A2 are similar. Designs B1 and B2 are similar. Other combos are distinct.
- **Distinct designs**: correlation is ρ_0 (intrinsic correlation)
- **Similar designs**: correlation is $\rho_0 + \triangle$ (additional correlation due to design similarity).

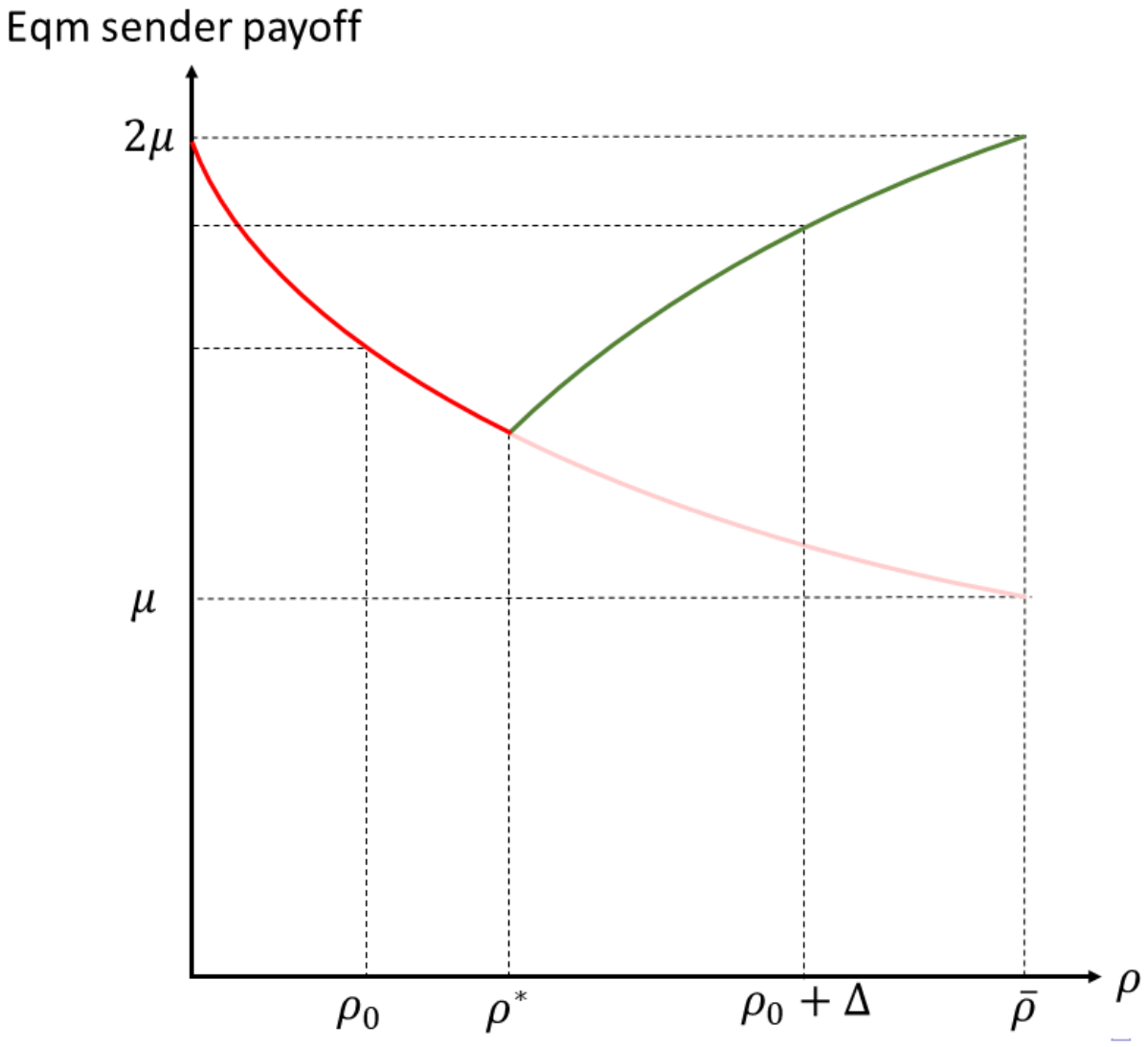
Corollary

Similar designs are adopted if the intrinsic correlation ρ_0 and/or the additional correlation $\triangle$ is sufficiently high.

