Correlated Persuasion

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Story

- **Governments (senders)** of two similar countries persuade their respective **citizens (receivers)** to take **covid vaccines**.
- The citizens can access the messages of both governments' public campaigns.
- The efficacies and side-effects are positively correlated.
- The decisions of citizens of country 1 are influenced by both governments' campaign messages. (Likewise for citizens of country 2)
  $\Rightarrow$ information spillover/ leakage

Another Story

- **Firms (senders)** selling similar products target different regional markets, say Hong Kong and Singapore.
- They persuade their own **customer base (receivers)** by designing their advertising + marketing campaigns.
- Their products' similarity suggests positive correlation in quality.
- HK customers' purchasing decisions are influenced by both firms' advertising campaigns. (Likewise for Singaporean customers.)
  $\Rightarrow$ information spillover/ leakage

Questions

- How would the correlation affect the **persuasion strategies**?
    - Compared to the independent benchmark, more informative or less informative?
- Does the correlation **benefit or hurt the senders**? What about the **receivers**?
- What is the implication for **product design**?
- How does **signal transparency** interact with correlated persuasion?
- What is the implication for **transparency design**?

Model

- Two ex-ante symmetric senders: Sender 1 and Sender 2
- Each sender i is endowed with a proposal with binary quality $U_i \in \{l, h\}$.
- Joint distribution

	$U_2 = l$	$U_2 = h$
$U_1 = l$	$(1-\mu)^2 + \rho$	$\mu(1-\mu) - \rho$
$U_1 = h$	$\mu(1-\mu) - \rho$	$\mu^2 + \rho$

- $\mu \in (0, 1/2)$: average quality.
- $\rho \in [0, \bar{\rho}]$: correlation parameter, where $\bar{\rho} = \mu(1-\mu)$.

Model

- Two receivers: Receiver 1 and Receiver 2
- **Receiver** i decides whether to adopt **sender** i**'s proposal or not.**
- His payoff depends **only** on U_i, but independent of U_j, $j \neq i$.
- For simplicity, suppose receiver i adopts iff sender i's proposal quality has a *posterior (that* $U_i = h$*)* **no less than** $1/2$.
- Sender i gets a positive payoff iff receiver i adopts her proposal.

Senders' Strategies

- Sender i persuades by costless design of signal (info structure) about U_i.
- She has no direct control over info revelation of U_j.
- Call the marginal posterior distribution over U_i based only on sender i's information **sender i's own signal realization** $p_i = \Pr(U_i = h | p_i)$.
- Wolog: a sender's strategy is a distribution over posteriors $F \in \triangle([0,1])$ such that its **mean equals the prior**: $\int_0^1 p_i dF(p_i) = \mu$.

Receivers' Strategies

- In the baseline model, both receivers have access to the signals and the realizations of **both senders**.
- Receiver i adopts sender i's proposal iff

$$\begin{aligned}
& \Pr\left(U_i = h | p_i, p_j\right) \\
= & \left(1 + \frac{\left((1-\mu)^2 + \rho\right) \frac{1-p_j}{p_j} \frac{\mu}{1-\mu} + \left(\mu\left(1-\mu\right) - \rho\right)}{\left(\mu\left(1-\mu\right) - \rho\right) \frac{1-p_j}{p_j} \frac{\mu}{1-\mu} + \left(\mu^2 + \rho\right)} \frac{1-p_i}{p_i} \frac{\mu}{1-\mu}\right)^{-1} \\
\geq & \frac{1}{2}.
\end{aligned}$$

