Objective and Subjective Indicators in Interim Performance Evaluations

Pak Hung Au* and Bin R. Chen†

November 2, 2016

Abstract

We study interim performance evaluations in a moral hazard setting with limited liability and two stages of production. At the end of the first stage, both an objective signal (publicly observed) and a subjective signal (privately observed by the principal) realize. We show that in the optimal contract, the two signals are used interactively with two notable features. First, whereas the objective signal is always used, the subjective signal is used only if the realized value of objective signal is below a cutoff. Second, the solicited effort may exhibit an upward distortion (relative to the first best) for an intermediate region of the objective signal realizations.

1 Introduction

In most organizations, performance evaluations are conducted in regular time intervals. According to Cleveland, Murphy, and Williams (1989), the purposes of performance evaluations include: (i) providing performance feedback, (ii) designing compensation packages, (iii) allowing workers to fine-tune or target their future efforts. Moreover, it is typical that workers are evaluated based on both objective performance and subjective judgement by their supervisors. For instance, the midterm review of a tenure-track assistant professor is usually based on both objective measures such as the publication record, and subjective measures such as the perceived quality and impact of conducted research. In this paper, we study optimal contracting in a dynamic moral hazard setting in which the interim performance evaluation

*Nanyang Technological University. Email: phau@ntu.edu.sg

†(Corresponding author) Sun Yat-sen University. Email: xiechenbin@gmail.com.

(IPE), based on both objective and subjective performance measures, is possible. Our analysis of the optimal contract illustrates the interaction of the three purposes mentioned above.

In our model, a principal hires an agent to undertake a project that entails two stages of production. In the first stage, the agent chooses an unobservable effort. The chosen effort stochastically determines the realization of two signals at the end of the first stage, which include an objective signal that is publicly observable and verifiable, as well as a subjective signal that is privately observed by the principal and non-verifiable. The principal can send the agent a message on the subjective signal, but she may not tell the truth. After observing the objective signal and possibly learning the principal's message about the subjective signal, the agent chooses another unobservable effort in the second stage. The second-stage effort, together with the interim signals, stochastically determines a verifiable final outcome, which is either a success or a failure. We assume that a higher value of objective signal or subjective signal, which is more indicative of effort exerted in the first stage, leads to a larger marginal product of effort in the second stage. This assumption implies that effort across stages is complementary. Moreover, throughout our analysis, we assume the principal always finds it optimal to solicit effort in the first stage.

We solve for the principal's optimal incentive contract offered to the agent, who is risk neutral and protected by limited liability.[1] We are also interested in comparing the optimal contract with the first-best contract. In general, the payment received by the agent depends on (i) the final outcome; (ii) the interim objective signal; and (iii) the principal's message. It is without loss to focus on the following contract implementation. For each objective signal realization, the contract stipulates a menu of incentive schemes, each of which consists of a pair of payments for the respective final outcomes. We say the menu is *pooling* if it is singleton, i.e., the incentive scheme is independent of the principal's message. We say the menu is *separating*, if it consists of more than one scheme and the principal selects one scheme from the menu depending on her subjective signal. Using this terminology, if and only if a separating menu is offered, the principal's report on the subjective signal will play the dual role of determining the agent's compensation and transmitting information about his future productivity. In this case, the agent can infer the subjective signal from the principal's message (or equivalently the offered incentive scheme), and tailor his effort decision in the second stage accordingly.

[1] In our model, contracting friction is introduced by assuming that the agent is protected by limited liability. This class of moral-hazard problem has been widely studied in the economic, accounting and finance literature. For instance, Innes (1990) shows that a standard debt contract can be optimal in such setting. Casamatta (2003) considers optimal financing in the venture capitalist industry in the presence of moral hazard problem. More recently, assuming risk neutrality and limited liability, Fong and Li (forthcoming) and Baldenius, Glover and Xue (2016) analyze relational contracting of a single agent and a team of agents, respectively.

It is worth noting that using a separating menu incurs an additional rent paid to the agent. Given a separating menu, motivating the agent to work in the second stage requires a lower bonus if he is made to believe that the subjective signal is favorable. As a result, if the principal aims at soliciting a high level of second-stage effort when an unfavorable subjective signal is received, she is tempted to overstate the subjective signal. In order that credible information on the subjective signal is transmitted, the menu should provide the principal with incentives to report truthfully. This can be achieved by promising a positive second-stage base wage to the agent—which is called as a *truth-telling rent*—whenever a favorable subjective signal is reported.

We begin the analysis of the principal's contract-design problem by assuming that the incentive compatibility (IC) constraint for the agent's first-stage effort is not binding, a scenario that can arise if the cost of the first-stage effort is sufficiently small. We find that in the optimal contract, the subjective signal is further used in determining the agent's incentive scheme, only if the realized value of objective signal is below a cutoff. There are two driving forces for this cutoff property. First, the truth-telling rent is increasing in the value of objective signal. Intuitively, a higher value of objective signal realized implies that future effort is more valuable to the principal, and that the necessary bonus for inducing future effort is lower. Therefore, the principal's temptation to overstate the subjective signal—which calls for a truth-telling rent—increases with the realized value of objective signal. Second, as mentioned above, a separating menu can fine-tune future effort to the subjective signal realization. We find that this effort-sorting benefit is decreasing in the value of objective signal. In sum, as a higher value of objective signal realizes, the benefit of adopting a separating menu goes down, whereas the associated cost—the truth-telling rent—goes up. As a result, a separating menu is adopted only if the realized value of objective signal is below a certain cutoff.

This simple cutoff structure of the optimal contract allows us to conduct comparative statics with respect to the degree of moral hazard problem. We find that as the second-stage moral hazard problem gets more severe, subjective signals are used for a larger region of objective signals. The reason is as follows. First, as inducing second-stage effort becomes more costly, the principal is more reliant on another (subjective) signal, which provides incremental information about second-stage productivity, in determining the effort to be solicited. Second, as the principal's purpose of overstating her subjective signal is to solicit a higher effort level, she finds it less appealing to misreport if inducing effort gets more costly. This in turn implies a smaller truth-telling rent. Both factors above encourage the principal to expand the use of separating menus.