Equilibrium Adoption of Distinct Designs



Equilibrium Adoption of Similar Designs



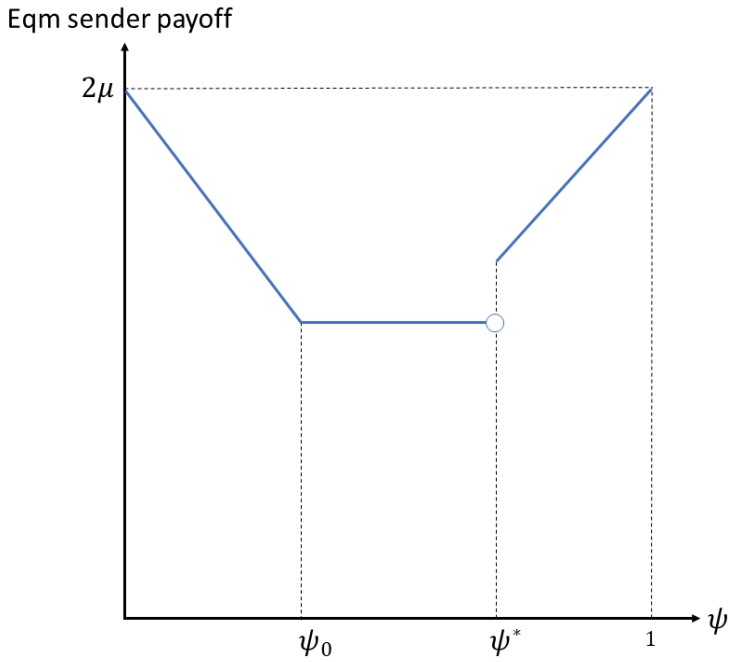
Implication for Transparency Design

- Will senders actively increase signal transparency to payoff-irrelevant receivers?
- Suppose receiver i can observe sender i's signal for sure, but can only see sender j's signal with probability ψ_i.
- If both ψ_1 and ψ_2 are very low, the sender may just focus on their own market. So let's focus on $\psi_1, \psi_2 \geq \psi_0$ — intrinsic transparency.
- Augment the game with an **initial stage of transparency design**.
- Simultaneously, sender 1 chooses $\psi_2 \in [\psi_0, 1]$ and sender 2 chooses $\psi_1 \in [\psi_0, 1]$, at a cost (of signal publicizing) that satisfies the standard properties.

Implication for Transparency Design

Lemma

With $\rho = \bar{\rho}$ and $(\psi_1, \psi_2) \in [\psi_0, 1]^2$,
(i) uncoordination is always a continuation equilibrium;
(ii) coordination is a continuation equilibrium iff
$(\psi_1, \psi_2) \in [\psi^*, 1]^2$, for some $\psi^* > \psi_0$.



Implication for Transparency Design

Proposition

Let $\rho = \bar{\rho}$. There exists a SPNE in which the senders choose (ψ^*, ψ^*) in the 1st stage and play the coordinated disclosure eqm in the 2nd stage, provided that $c(\psi^*)$ is sufficiently low.

- Senders attempt to coordinate on weak disclosure eqm.
- Sender 1: if I set $\psi_2 < \psi^*$, my signal is not influential enough on receiver 2.
  $\Rightarrow$ my promise of **weak disclosure is not credible**
  $\Rightarrow$ aggressive response by sender 2.
- If I set $\psi_2 = \psi^*$, my signal is influential enough on receiver 2.
  $\Rightarrow$ my promise of **weak disclosure is credible**
  $\Rightarrow$ friendly response by sender 2.
- **Fat-cat strategy**: strategic incentive to over-invest in publicizing signal to the payoff-irrelevant market.

Summary

- How would the correlation affect the persuasion strategies?
  - Low correlation $\Rightarrow$ more revealing
  - High correlation $\Rightarrow$ less revealing
- Does correlation benefit or hurt the senders?
  - Correlation hurts senders, but the effect is non-monotone
- What about receivers?
  - Benefit only if correlation is weak.
- Under the shadow of correlated persuasion, senders may find it in their own interest to
  (i) adopt product designs similar to others.
  (ii) publicize their signals to payoff-irrelevant receivers.