Correlated Persuasion

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Stories

- Governments (senders) of two neighboring countries persuade their respective citizens (receivers) to take covid vaccines.
- The citizens can access the messages of both governments' public campaigns.
- The efficacies and side-effects are positively correlated.
- The decisions of citizens of country 1 are influenced by both governments' campaign messages. (Likewise for citizens of country 2)
  $\Rightarrow$ information spillover/ leakage

Stories

- Firms (senders) selling similar products target different regional markets, say Hong Kong and Singapore.
- They persuade their own customer base (receivers) by designing their advertising + marketing campaigns.
- Their products' similarity suggests positive correlation in quality.
- HK Customers' purchasing decisions are influenced by both firms' advertising campaigns. (Likewise for Singaporean customers.)
  $\Rightarrow$ information spillover/ leakage

Questions

- How would the correlation affect the **persuasion strategies**?
  - Compared to the benchmark independent case, more informative or less informative?
- Does the correlation **benefit or hurt the senders**? What about **receivers**?
  - The equilibrium level of **information revelation**
- What are the implications for *product design* and *transparency*?

Model

- Two (ex-ante symmetric) senders: Sender 1 and Sender 2
- Each sender i is endowed with a proposal with binary quality $U_i \in \{l, h\}$ with $h > l$.
- Joint distribution

	$U_2 = l$	$U_2 = h$
$U_1 = l$	$(1-\mu)^2 + \rho$	$\mu(1-\mu) - \rho$
$U_1 = h$	$\mu(1-\mu) - \rho$	$\mu^2 + \rho$

- $\mu \in (0, 1/2)$: average quality.
- $\rho \in [0, \bar{\rho}]$: correlation parameter, where $\bar{\rho} = \mu(1-\mu)$.

Model

- Two receivers: Receiver 1 and Receiver 2
- **Receiver** i decides whether to adopt **the proposal of sender** i.
- His payoff depends only on U_i, but independent of U_j, $j \neq i$.
- For simplicity, suppose receiver i adopts iff sender i's proposal quality has a *posterior (that* $U_i = h$*)* **no less than** $1/2$.
- Sender i gets a positive payoff iff Receiver i adopts her proposal.

Strategies

- Sender i persuades by costless design of signals (info structure) about U_i.
  - It has no direct control over info revelation of U_j.
- The marginal distribution over U_i **conditional only on sender i's own signal realization** m_i is generically denoted by **posterior** $p_i = \Pr(U_i = h | m_i)$.
- Wolog: denote her strategy as a distribution over posteriors $F \in \triangle([0,1])$ such that its **mean equals the prior**: $\int_0^1 p_i dF(p_i) = \mu$.
- Both receivers have access to the signal realizations/posteriors of **both senders**.
- Receiver i adopts Sender i's proposal iff