An interesting implication of the truth-telling rent is that the solicited effort in the optimal contract is *upwardly distorted*. Specifically, there exists a region of objective signals in which the efficient contract calls for second-stage effort only if the subjective signal is favorable, but the optimal contract solicits second-stage effort even if the subjective signal is unfavorable. The reason for the upward distortion in effort is to save the truth-telling rent: separate treatment of different subjective signals necessarily incurs the truth-telling rent, whereas a pooling menu frees the principal from paying such a rent.

Our analysis in Section 4 focuses on the case where the IC constraint for the agent's first-stage effort is binding. This case arises if the cost of the first-stage effort is sufficiently large. The analysis of this section is more involved, as the choice between pooling and separating menus depends on an additional factor: its effectiveness in motivating the first-stage effort. There are three main results. First, as long as either the objective or the subjective signal is sufficiently informative about the agent's first-stage effort, the optimal contract retains a simple cutoff structure: the separating menu is not used in the upper tail of objective signal realizations. Second, the introduction of a binding first-stage moral hazard problem strictly expands the region of objective signal realizations with which a separating menu is used. Loosely speaking, the reason for this finding is that at the margin, a properly-designed separating menu is more cost-effective in providing first-stage incentive than a pooling menu. Third, it is still possible that the solicitation of second-stage effort is upwardly distorted in the optimal contract.

The paper is organized as follows. The next subsection reviews the relevant literature. The model is set up in Section 2. In Sections 3 and 4, we study the principal's contract-design problem by considering two cases: the agent's first-stage IC constraint is binding or not binding. The final section concludes. All proofs are relegated to the appendix.

Literature Review

Incentive provision with subjective performance measures The literature on subjective performance evaluations (e.g., Baker, Gibbons and Murphy 1994; Baiman and Rajan 1995; MacLeod 2003; Levin 2003; Rajan and Reichelstein 2006, 2009; Baldenius, Glover and Xue 2016) examines how a principal uses nonverifiable measures to create work incentives for either an individual agent or a team of agents. In a static moral hazard setting, MacLeod (2003) shows that the use of the subjective signal in contracting necessitates a destruction of surplus (e.g., quitting). He finds that the agent receives a constant discretionary bonus except for *the worst* possible evaluation, in which case the bonus is diverted to a third party (or destroyed). Rajan and Reichelstein (2009) extend his analysis by incorporating an objective performance measure, and find that the agent receives a constant bonus except for the case where *both* the realizations of the objective and the subjective measures are the worst possible. Fuchs (2007) extends MacLeod (2003) to a repeated setting, and finds that the agent receives a constant discretionary bonus except for the case where the realizations of subjective measures are the worst possible *in every period*. The general message of this line of investigation is therefore that the use of subjective performance measures in optimal contracting is *highly compressed*. In contrast, we find that when the subjective evaluation reveals information about next-stage productivity—an assumption that is natural in an IPE setting—the discretionary bonus typically varies with subjective measures *for a range of objective signal realizations at the lower tail*.[2]

Interim performance evaluations Our paper belongs to the literature that investigates whether IPEs improve the social welfare or the principal's payoff in two-stage principal-agent models. Chen and Chiu (2013) consider a setting similar to us with the notable exception that the signal about the interim outcome is either subjective or objective. They identify conditions under which the principal finds it profitable to conduct an IPE. We differ by considering a setting in which both objective and subjective signals about the interim outcome are available, and show that the optimal contract combines the use of both signals.[3] Zábojník (2014) considers a setting in which the agent has an innate ability and engages in serially-independent production. Similar to our setting, the IPE—which plays the dual role of providing feedback and incentives—involves both objective and subjective measures. In contrast to our setting, the agent in his model faces multi-tasking. He shows in the special case with a uniform distribution in ability, the use of subjective and objective incentives are separable. Restricting to a simpler setting with binary effort allows us to obtain a sharp characterization and clear economic intuition about the optimal contract. Moreover, we find that subjective and objective incentives are generally inseparable in the optimal contract.[4]

[2] As pointed out by recent papers, the information-transmission (feedback) role of the subjective evaluation can significantly change the principal's optimal contract. Suvorov and van de Ven (2009) show that if the agent has intrinsic motivation, the feedback role of subjective evaluation makes a discretionary bonus incentive-compatible for the principal without breaking the budget balance condition. In a setting without moral hazard, Fuchs (2015) shows that the principal can credibly communicate to the agent about his match quality if a discretionary bonus is used.

[3] This point also explains the distinction between Silvers's (2013) model and ours. Silvers considers a model similar to the setting analyzed in Section 3, but focuses on whether the principal prefers private or public information, as well as how the principal's payoff responds to the quality of each type of information.

[4] Notable articles in this line of literature are briefly discussed below. Ray (2007) finds that the IPE enhances the efficiency by providing the option of ending a project with low expected returns. Lizzeri, Meyer, and Persico (2002) and Manso (2011)

Conditional variance investigation Our result that the subjective evaluation is used only if the objective signal is in a lower tail is related to the literature on conditional variance investigation in managerial accounting. Conditional variance investigation refers to the policy of seeking additional information if a primary signal yields an outcome below (or above) a certain threshold. Models in this literature typically assume that the principal commits to incur an exogenous cost to generate an additional objective (and verifiable) signal, conditional on the realization of the primary signal.[5] In our setting, the second (subjective) signal is costly to use because additional truth-telling constraints are imposed on the principal.

As pointed out by Ederhof, Rajan, and Reichelstein (2011), a central point of interest in the literature of conditional variance investigation is whether the investigation policy is lower-tailed, upper-tailed, or two-tailed in nature. Baiman and Demski (1980) show that the optimal investigation policy is single-tailed, but may occur at either the upper tail or the lower tail (but not both). Lambert (1985) and Young (1986) show that when either the assumption of conditionally independent signals or the assumption of a HARA utility function is dropped, the optimal investigation policy can be two-tailed. Ederhof (2010) extends the analysis of Rajan and Reichelstein (2009) to allow for correlation in the objective and subjective performance measures. She finds that the subjective measure is used at either the best or the worst possible objective signal, thus showing that a single-tailed investigation policy is optimal.[6] In all articles above, evaluations play a single role of providing incentives for exerting effort. In a setting in which the key roles of evaluations are providing performance feedback and fine-tuning the agent's subsequent effort, we show that the optimal investigation policy calls for using an additional (subjective) signal in a lower tail of the primary (objective) signal. Moreover, subjective measures are often used beyond the worst possible objective signal.