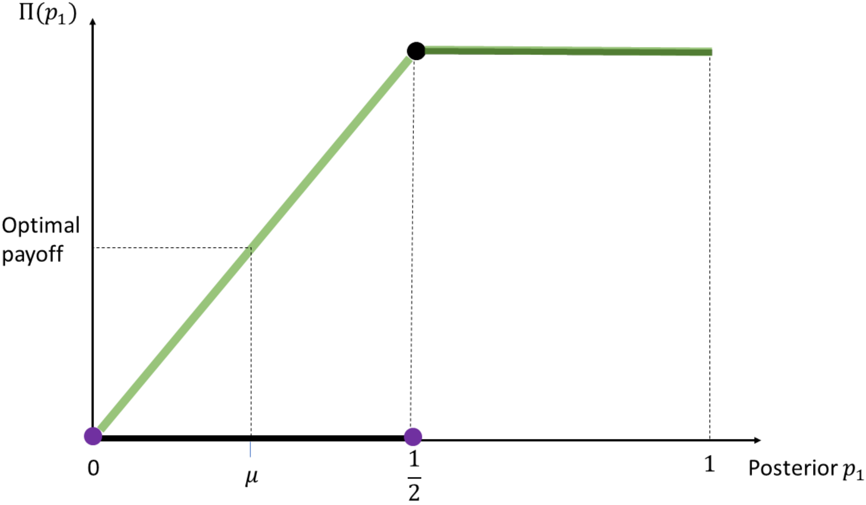
Timeline

1. The two senders simultaneously post their signals (info structures) about their respective U_i.
2. The two receivers observe the signal realizations of both senders.
3. Receiver i adopts sender i's proposal iff the combined posterior of U_i is no less than $1/2$.
4. The players collect their respective payoffs.

Equilibrium

- Focus on the **symmetric equilibria** between the senders' play.
- If there are multiple symmetric equilibria, pick the sender-optimal one.

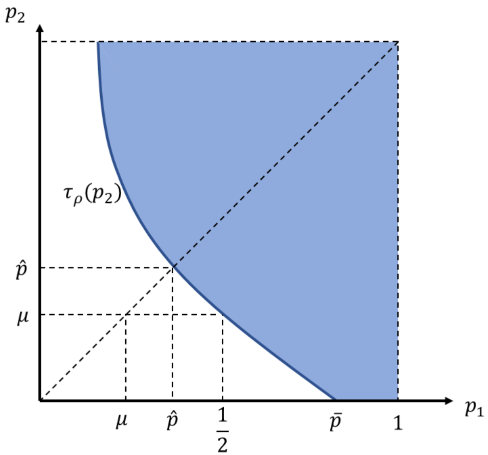
The Independence Benchmark



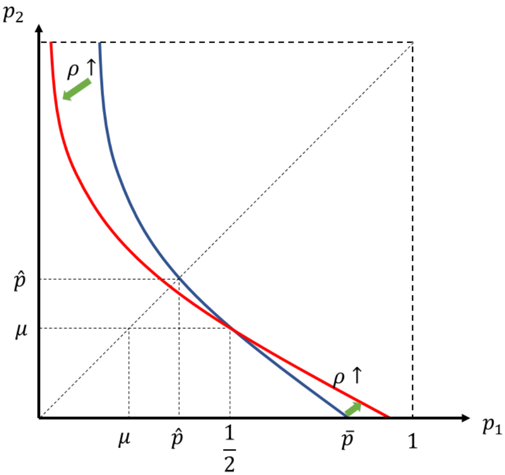
Persuasion under Correlation

Let $\rho \in (0, \mu(1-\mu)]$. Sender 1 succeeds in persuasion iff

$$\Pr(U_1 = h | p_1, p_2) \geq \frac{1}{2}.$$



Persuasion under Correlation: Increase in Correlation



Some Key Posteriors

- If sender 2 reveals nothing, then sender 1 can successfully persuade by realizing $p_1 = 1/2$:

$$\Pr\left(U_1 = h \middle| \frac{1}{2}, \mu\right) = \frac{1}{2}.$$

- By realizing posterior $\bar{p}$, a sender can guarantee success regardless of what happens at the fellow sender:

$$\Pr\left(U_1 = h | \bar{p}, 0\right) = \frac{1}{2}.$$

- The minimum common posterior that yields successful persuasion is denoted by $\hat{p}$:

$$\Pr\left(U_1 = h | \hat{p}, \hat{p}\right) = \frac{1}{2}.$$

- Intuitively,

$$0 < \mu < \hat{p} < \frac{1}{2} < \bar{p} < 1.$$

Payoff Function

- Denote the strategy of sender 2 by σ_2. The payoff function of sender 1 in his own posterior:

$$\Pi\left(p_1;\sigma_2\right)=\sum_{\{p_2\in supp\{\sigma_2\}:\Pr(U_1=h|p_1,p_2)\geq 1/2\}}\Pr\left(p_2|p_1,\sigma_2\right),$$