$$\Pr(U_i = h | p_i, p_j) \geq \frac{1}{2}.$$

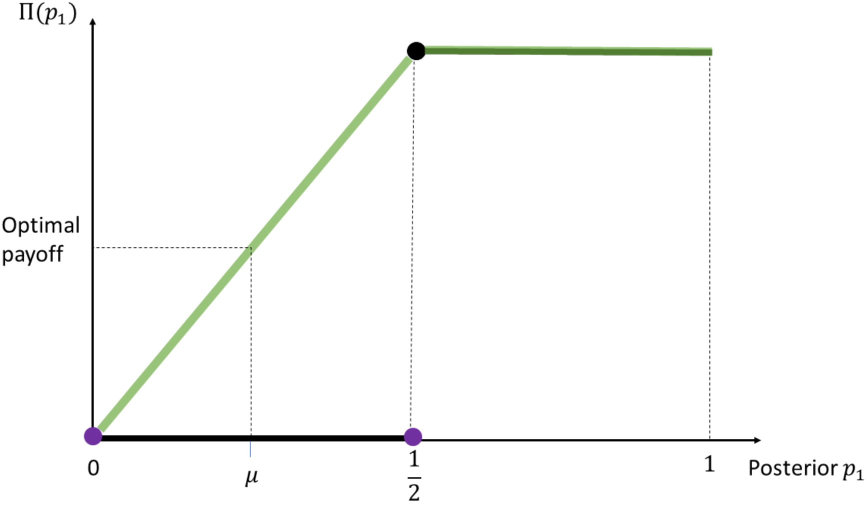
Timeline

1. Sender 1 and 2 simultaneously post their signals / info structure about their respective U_i.
2. Receiver 1 and Receiver 2 observe the signal realizations by both senders.
3. Receiver 1 adopts Sender 1's proposal iff the combined posterior of U_1 is no less than $1/2$.
   Receiver 2 adopts Sender 2's proposal iff the combined posterior of U_2 is no less than $1/2$.
4. The players collect their respective payoffs.

Equilibrium

- Focus on the **symmetric equilibria** between the senders' play.
- If the symmetric equilibria can be Pareto ranked, the Pareto optimal one is selected.

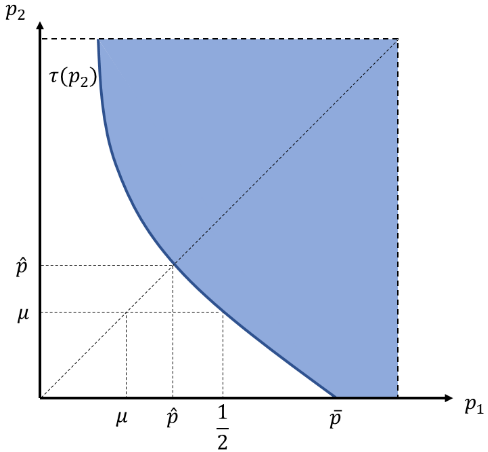
The Independence Benchmark



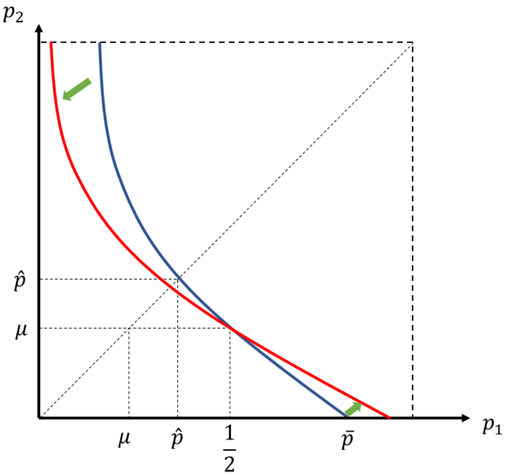
Persuasion under Correlation

Let $\rho \in (0, \mu(1-\mu)]$. Sender i is successful in persuasion iff

$$\Pr(U_i = h | p_i, p_j) \geq \frac{1}{2}.$$



Persuasion under Correlation: Increase in Correlation



Some Key Posteriors

- If the other sender reveals nothing, $p_2 = \mu$, then Sender 1 can successfully persuade by realizing $p_1 = \mu$:

$$\Pr\left(U_1 = h | \frac{1}{2}, \mu\right) = \frac{1}{2}.$$

- By realizing posterior $\bar{p}$, a sender can guarantee success regardless of what happens at the fellow sender:

$$\Pr\left(U_1 = h | \bar{p}, 0\right) = \frac{1}{2}.$$

- The minimum common posterior that yields successful persuasion is denoted by $\hat{p}$:

$$\Pr\left(U_1 = h | \hat{p}, \hat{p}\right) = \frac{1}{2}.$$

- Intuitively,

$$0 < \mu < \hat{p} < \frac{1}{2} < \bar{p} < 1.$$

Payoff Function

- KG11: The best signal can be identified by constructing the concavification of payoff function in own posterior.
- Denote the strategy of Sender 2 by σ_2. Payoff function of Sender 1 takes the form:

$$\Pi\left(p_1;\sigma_2\right)=\sum_{\{p_2\in supp\{\sigma_2\}:\Pr(U_i=h|p_1,p_2)\geq 1/2\}}\Pr\left(p_2|p_1,\sigma_2\right),$$