2 Model

A principal (she) hires an agent (he) to complete a two-stage project, in which the agent makes two unobservable effort decisions. Both parties are risk-neutral and fully patient (i.e., they do not discount future payoffs). In Stage 1, the agent chooses between a high effort ($e_1 = 1$) at a cost $\psi > 0$, and a low

find that if the effort plan is independent of an IPE, it is optimal not to provide feedback. Fuchs (2007) shows in a repeated moral hazard setting that to exploit the "reusability of punishment," it is optimal not to conduct an IPE.

[5] See, for instance, Fagart and Sinclair-Desgagné (2007).

[6] Ederhof (2010) also provides empirical evidence that senior-level executives are more likely to receive a discretionary bonus if the company's objective performance is at either the upper or the lower tail.

effort ($e_1 = 0$) at no cost. After effort e_1 is chosen, two conditionally independent signals of e_1 realize. The first signal $\sigma \in \Omega \subset \mathbb{R}_+$ is objective and its realization is publicly known. The second signal $s \in \{h, l\}$ is subjective and its realization is privately known to the principal. The objective signal σ is assumed to have a finite support of size n, and is distributed according to conditional probability function $f(\sigma|\, e_1)$. By relabeling the signals, it is without loss of generality to assume $\sigma = \frac{f(\sigma|1)}{f(\sigma|0)}$, i.e., each objective signal σ is labelled as its likelihood ratio. With this relabelling, a higher value of signal is more indicative of $e_1 = 1$. The maximum and the minimum signals are denoted by $\sigma_{\max}$ and $\sigma_{\min}$ respectively. On the other hand, the subjective signal is binary, $s \in \{h, l\}$. The probability of s given effort e_1 is $g(s|\, e_1)$. Moreover, $s = h$ is more indicative of $e_1 = 1$:

$$k_h \equiv \frac{g(h|\,0)}{g(h|\,1)} < k_l \equiv \frac{g(l|\,0)}{g(l|\,1)}.$$

Note that by definition, $k_l > 1 > k_h$.

In Stage 2, the agent chooses effort $e_2 \in \{0, 1\}$ at a cost ce_2. At the end of Stage 2, a contractible final outcome $x \in \{0, 1\}$ realizes, and the principal gains a payoff normalized to 1 if and only if $x = 1$. The final outcome is determined stochastically by interim signals and the agent's Stage-2 effort according to the following distribution:

$$\Pr(x = 1|\sigma, s, e_2) = t + y(\sigma, s)\, e_2, \tag{1}$$

where t denotes the default probability of success, and $y(\sigma, s)$ measures the marginal product of e_2 and is determined by the interim signals σ and s. The following assumptions are imposed:

A1 $t \in (0, 1)$ and $0 < y(\sigma, s) \leq 1 - t$.

A2 $y(\sigma'', s) > y(\sigma', s)$ for all $s \in \{h, l\}$, and $\sigma'', \sigma' \in \Omega$ such that $\sigma'' > \sigma'$; and $y(\sigma, h) > y(\sigma, l)$ for all $\sigma \in \Omega$.

A3 $y(\sigma_{\min}, h), y(\sigma_{\max}, l) > c > y(\sigma_{\min}, l)$.

A4 The ratio $\frac{y(\sigma,l)}{y(\sigma,h)}$ is non-decreasing in σ.

A5 $y(\sigma, s)$ is continuous in σ.

Assumption **A1** ensures the probability distribution of the final outcome is well-defined. Assumption

A2 implies that efforts in the two stages are complement in achieving a successful final outcome. The complementarity may result from the fact that satisfactory completion of the previous phase lays the foundation for the next phase.[7] Assumption **A3** implies that high effort in the second-stage production is efficient whenever either $\sigma = \sigma_{\max}$ or $s = h$. Moreover, if $\sigma = \sigma_{\min}$ and $s = l$, it is inefficient to solicit the second-stage effort. Assumption **A4** states that $\ln y(\sigma, s)$ is submodular. Note that this allows $y(\sigma, s)$ to be submodular or mildly supermodular. For instance, each $y(\sigma, s)$ with constant elasticity of substitution (including Cobb-Douglas, Leontief, etc.) satisfies the assumption. In Appendix D, we show that Assumption **A4** can be implied by monotone likelihood ratio conditions. Assumption **A5** is made for simplifying our exposition.[8]

Contract Before Stage 1, the principal proposes a long-term contract, denoted by

$$\Gamma \equiv \{w(\sigma, m), b(\sigma, m)\}_{\sigma \in \Omega, m \in M}$$

Contingent on the realization of the objective signal $\sigma \in \Omega$, the contract specifies a menu of incentive schemes, $(w(\sigma, m), b(\sigma, m))_{m \in M}$; the incentive scheme implemented is further determined by a message $m \in M$ disclosed by the principal. Here, M is the principal's message space specified in the long-term contract. An incentive scheme consists of a base wage $w(\sigma, m)$ that is paid to the agent regardless of the final outcome, as well as a bonus $b(\sigma, m)$ that is paid if and only if $x = 1$.

We assume that the agent is protected by limited liability.[9] Specifically,

$$w(\sigma, m) \geq 0, \text{ and } w(\sigma, m) + b(\sigma, m) \geq 0.$$

Given an objective signal $\sigma \in \Omega$, if the wage $w(\sigma, m)$ and the bonus $b(\sigma, m)$ do not vary with the message m, the menu contingent on σ is called as a *pooling menu*; otherwise, it is called as a *separating menu.* To simplify notation, we express a pooling menu as $(w(\sigma), b(\sigma))$.

Timeline The timing of the game is described as follows. First, the principal proposes a contract Γ. After learning contract Γ proposed, the agent either accepts or rejects it. If he rejects, both the principal

[7] Manso (2011) argues that complementarity naturally arises when the principal tries to design a contract for "exploration."

[8] It is immediate that for every function $y' : \Omega \times \{h, l\} \to [0, 1-t]$, there exists a function $y : [\sigma_{\min}, \sigma_{\max}] \times \{h, l\} \to [0, 1-t]$ such that $y(\sigma, s) = y'(\sigma, s)$ for all $\sigma \in \Omega$ and $s \in \{h, l\}$.