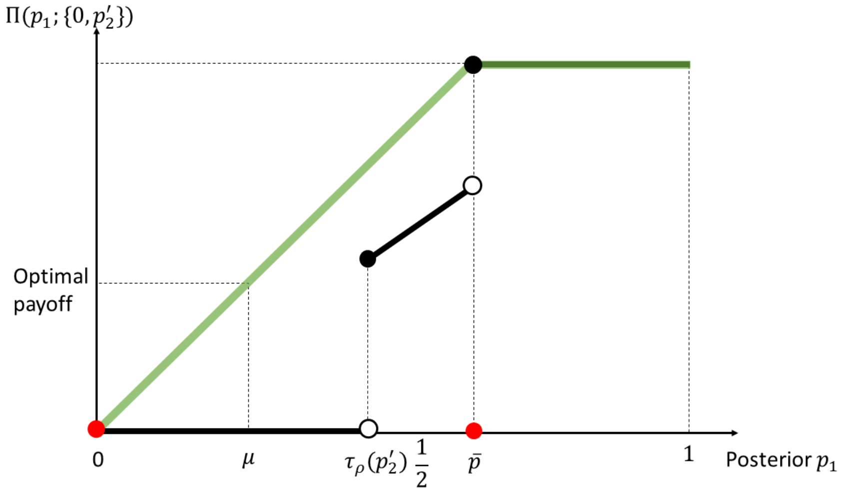
where

$$\Pr\left(p_2|p_1,\sigma_2\right)=\sigma_2\left(p_2\right)\left(1+\frac{\rho}{\mu^2\left(1-\mu\right)^2}\left(p_2-\mu\right)\left(p_1-\mu\right)\right).$$

- Fixing strategy σ_2, good news by sender 1 implies sender 2 is more likely to bring good news too.
- This effect is more salient if ρ is large.

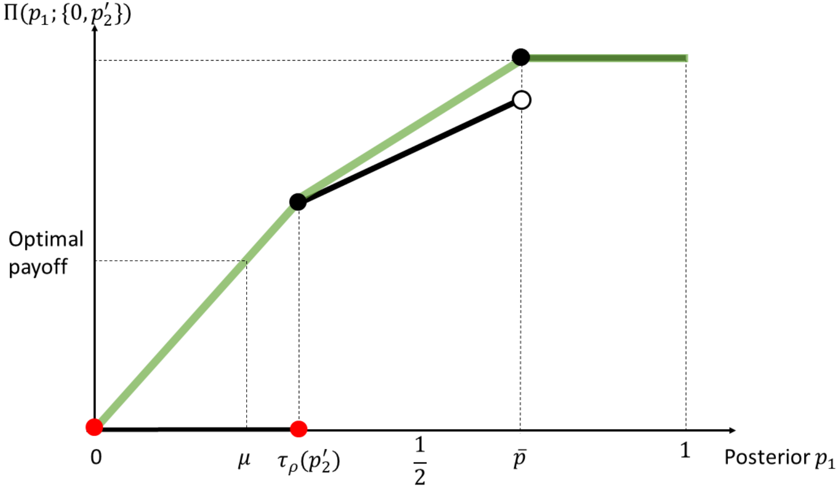
Payoff Function

Say σ_2 has support $\{0, p_2'\}$.