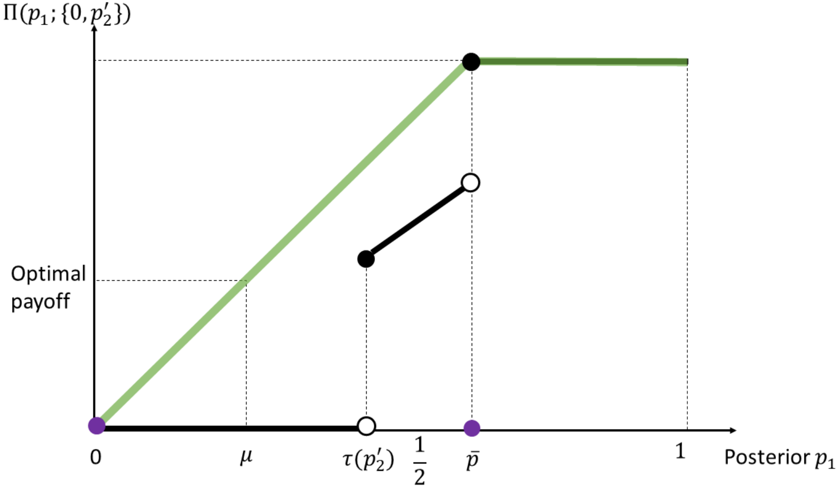
where

$$\Pr\left(p_2|p_1,\sigma_2\right)=\sigma_2\left(p_2\right)\left(1+\frac{\rho}{\mu^2\left(1-\mu\right)^2}\left(p_2-\mu\right)\left(p_1-\mu\right)\right).$$

- Fixing strategy σ_2, good news by Sender 1 implies Sender 2 is more likely to bring good news too.
- This effect is more salient if ρ is large.

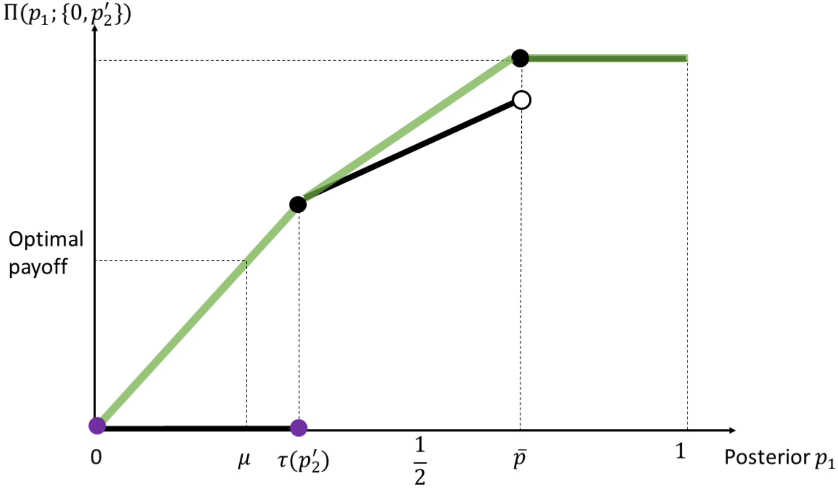
Payoff Function

Say σ_2 has support $\{0, p_2'\}$. Sender 1's payoff function:



Payoff Function

With higher correlation:



Structure of Symmetric Equilibria

Lemma

There are only two possible classes of symmetric equilibria. The first class has a support $\{0, \hat{p}\}$*. The second class has a support containing* $\{0, \bar{p}\}$ *(and possibly more).*

- Coordinated equilibrium: $\{0, \hat{p}\}$
- Uncoordinated equilibrium: $\{0, \bar{p}\}$ (and possibly more)

Structure of Symmetric Equilibria

(i) 0 must be on the support.