[9] Because of limited liability, the agent's individual rationality constraint is not binding.

and the agent get their respective outside options, both normalized to zero. If the agent accepts, the first-stage production begins, and he privately exerts effort e_1. At the end of the first stage, the objective signal σ and the subjective signal s realize. While σ is publicly known, s is privately observed by the principal. After learning s, the principal sends a public message m, which, together with σ, determines the incentive scheme $(w(\sigma, m), b(\sigma, m))$ for the agent. In Stage 2, knowing the specific incentive scheme, the agent privately chooses effort e_2. Finally, outcome x realizes and the principal pays the agent according to the specified incentive scheme.[10]

Before ending the depiction of our model, several remarks are in order. First, each contract Γ defines a dynamic Bayesian game between the principal and the agent. A generic strategy profile in this game is denoted by $\tau \equiv (\beta, \alpha)$, where the principal's behavioral strategy $\beta : \Omega \times \{h, l\} \to \triangle M$ is a mapping from the objective signal and the subjective signal to a randomization over messages, and the agent's strategy α specifies the choices of e_1 and e_2 conditional on relevant histories. Second, we say that τ is incentive compatible to Γ if and only if τ constitutes a perfect Bayesian equilibrium in the associated game. Third, randomizing effort decision cannot improve the payoff of the agent and the principal, so it is without loss to assume that the agent's strategy α is pure in the optimal contract. Fourth, if the agent knows the interim signals, σ and s, his effort decision e_2 is independent on e_1. However, as the subjective signal s is not directly observed by the agent, if the principal does not fully reveal s (e.g., a pooling menu is offered), the agent may condition the choice of e_2 on his previous choice of e_1.

The principal's expected profit of a contract $U(\Gamma, \tau)$ is jointly determined by the contract terms Γ and the incentive-compatible strategy profile τ associated with it. Our goal is to look for the optimal contract of the principal, i.e., the contract that maximizes $\{U(\Gamma, \tau) : \tau \text{ is incentive compatible to } \Gamma\}$. Throughout our analysis, we assume that the principal would like to solicit $e_1 = 1$.[11]

3 Non-binding First-stage Moral Hazard

In this section, we consider the contracting problem that *ignores* the constraint for inducing the agent's first-stage effort. The purpose of this analysis is twofold. First, as the second-stage production provides a positive expected rent to the agent, it is possible that the agent's first-stage IC constraint is not binding.

[10] As neither the principal nor the agent discounts deferred payoffs, it is without loss of generality to assume all transfers are settled at the end of the relationship.

[11] The alternative case in which first-stage effort is not solicited can be easily obtained by modifying the analysis in the subsequent sections.

In this case, the assumption made is without loss of generality. Second, the assumption allows us to illustrate, in the simplest and most transparent setting, the effect of principal's incentive constraint—which ensures her truthful report of the subjective signal s—on the effort distortion. The analysis below fully characterizes the optimal contract and contrasts it with the efficient contract. It is shown that the principal's truth-telling incentive constraints result in an upward distortion in the solicitation of second-stage effort and an under-utilization of subjective signals in contracting. Moreover, the optimal contract has a simple cutoff structure.

We begin our analysis with a couple of observations. First, in the absence of the IC constraint for inducing $e_1 = 1$, the contracting problem for each objective signal $\sigma \in \Omega$ is independent of each other. Thus, the principal's optimal contract must maximize her expected profit for each individual signal σ. Second, following a replication argument similar to that of the revelation principle,[12] it is without loss to assume that the message space M is binary.

Lemma 1 *Suppose that the IC constraint for the agent's first-stage effort is not binding. In the search for an optimal contract, it is without loss to assume that the message space M is binary, and the principal's strategy β is pure.*

According to the lemma above, it is without loss to focus on the contracts that stipulate, following each objective signal σ, a menu consisting of either one or two incentive schemes for the principal to choose. If there is only one scheme, the menu is pooling and is represented by $(w(\sigma), b(\sigma))$. If there are two schemes, without loss of generality, we only consider the separating menus that induce the principal to truthfully disclose $m = s$. In this case, the two schemes on the menu is respectively written as $(w(\sigma, h), b(\sigma, h))$ and $(w(\sigma, l), b(\sigma, l))$. Depending on the solicitation of effort, the set of relevant menus can be classified into one of the following categories.[13]

P0 A pooling menu that induces $e_2 = 0$;

P1 A pooling menu that induces $e_2 = 1$;

S10 A separating menu that induces $e_2 = 1$ if and only if $s = h$;

[12] See, for instance, Myerson (1979).

[13] A brief note on notations of the menu types may be helpful. The letter in the name tells whether it is separating (S) or pooling (P). For a pooling menu, the number indicates whether second-stage effort is solicited (P1) or not (P0). For a separating menu, the first number indicates whether second-stage effort is solicited following $s = h$, whereas the second number indicates whether second-stage effort is solicited following $s = l$.

S11 A separating menu that induces $e_2 = 1$ for both $s = h, l$;

S01 A separating menu that induces $e_2 = 1$ if and only if $s = l$.

The optimal P0 menu is given by $w(\sigma) = b(\sigma) = 0$, and the principal's expected profit of using this menu is t. This menu is also payoff equivalent to a separating menu with $w(\sigma, h) = b(\sigma, h) = w(\sigma, l) = b(\sigma, l) = 0$. In other words, if the second-stage effort is not solicited for any subjective signal s, the principal finds it indifferent between revealing and concealing s. Moreover, it is immediately clear that a S01 menu can never be optimal because if soliciting $e_2 = 1$ following $s = l$ is profitable, such a scheme would be dominated by a corresponding P1 menu.

It remains to consider menus from the categories P1, S10, and S11. First consider the agent's incentive for effort exertion under various menus. Suppose a pooling menu is offered after the objective signal σ is realized. The agent is willing to choose $e_2 = 1$ if and only if

$$b(\sigma) \geq \frac{c}{g(h|e_1)\, y(\sigma, h) + g(l|e_1)\, y(\sigma, l)}. \tag{2}$$

On the other hand, suppose the agent is offered a separating menu after the objective signal σ is realized, and he believes the principal has truthfully reported the subjective signal s. Then the agent is willing to choose $e_2 = 1$ if and only if

$$b(\sigma, s) \geq \frac{c}{y(\sigma, s)}. \tag{3}$$

As the first-stage IC constraint is not binding, the pooling menu P1, if offered contingent on σ, only need to satisfy (2). Thus, the optimal P1 menu is given by

$$w(\sigma) = 0, b(\sigma) = \frac{c}{g(h|1)\, y(\sigma, h) + g(l|1)\, y(\sigma, l)}, \tag{4}$$

and the principal's expected profit is

$$\pi^P(\sigma) \equiv [t + g(h|1)\, y(\sigma, h) + g(l|1)\, y(\sigma, l)] \left[1 - \frac{c}{g(h|1)\, y(\sigma, h) + g(l|1)\, y(\sigma, l)}\right]. \tag{5}$$

Observe that if $t > 0$, the agent would receive a positive rent:

$$[t + g(h|1)\, y(\sigma, h) + g(l|1)\, y(\sigma, l)]\, b(\sigma) - c = \frac{tc}{g(h|1)\, y(\sigma, h) + g(l|1)\, y(\sigma, l)}.$$

This agency rent arises because the agent is protected by limited liability. Following standard terminology, we call this rent the *limited-liability rent*.