Payoff Function

With higher correlation:



Structure of Symmetric Equilibria

Lemma

Every symmetric equilibrium has the following properties.
(i) There is an atom at 0.
(ii) The support is either $\{0, \hat{p}\}$ *or it includes* $\bar{p}$ *as its maximum.*

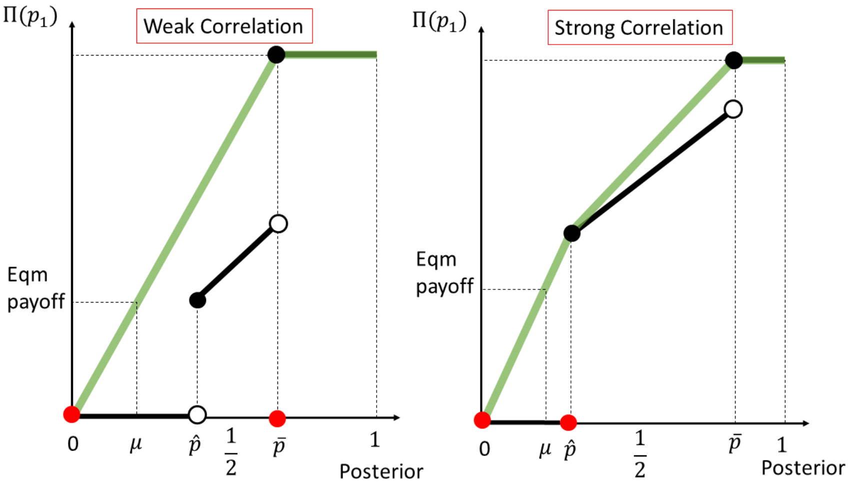
Equilibrium Existence

- **Coordinated** equilibrium has a support $\{0, \hat{p}\}$.
- **Uncoordinated** equilibrium has a support of the form: $\{0, ..., \bar{p}\}$.

Lemma

An uncoordinated equilibrium always exists.
A coordinated equilibrium exists if and only if $\rho \geq \rho^*$ for some $\rho^* \in (0, \bar{\rho})$.

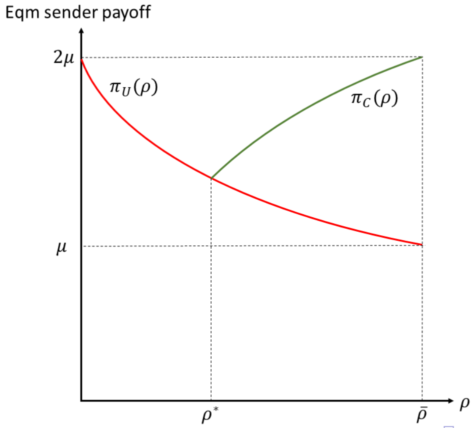
The Effect Of Correlation



Equilibrium Payoff

Payoffs of uncoordinated and coordinated equilibrium are

$$\pi_U(\rho) = \frac{1}{\bar{p}} \text{ and } \pi_C(\rho) = \frac{\Pi(\hat{p}; \{0, \hat{p}\})}{\hat{p}}.$$



Sender-Optimal Symmetric Equilibrium

Proposition

If $\rho < \rho^*$, the sender-optimal symmetric equilibrium is uncoordinated supported on $\{0, \bar{p}\}$.
If $\rho \geq \rho^*$, the sender-optimal symmetric equilibrium is coordinated supported on $\{0, \hat{p}\}$.

The Effect of Correlation on Info Revelation

Exploit fellow sender's good news (calls for weak disclosure) or overcome his bad news (calls for strong disclosure)?

If the **correlation is low**,

- my good signal realization $\nRightarrow$ his is likely to be good;
- his good signal realization is not that helpful anyway;
- not too costly to counter his bad realization.

$\Rightarrow$ **More informative disclosure to counter correlation**.

If the **correlation is high**,

- my good signal realization $\Rightarrow$ his is likely to be good;
- his good realization is very helpful;
- very costly to counter his bad realization.

$\Rightarrow$ **Less informative disclosure to exploit correlation**.