- Denote by $p_{\inf}$ the inf of the support and by $p_{\sup}$ the sup of the support.
- If $p_{\inf} > 0$, it yields persuasion with positive prob,
  $\Rightarrow \Pr\left(U_1 = h \middle| p_{\inf}, p_{\sup}\right) \geq 1/2$
  $\Rightarrow \Pr\left(U_1 = h \middle| p_{\sup}, p_{\inf}\right) > 1/2$
  $\Rightarrow \Pr\left(U_1 = h \middle| p_{\sup} - \delta, p_{\inf}\right) \geq 1/2$ for some $\delta > 0$
  $p_{\sup}$ is an overkill.

Structure of Symmetric Equilibria

(ii) If $\bar{p}$ is not on the support, the support must be $\{0, \hat{p}\}$.

- Suppose $p_{\text{sup}} \in (\hat{p}, \bar{p})$.
- Let p' be the smallest non-zero posterior on the support.
  $\Rightarrow \Pr(U_1 = h | p', p_{\text{sup}}) \geq 1/2$
  $\Rightarrow \Pr(U_1 = h | p_{\text{sup}}, p') > 1/2$
  $\Rightarrow \Pr(U_1 = h | p_{\text{sup}} - \delta, p') \geq 1/2$ for some $\delta > 0$
  $\Rightarrow p_{\text{sup}}$ is again an overkill.

Equilibrium Existence

Lemma

An uncoordinated equilibrium always exists.
A coordinated equilibrium exists if and only if $\rho \geq \rho^*$ *for some* $\rho^* \in (0, \bar{\rho})$.

The Effect Of Correlation

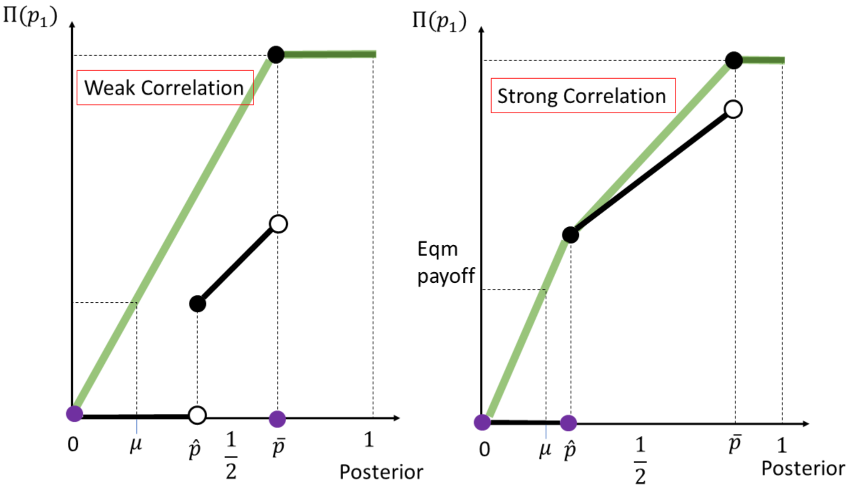
Why is high correlation needed to support coordinated eqm?

$$\frac{\Pi\left(\hat{p};\{0,\hat{p}\}\right)}{\hat{p}} \geq \frac{1}{\bar{p}}.$$

When correlation ρ goes up,....

- ...Sender 2's bad news hurts more.
  $\Rightarrow \bar{p}$ goes up.
- ... Sender 2's good news helps more.
  $\Rightarrow \tau(\cdot)$ goes down for all $p_2 > \mu \Rightarrow \hat{p}$ goes down.
- ...Conditional on Sender 1's good news, Sender 2 is able to bring good news with a higher chance.
  $\Rightarrow \Pi(p_1;\sigma)$ goes up for $p_1 > \mu$.