If the principal replaces a P1 menu with a S11 menu contingent on σ, the same level of second-stage effort is solicited. However, the separating menu S11 should satisfy (3) with $s = h$ and $s = l$, as well as the constraints that induce the principal to truthfully report $m = s$:

$$(t + y(\sigma, h))(1 - b(\sigma, h)) - w(\sigma, h) \geq (t + y(\sigma, h))(1 - b(\sigma, l)) - w(\sigma, l); \quad (6)$$

$$(t + y(\sigma, l))(1 - b(\sigma, l)) - w(\sigma, l) \geq (t + y(\sigma, l))(1 - b(\sigma, h)) - w(\sigma, h). \quad (7)$$

Constraints (6) and (7) can be rearranged into

$$(t + y(\sigma, l))(b(\sigma, l) - b(\sigma, h)) \leq w(\sigma, h) - w(\sigma, l) \leq (t + y(\sigma, h))(b(\sigma, l) - b(\sigma, h)).$$

This clearly implies that $b(\sigma, l) \geq b(\sigma, h)$ and that $w(\sigma, h) \geq (t + y(\sigma, l))(b(\sigma, l) - b(\sigma, h))$. Thus, the truth-telling constraints result in a strictly positive agency rent, in the form of a positive $w(\sigma, h)$. We call this rent the *truth-telling rent*. As a P1 menu can induce the same level of efficiency without incurring a truth-telling rent, the principal strictly prefers a P1 menu to a S11 menu:

Lemma 2 *Suppose that the IC constraint for the agent's first-stage effort is not binding. A S11 menu is never offered contingent on any σ.*

We now consider the separating menu S10. Given that the agent's first-stage IC constraint is not binding, the principal's optimal S10 menu contingent on σ can be found by solving the problem:

$$\max_{(w(\sigma,m), b(\sigma,m))_{m \in \{h,l\}}} g(h|1)\left[(t + y(\sigma, h))(1 - b(\sigma, h)) - w(\sigma, h)\right] + g(l|1)\left[t(1 - b(\sigma, l)) - w(\sigma, l)\right],$$

subject to (3) with $s = h$, and the following constraints that induce the principal to truthfully report $m = s$,

$$(t + y(\sigma, h))(1 - b(\sigma, h)) - w(\sigma, h) \geq t(1 - b(\sigma, l)) - w(\sigma, l), \quad (8)$$

$$t(1 - b(\sigma, l)) - w(\sigma, l) \geq (t + y(\sigma, l))(1 - b(\sigma, h)) - w(\sigma, h). \quad (9)$$

Inequality (8) states that if $s = h$, the principal's profit of reporting h is no less than that of reporting

l. Note that by reporting h, the agent believes that $s = h$ and is willing to put in effort. An analogical explanation applies to constraint (9) that induces the principal to truthfully report $m = s = l$. The solution to the problem above is[14]

$$\begin{aligned} w(\sigma, l) &= b(\sigma, l) = 0, b(\sigma, h) = \frac{c}{y(\sigma, h)}, \\ w(\sigma, h) &= \max\left\{0, y(\sigma, l)\left(1 - \frac{c}{y(\sigma, h)}\right) - t\frac{c}{y(\sigma, h)}\right\}, \end{aligned} \tag{10}$$

and the principal's expected profit is

$$\pi^S(\sigma) \equiv t + g(h|1)\left[y(\sigma, h) - c - t\frac{c}{y(\sigma, h)} - \max\left\{0, y(\sigma, l)\left(1 - \frac{c}{y(\sigma, h)}\right) - t\frac{c}{y(\sigma, h)}\right\}\right]. \tag{11}$$

Note that in the optimal S10 menu, the truth-telling constraint for $s = h$ always slacks, whereas that for $s = l$ binds if the realized value of σ is high enough. Intuitively, the principal who observes $s = l$ may be inclined to overstate her private signal, because reporting $m = h$ boosts the agent's belief about second-stage productivity and reduces the bonus that is required for inducing $e_2 = 1$. However, overstating may lead to an efficiency loss, as soliciting high effort may be undesirable following $s = l$. An increase in σ, which mitigates the latter efficiency loss and hence enhances the principal's incentive to overstate s, leads to a weakly increase in the base wage, $w(\sigma, h)$, paid to deter upward cheating.

Summarizing the discussion above, given an objective signal σ, the candidates for the optimal menu are (i) a P0 menu with $w(\sigma) = b(\sigma) = 0$; (ii) a P1 menu given by (4); (iii) a S10 menu given by (10). Proposition 1 compares the expected profit of each menu, and shows that the principal's optimal contract has a simple cutoff structure.

Proposition 1 *Suppose the IC constraint for the agent's first-stage effort is not binding. There exist a pair of cutoffs $\sigma_0, \sigma^* \in [\sigma_{\min}, \sigma_{\max}]$ with $\sigma_0 < \sigma^*$ such that the optimal menu takes the following form.*

(i) If $\sigma \geq \sigma^*$, the pooling menu characterized by (4) is offered.

(ii) If $\sigma_0 \leq \sigma < \sigma^*$, the separating menu characterized by (10) is offered.

(iii) If $\sigma < \sigma_0$, either a separating menu or a pooling menu, which is characterized by zero payments, is offered.