The Effect of Correlation on Sender Payoff

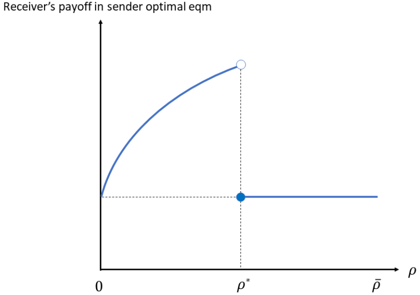
- The overall effect of info spillover/leakage is a **negative externality** — a loss of control over the signal received by target receiver.
- The eqm magnitude of negative externality is **non-monotone in the degree of correlation**.
- At $\rho < \rho^*$, senders counter correlation by more aggressive revelation, exacerbating the info leakage problem.
- At $\rho > \rho^*$, senders are able to coordinate with less informative revelation, mitigating the info leakage problem.

The Effect of Correlation on Receiver Payoff

Suppose the receiver gets a positive payoff iff she makes the right ex-post decision.

Corollary

Relative to the independence benchmark, the receiver benefits from correlated persuasion iff $\rho \in (0, \rho^*)$.



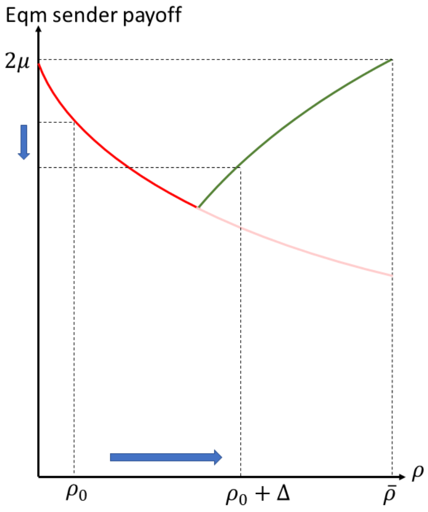
Implication for Proposal Designs

- Will senders homogenize or differentiate proposal designs?
- Augment the baseline game with an initial stage of proposal design by senders.
- Distinct designs: correlation is ρ_0 (intrinsic correlation)
- Similar designs: correlation is $\rho_0 + \triangle$ (additional correlation due to design similarity).

Corollary

Similar designs are adopted if the intrinsic correlation ρ_0 and/or the additional correlation $\triangle$ is sufficiently high.

Equilibrium Adoption of Distinct vs Similar Designs



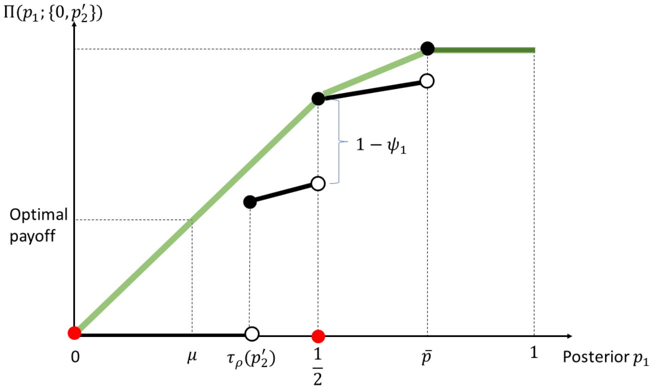
The Effect of Signal Transparency

Modify the model as follows:

- Receiver i always has access to the signal and its realization of his matched sender i.
- Receiver i has access to those of unmatched sender j with independent probability $\psi \in (0, 1]$.
- In case the other sender's signal fails to reach him, it is without loss that suppose receiver i observes $p_j = \mu$.

The Effect of Signal Transparency

$$\Pi\left(p_1;\sigma_2\right) = (1-\psi)\,\mathbb{1}_{\left[p_1\geq\frac{1}{2}\right]} + \psi \sum_{\{p_2\in supp\{\sigma_2\}:\Pr(U_1=h|p_1,p_2)\geq 1/2\}} \Pr\left(p_2|p_1,\sigma_2\right)$$