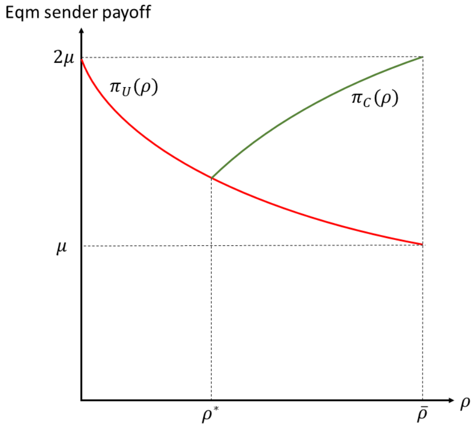
The Effect Of Correlation



Equilibrium Payoff

Payoffs of uncoordinated and coordinated equilibrium are

$$\pi_U(\rho) = \frac{1}{\bar{p}} \text{ and } \pi_C(\rho) = \frac{\Pi(\hat{p}; \{0, \hat{p}\})}{\hat{p}}.$$

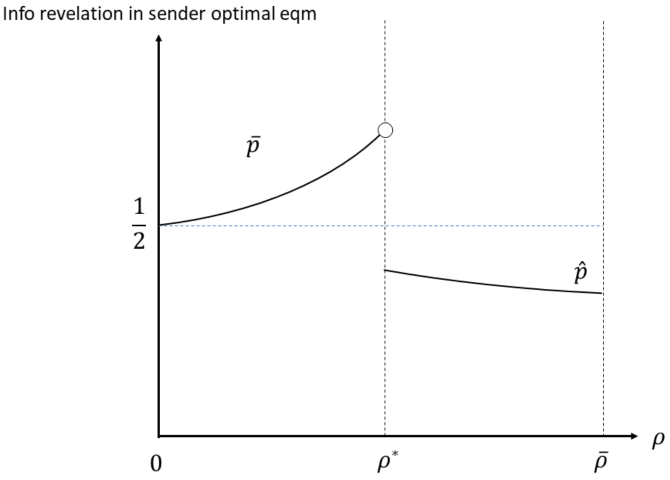


Optimal Symmetric Equilibrium

Proposition

If $\rho < \rho^*$, the optimal symmetric equilibrium is uncoordinated supported on $\{0, \bar{p}\}$.
If $\rho \geq \rho^*$, the optimal symmetric equilibrium is coordinated supported on $\{0, \hat{p}\}$.

The Effect of Correlation on Info Revelation



The Effect of Correlation on Info Revelation

- Exploit fellow sender's good news (calls for weak disclosure) or overcome his bad news (calls for strong disclosure)?
- If the **correlation is low**,
  - my good signal realization does not imply his is likely to be good;
  - his good signal realization is not that helpful anyway;
  - not too costly to counter his bad realization.

$\Rightarrow$ **More informative disclosure to counter correlation**.

The Effect of Correlation on Info Revelation

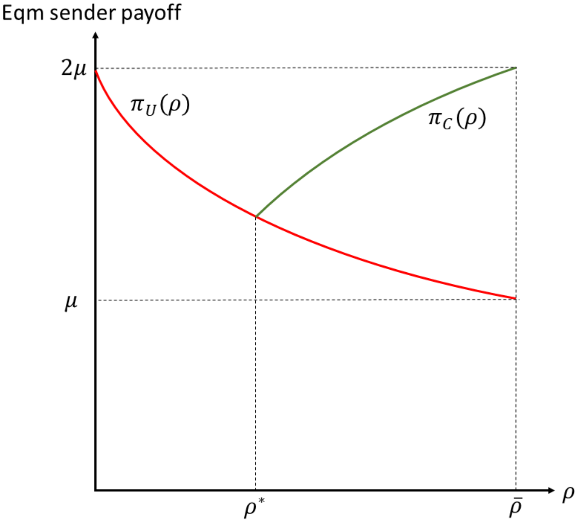
- Exploit fellow sender's good news (calls for weak disclosure) or overcome his bad news (calls for strong disclosure)?
- If the **correlation is high**,
  - my good signal realization does imply his is likely to be good;
  - his good realization is very helpful;
  - very costly to counter his bad realization.