If the realized value of σ is very low, the optimal contract calls for $e_2 = 0$ regardless of the subjective

[14] Constraints (8) and (9) require that $b(\sigma, h) \leq 1$. As guaranteed by Assumption **A3**, the solution characterized by (10) satisfies this condition.

signal. Although Assumption **A3** guarantees the efficiency of soliciting $e_2 = 1$ following $s = h$, the presence of the limited-liability rent (in the case $t > 0$) can make it unprofitable to do so if σ is low.[15] In this region of σ, learning and/or using the subjective signal conveys no strict benefit. As σ increases beyond σ_0, the principal finds it profitable to use the separating menu to solicit $e_2 = 1$ only upon $s = h$. Finally, as σ increases beyond σ^*, the principal induces $e_2 = 1$ for all subjective signals. As shown in Lemma 2, the principal prefers using a pooling menu to achieve this. The cutoff σ^* is given by the unique solution to $\pi^S(\sigma) = \pi^P(\sigma)$, whereas σ_0 is given by the unique solution to $\pi^S(\sigma) = t$.[16]

Proposition 1 asserts that there is a unique cutoff in the objective signal, above which revealing the subjective signal is strictly undesirable for the principal. The intuition can be obtained from the classical efficiency-rent tradeoff. On the one hand, the efficiency of a P1 menu relative to a S10 menu increases with the realized value of σ. On the other hand, the agency rent of the optimal S10 menu relative to that of the optimal P1 menu also increases with σ, due to the following facts: First, the optimal P1 menu pays a limited-liability rent in both cases of $s = h$ and $s = l$; the optimal S10 menu pays, in the case of $s = h$, both a limited-liability rent and a truth-telling rent (in the form of a positive $w(\sigma, h)$). Second, the limited-liability rent is decreasing in σ, as a higher σ reduces the required bonus for inducing $e_2 = 1$; however, the truth-telling rent is increasing in σ, because a higher σ makes the principal more tempted to induce $e_2 = 1$ by misreporting $m = h$ upon learning $s = l$. These two observations both point to an increase in the agency rent of a S10 menu relative to a P1 menu as σ increases. As a result, both the efficiency concern and rent saving favor a P1 menu over a S10 menu as a higher σ realizes.

The result on the unique cutoff (in the choice between a separating menu and a pooling menu) is robust to alternative model specifications, such as non-binary effort and/or subjective signals. In Appendix B, we show that the optimal contract retains a simple cutoff structure even if the agent can choose from a continuous range of effort in the second stage. The key driving force for this result is that the truth-telling rent of a separating menu increases in σ at a rate higher than that of its efficiency gain.

Proposition 1 sheds new light on the notion of conditional variance investigations. The existing literature in this area has focused on using investigations to motivate the agent's previous-stage effort. However, in our model, evaluations also transmit information about subsequent-stage productivity, so an additional signal is more valuable if the primary signal alone is not sufficiently favorable to warrant high

[15] Note that if $t = 0$, then $\sigma_0 < \sigma_{\min}$, and the region corresponding to part (iii) of Proposition 1 vanishes.

[16] Because of the discrete nature of the objective signal space Ω, it is possible that no objective signal takes the exactly same value as σ^*. However, it is without loss to define σ^* as the upper bound on the set of objective signals in which separating menus are offered. A similar remark applies to σ_0. Moreover, given Assumption **A3**, it is possible that $\sigma_0 \in [\sigma_{\min}, \sigma_{\max}]$.

effort in the subsequent stage. Moreover, the cost of learning and/or using the additional (subjective) signal arises endogenously from the principal's truth-telling constraints, and increases with the realization of the primary (objective) signal. As a result, both the information-transmission role, as well as the monotonicity of the truth-telling rent, favors the additional (subjective) signal being used only if the primary signal is sufficiently unfavorable.

In our model, the use of subjective signals in the optimal contract is compressed. Specifically, if the signal s is observable and verifiable, s would be used in contracting for a range of objective signals that strictly include $[\sigma_0, \sigma^*]$. The subjective nature of the signal s reduces its use in contracting, because a truth-telling rent is incurred. However, it is worth noting that the degree of compression in our setting is mild, compared with the setting in which the subjective signal does not convey information about next-stage productivity (e.g., Rajan and Reichelstein (2009) find that the subjective signal is used only at the worst possible objective signal).

The simple cutoff structure of the optimal contract derived in Proposition 1 allows us to investigate factors that affect the extent of using subjective signals in optimal contracting. The following corollary reports the comparative statics of cutoff σ^*, the upper bound on the set of objective signals at which separating menus are offered.

Corollary 1 *An increase in either c or t leads to an increase in σ^*.*

As c or t increases, the agent must be given a larger limited-liability rent in order to motivate him to exert effort in Stage 2. Therefore, it is more beneficial for the principal to fine-tune effort solicitation to an additional (subjective) signal that informs about the agents's second-stage productivity. Moreover, the truth-telling rent, which is paid to prevent the principal from overstating s under the separating menu, is decreasing in c and t. This is because an exacerbated second-stage moral hazard problem discourages the principal who receives $s = l$ from soliciting $e_2 = 1$ (as the principal who receives $s = h$ would do). As a result, an increase in either c or t favors the use of a separating menu relative to a pooling menu contingent on every σ.

In the remainder of this section, we compare the optimal contract derived above with the first-best contract. The efficient level of second-stage effort maximizes the total surplus of the principal and the agent. As σ increases, the marginal product of e_2, which is defined by $y(\sigma, s)$, increases, and it becomes more desirable to implement $e_2 = 1$ from the perspective of a social planner. Define σ_l as the unique solution to $y(\sigma, l) = c$. Under assumptions **A2** and **A3**, we have $\sigma_l \in [\sigma_{\min}, \sigma_{\max}]$. The efficient effort profile is as follows.

Efficient effort profile	Objective signal
$e_2 = 1$ regardless of s	if $\sigma \geq \sigma_l$;
$e_2 = 1$ if and only if $s = h$	if $\sigma_{\min} \leq \sigma < \sigma_l$;

If $t > 0$, it is possible that $\sigma_{\min} \leq \sigma_0$, suggesting that the second-stage effort is under-incentivized following $\sigma \in [\sigma_{\min}, \sigma_0)$. This downward distortion in effort solicitation is caused by the principal's motive to save the limited-liability rent, and is typical in the second-best contract in moral hazard models with limited liability. The comparison of σ_l with σ^* is more interesting. Whether there is an upward distortion of effort solicitation (i.e., $\sigma_l > \sigma^*$) or a downward distortion ($\sigma_l < \sigma^*$) depends on the magnitude of t.