The Effect of Signal Transparency: Equilibrium

- **Coordinated** equilibrium has a support $\{0, \hat{p}\}$.
- Uncoordinated equilibrium has a support of the form:

$$\left\{0, ..., \frac{1}{2}\right\} \text{ or } \{0, ..., \bar{p}\}.$$

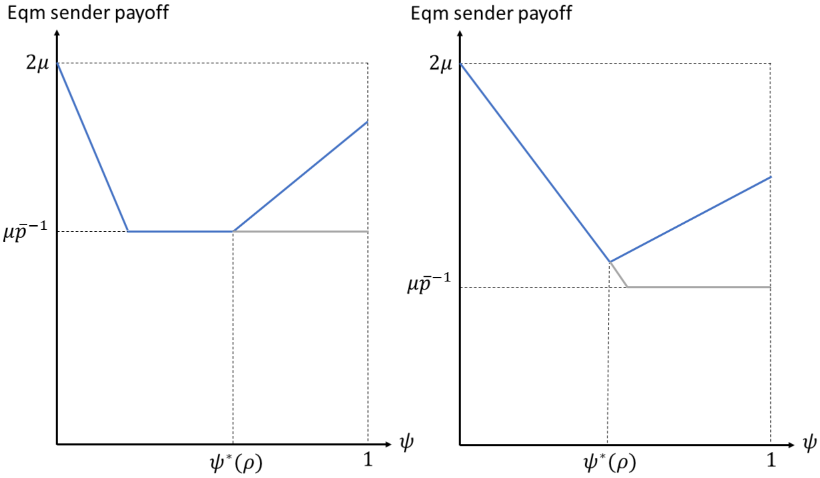
- The former, when exists, gives a higher sender payoff.

Proposition

An uncoordinated equilibrium always exists.
A coordinated equilibrium exists if $\rho \geq \rho^*$ and $\psi \geq \psi^*(\rho)$ for some cutoff function $\psi^*(\cdot)$.

The Effect of Signal Transparency on Senders' Payoffs

At high correlation $\rho > \rho^*$, senders' payoff is non-monotone in transparency.



The Effect of Signal Transparency on Receivers' Payoffs

Corollary

Focus on the sender-optimal equilibrium.
Compared with complete opaqueness $\psi = 0$*, the receiver strictly benefits from partial transparency only iff* $\psi \in (0, \psi^* (\rho))$*.*
Consequently, the receiver's payoff can go up with a decrease in the level of transparency ψ *from above* $\psi^* (\rho)$ *to below.*

Implication for Transparency Designs

- Extension 1: Sender i can inform her **own receiver** about sender j's signal.
- Sender i chooses between
  informing: $\psi_i = \underline{\psi} + \triangle$ vs
  not informing: $\psi_i = \underline{\psi}$.

Corollary

If ρ, $\underline{\psi}$ and $\triangle$ are sufficiently high, it is a symmetric equilibruim for both senders to inform her own receiver and play the coordinated dislcosure $\{0, \hat{p}\}$.

Implication for Transparency Designs

- Extension 2: Sender i can inform the **unpaired receiver** about her own signal **at a personal cost**
- Sender i chooses between
  informing: $\psi_j = \underline{\psi} + \triangle$ (at cost k) vs
  not informing: $\psi_j = \underline{\psi}$ (at cost 0).

Corollary

If ρ, $\underline{\psi}$ and $\triangle$ are sufficiently high and k is sufficiently low, it is a subgame perfect equilibrium for both senders to pay a cost to inform the unpaired receiver and play the coordinated dislcosure $\{0, \hat{p}\}$.

- Information sharing as a strategic commitment to induce a weak disclosure by both senders in the persuasion stage.

Summary

How would the correlation affect the persuasion strategies?

- Low correlation $\Rightarrow$ more revealing
- High correlation $\Rightarrow$ less revealing

Does correlation benefit or hurt the senders?

- Correlation hurts senders, but the effect is non-monotone

What about the receivers?

- Benefit only if correlation is weak and/or transparency is not too high

Summary

What is the implication for product design?

- Senders may find it in the common interest to adopt similar product designs.

How does signal transparency interact with correlated persuasion?

- For senders, transparency and correlation can be complementary.
- For receivers, they may benefit from a reduction in signal transparency when correlation is high.

What is the implication for transparency choice?

- Senders may find it in the own interest to increase the signal transparency, including publicizing signals to payoff-irrelevant receivers.