$\Rightarrow$ **Less informative disclosure to exploit correlation**.

The Effect of Correlation on Sender Payoff

- The overall effect of info spillover/leakage is a **negative externality** – a loss of control over the signal received by target receiver.
- The eqm magnitude of negative externality is **non-monotone in the degree of correlation**.
- At $\rho < \rho^*$, senders counter correlation by more aggressive revelation, exacerbating the info leakage problem.
- At $\rho > \rho^*$, senders are able to coordinate with less informative revelation, mitigating the info leakage problem.

The Effect of Correlation on Sender Payoff

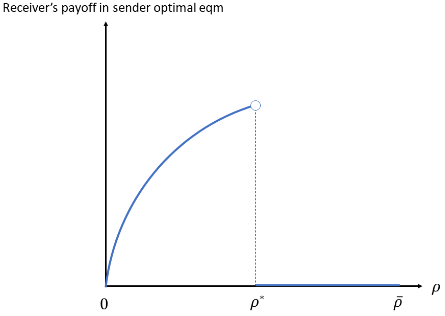


The Effect of Correlation on Receiver Payoff

Suppose receiver gets payoff iff she adopts a worthy proposal.

Corollary

Relative to the independence benchmark, the receiver benefits from correlated persuasion iff $\rho < \rho^*$.



Implication for Proposal Designs

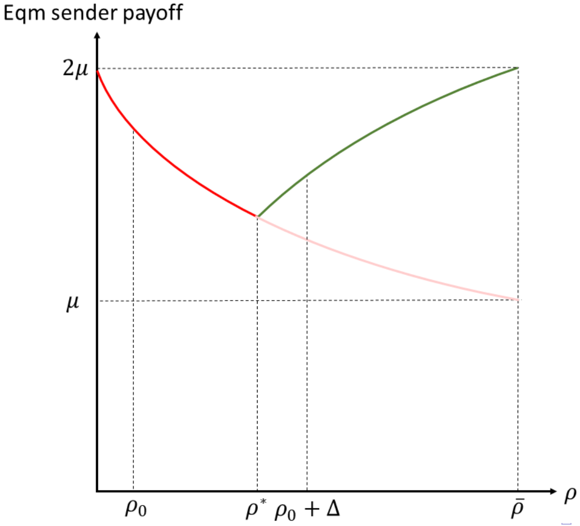
- Will senders homogenize or differentiate proposal designs?
- Augment the baseline game with an initial stage of proposal design.
- Sender 1 chooses between design A1 and B1, whereas sender 2 simultaneously chooses between design A2 and B2.
- Designs A1 and A2 are similar. Designs B1 and B2 are similar. Other combos are distinct.
- Distinct designs: correlation is ρ_0 (intrinsic correlation)
- Similar designs: correlation is $\rho_0 + \triangle$ (additional correlation due to design similarity).

Implication for Proposal Designs

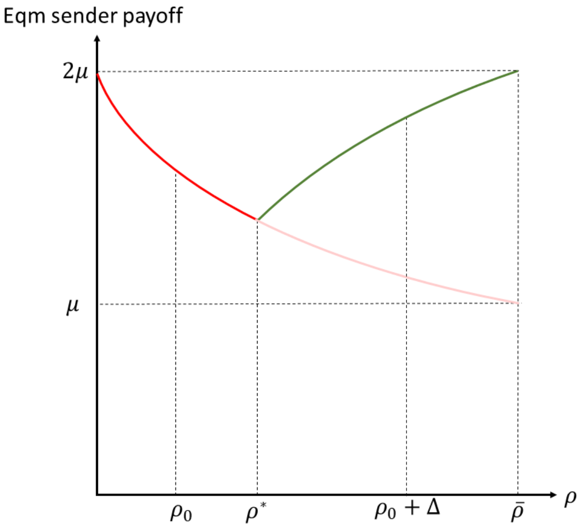
Corollary

Similar designs are adopted if the intrinsic correlation ρ_0 and/or the additional correlation $\triangle$ is sufficiently high.