Proposition 2 *$\sigma^* < \sigma_l$ if and only if*

$$t < [g(h|1)\, y(\sigma_l, h) + g(l|1)\, c]\, g(h|1) \left(1 - \frac{c}{y(\sigma_l, h)}\right). \tag{12}$$

The intuition is as follows. On the one hand, a limited-liability rent, which increases with t, calls for a downward distortion in effort e_2. On the other hand, the presence of a truth-telling rent, which disfavors the usage of the separating menu S10 relative to the pooling menu P1, calls for an upward distortion in effort e_2 for $s = l$. Thus, if t is small enough, the latter rent that drives an upward distortion in effort e_2 is dominant; in this case, following $\sigma \in (\sigma^*, \sigma_l)$, the principal solicits $e_2 = 1$ regardless of s, but it is socially inefficient to do so for $s = l$.

Our finding that the second-stage effort can be upwardly distorted in the optimal contract is in sharp contrast to that in typical screening models.[17] In standard screening models—where the agent holds private information—the principal's tradeoff between efficiency and information rent *always results in a downward distortion* in quality provided or effort solicited. In contrast, in our setting—where the principal holds private information—the efficiency-rent tradeoff can result in an upward distortion in effort. It is worth noting that in a related setting, Chen and Chiu (2013) provide a different reason for an upward distortion in effort, namely, implementing a S10 menu may be infeasible.[18]

The literature on informed principal has also investigated effort distortion. Considering moral hazard settings in which the principal has private information about the agent's productivity before offering a contract, Beaudry (1994) and Inderst (2001) find that the effort of the efficient type is downwardly

[17] See Chapter 2 of Bolton and Dewatripont (2005) for a survey.

[18] In contrast, implementing a S10 menu is always feasible in our setting. See footnote 14 for an explanation.

distorted. In a setting without moral hazard, Laffont and Martimort (2002) find that there is an upward distortion for the efficient type. Chade and Silvers (2002) consider a moral hazard setting in which the principal has private information about the informativeness of the monitoring technology. They identify a pooling equilibrium in which effort can be upwardly distorted. However, they point out that this equilibrium does not survive the Cho-Kreps intuitive criterion. Moreover, they do not analyze whether the pooling equilibrium gives the highest ex-ante payoff to the principal. In contrast to the articles above, the principal in our setting does not have any private information at the time of offering the contract, so our analysis is free from equilibrium multiplicity. Furthermore, an upward distortion in effort occurs in the principal's *optimal* contract.[19]

In Appendixes B and C, we consider two extensions of the basic model of this section. The first extension considers continuous effort choice of the agent. The second extension allows for a richer space of subjective signals. It is shown that the findings of (i) the simple cutoff structure of the optimal contracts; (ii) a possible upward distortion in effort e_2 and (iii) the under-utilization of subjective signals are robust to the two extensions.

4 Binding First-stage Moral Hazard

In this section, we investigate how the principal's optimal design of contract is affected by the need of providing incentives for the agent to exert effort in the first stage. To simplify our analysis, we assume that $t = 0$. In this setting, the principal does not need to pay limited-liability rents to the agent for satisfying the latter party's second-stage IC constraint; the rents, which are paid by the principal in addition to the effort cost, result from either the principal's truth-telling constraints or the agent's first-stage moral hazard problem. There are two main results of this section. First, our previous findings, e.g., the simple cutoff structure of the optimal contract and an upward distortion in second-stage effort solicitation, remain robust, provided that the first-stage moral hazard problem is not very severe. Second, the introduction of a binding first-stage moral hazard problem could expand the region of objective signals with which a separating menu is offered.

We begin the analysis by noting that a result similar to Lemma 1 applies.

[19] As pointed out in Silvers (2012), the assumption that the principal's information concerns the agent's productivity leads to results distinct from the assumption that the information concerns the monitoring technology.

Lemma 3 *Suppose $t = 0$. In the search for an optimal contract, it is without loss to assume that the message space M is binary, and the principal's strategy β is pure.*

Using the lemma above, the set of relevant menus that the principal may offer contingent on the objective signal σ still consists of the five types mentioned in Section 3. Denote by $\Pi_i(\sigma)$ the principal's payoff of using menu $i \in \{\text{P0, P1, S10, S01, S11}\}$. We can express these profit functions as follows.

$$\begin{aligned}
\Pi_{P0}(\sigma) &= -w(\sigma); \\
\Pi_{P1}(\sigma) &= [g(h|1)\,y(\sigma,h) + g(l|1)\,y(\sigma,l)]\,(1-b(\sigma)) - w(\sigma); \\
\Pi_{S10}(\sigma) &= g(h|1)\,[y(\sigma,h)\,(1-b(\sigma,h)) - w(\sigma,h)] - g(l|1)\,w(\sigma,l); \\
\Pi_{S01}(\sigma) &= -g(h|1)\,w(\sigma,h) + g(l|1)\,[y(\sigma,l)\,(1-b(\sigma,l)) - w(\sigma,l)]; \\
\Pi_{S11}(\sigma) &= g(h|1)\,[y(\sigma,h)\,(1-b(\sigma,h)) - w(\sigma,h)] + g(l|1)\,[y(\sigma,l)\,(1-b(\sigma,l)) - w(\sigma,l)].
\end{aligned}$$

For each $i \in \{\text{P0, P1, S10, S01, S11}\}$, denote by $\rho_i(\sigma)$ the decision of offering menu i conditional on the realization of σ. Specifically, $\rho_i(\sigma) = 1$ stands for menu i being offered contingent on σ; whereas $\rho_i(\sigma) = 0$ stands for menu i being not offered contingent on σ.[20] Using these notations, the principal's expected profit can be expressed as

$$\sum_{\sigma\in\Omega} f(\sigma|1)\,[\Pi_{P0}(\sigma)\,\rho_{P0}(\sigma) + \Pi_{P1}(\sigma)\,\rho_{P1}(\sigma) + \Pi_{S10}(\sigma)\,\rho_{S10}(\sigma) + \Pi_{S01}(\sigma)\,\rho_{S01}(\sigma) + \Pi_{S11}(\sigma)\,\rho_{S11}(\sigma)]. \tag{13}$$

In designing her contract, i.e., choosing $\rho_i(\sigma)$ and the corresponding payment scheme(s) to maximize (13), the principal is subject to the following three groups of constraints.