Equilibrium Adoption of Distinct Designs



Equilibrium Adoption of Similar Designs



Implication for Transparency Design

- Will senders actively increase signal transparency to payoff-irrelevant receivers?
- Modify the baseline model by assuming that
  Receiver 1 observes Sender 2's signal realization with probability $\psi < 1$;
  Receiver 2 observes Sender 1's signal realization with probability $\psi < 1$.

Implication for Transparency Design

Lemma

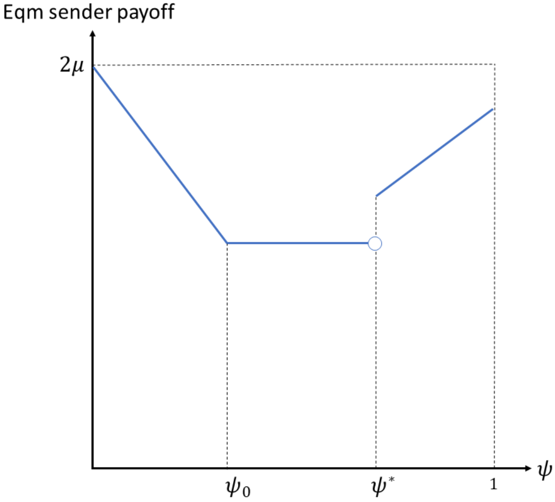
With partial observability, there exists a symmetric equilibrium supported on $\{0, 1/2\}$ if $\psi \leq \psi_0$, and one supported on $\{0, \mu, 1/2\}$ if $\psi \geq \psi_0$.

- We term it the partially coordinated equilibrium.

Proposition

Let $\rho > \rho^*$. There exists a cutoff $\psi^* \in (\psi_0, 1)$ such that the optimal symmetric equilibrium is
(i) the partially coordinated equilibrium if $\psi < \psi^*$
(ii) the coordinated equilibrium if $\psi \geq \psi^*$.

Implication for Transparency Design



Implication for Transparency Design

- Augment the game with an initial stage of transparency design.
- Sender 1 chooses $\psi_2 \in [\psi_0, 1]$, the probability that Receiver 2 can observe her signal realization.
- Sender 2 chooses $\psi_1 \in [\psi_0, 1]$, the probability that Receiver 1 can observe her signal realization.
- The cost of publicizing own signal realizations $c(\psi)$ satisfies the standard properties.

Implication for Transparency Designs

Proposition

Let $\rho > \rho^*$. There exists a SPNE in which the senders choose (ψ^*, ψ^*) in the 1st stage and play the coordinated disclosure eqm in the 2nd stage, provided that $c(\psi^*)$ is sufficiently low.

- In the SPNE, deviation from ψ^* is "penalized" by the partially coordinated eqm in the continuation play.

Implication for Transparency Designs

- Senders attempt to coordinate on weak disclosure eqm.
- Sender 1: if I choose $\psi_2 < \psi^*$, my own signal is not influential enough on Receiver 2, making my promise of weak disclosure non-credible
  $\Rightarrow$ aggressive response by Sender 2.
  - With low ψ_2, Sender 2 does not find it worthwhile to ride on my good signal realization $\Rightarrow$ she responds by aggressive disclosure $\Rightarrow$ coaxes me into aggressive disclosure.
- If I choose $\psi_2 = \psi^*$, my own signal is influential enough on Receiver 2, making my promise of weak disclosure credible
  $\Rightarrow$ friendly response by Sender 2.

Summary

- How would the correlation affect the persuasion strategies?
  - Low correlation $\Rightarrow$ more revealing
  - High correlation $\Rightarrow$ less revealing
- Does correlation benefit or hurt the senders?
  - Correlation hurts senders, but the effect is non-monotone
- What about receivers?
  - Benefit only if correlation is weak.
- Senders may find it in the common interests to adopt similar product designs.
- Senders may find it in the own interest to publicize their signals to payoff-irrelevant receivers.