(I) The feasibility constraint on the choice of menu:

$$\rho_{P0}(\sigma) + \rho_{P1}(\sigma) + \rho_{S10}(\sigma) + \rho_{S01}(\sigma) + \rho_{S11}(\sigma) = 1. \tag{14}$$

[20] It is straightforward to show that even if randomization over menu offer is allowed, the optimal contract involves only deterministic menus. Thus, the restriction of $\rho_i(\sigma) \in \{0,1\}$ is without loss of generality.

(II) The agent's first-stage IC constraint:

$$\sum_{\sigma\in\Omega} f(\sigma|1)\left[C_{P0}(\sigma)\rho_{P0}(\sigma)+C_{P1}(\sigma)\rho_{P1}(\sigma)+C_{S10}(\sigma)\rho_{S10}(\sigma)+C_{S01}(\sigma)\rho_{S01}(\sigma)+C_{S11}(\sigma)\rho_{S11}(\sigma)\right]\geq\psi, \tag{15}$$

where

$$\begin{aligned}
C_{P0}(\sigma) &= 0,\\
C_{P1}(\sigma) &= \left(1-\frac{1}{\sigma}\right)w(\sigma)+\left[g(h|1)\,y(\sigma,h)+g(l|1)\,y(\sigma,l)\right]\left(b(\sigma)-\frac{c}{g(h|1)\,y(\sigma,h)+g(l|1)\,y(\sigma,l)}\right)\\
&\quad -\frac{1}{\sigma}\left[g(h|0)\,y(\sigma,h)+g(l|0)\,y(\sigma,l)\right]\max\left\{b(\sigma)-\frac{c}{g(h|0)\,y(\sigma,h)+g(l|0)\,y(\sigma,l)},0\right\},\\
C_{S10}(\sigma) &= g(h|1)\left(1-\frac{k_h}{\sigma}\right)\left[w(\sigma,h)+y(\sigma,h)\,b(\sigma,h)-c\right]+g(l|1)\left(1-\frac{k_l}{\sigma}\right)w(\sigma,l),\\
C_{S01}(\sigma) &= g(h|1)\left(1-\frac{k_h}{\sigma}\right)w(\sigma,h)+g(l|1)\left(1-\frac{k_l}{\sigma}\right)\left[w(\sigma,l)+y(\sigma,l)\,b(\sigma,l)-c\right],\\
C_{S11}(\sigma) &= g(h|1)\left(1-\frac{k_h}{\sigma}\right)\left[w(\sigma,h)+y(\sigma,h)\,b(\sigma,h)-c\right]+g(l|1)\left(1-\frac{k_l}{\sigma}\right)\left[w(\sigma,l)+y(\sigma,l)\,b(\sigma,l)-c\right];
\end{aligned}$$

(III) The IC constraints for the agent's second-stage effort and the truth-telling constraints for the principal (if a separating menu is offered). Specifically, for a P1 menu, (2) are imposed; for a S10 menu, (3) with $s=h$, (8), and (9) are imposed; for a S11 menu, (3) with $s=h$ and $s=l$, (6), and (7) are imposed; for a S01 menu, besides (3) with $s=l$, the following constraints that guarantee the principal's truthful report of s are imposed.

$$-w(\sigma,h)\geq y(\sigma,h)\left(1-b(\sigma,l)\right)-w(\sigma,l); \tag{16}$$

$$y(\sigma,l)\left(1-b(\sigma,l)\right)-w(\sigma,l)\geq -w(\sigma,h). \tag{17}$$

To understand (15), it is worth noting that (i) the agent's first-stage effort increases the probability of generating the signal profile (σ,s) by $f(\sigma|1)\,g(s|1)\left(1-\frac{k_s}{\sigma}\right)$; (ii) contingent on the realization of σ and s, the agent's rent, determined by the offered menu, is given by $w(\sigma,s)+\max\{y(\sigma,s)\,b(\sigma,s)-c,0\}$. Thus, the left hand side (LHS) of (15) is the increase in the agent's rent resulting from his choice of $e_1=1$, while the right hand side (RHS) is the effort cost.

We solve the principal's contract-design problem above in two steps. First, we characterize the optimal payment scheme for each menu, given that this menu is offered contingent on $\sigma \in \Omega$, i.e., $\rho_i(\sigma) = 1$. Second, we solve the global optimization problem by pinning down $\rho_i(\sigma)$, thus identifying the regions in which each type of menu is used. The following proposition reports the result from the first step of the analysis.

Proposition 3 *Suppose $t = 0$. Define*

$$Z(\sigma) \equiv \frac{g(h|1)\,y(\sigma,h) + g(l|1)\,y(\sigma,l)}{g(h|1)\left(1 - \frac{k_h}{\sigma}\right) y(\sigma,h) + g(l|1)\left(1 - \frac{k_l}{\sigma}\right) y(\sigma,l)}.$$

In the optimal contract, the menu that is offered contingent on $\sigma \in \Omega$ should take one of the following forms:

(i) a P0 menu with $w(\sigma) = b(\sigma) = 0$.

(ii) a P1 menu with $w(\sigma) = 0$ and $b(\sigma)$ satisfying the following properties:

$$\begin{aligned} \text{for } \sigma &= \arg\min_{\sigma' \in \Omega} Z(\sigma'),\ b(\sigma) \geq \frac{c}{g(h|0)\,y(\sigma,h) + g(l|0)\,y(\sigma,l)}; \\ \text{for } \sigma &\neq \arg\min_{\sigma' \in \Omega} Z(\sigma'),\ b(\sigma) = \frac{c}{g(h|1)\,y(\sigma,h) + g(l|1)\,y(\sigma,l)},\ \text{or } b(\sigma) = \frac{c}{g(h|0)\,y(\sigma,h) + g(l|0)\,y(\sigma,l)}. \end{aligned}$$

(iii) a S10 menu with

$$\begin{aligned} w(\sigma,l) &= b(\sigma,l) = 0, b(\sigma,h) = \frac{c}{y(\sigma,h)}, \\ w(\sigma,h) &= \begin{cases} y(\sigma,h) - c & \text{if } \sigma \text{ is sufficiently high,} \\ y(\sigma,l)\left(1 - \frac{c}{y(\sigma,h)}\right) & \text{otherwise.} \end{cases} \end{aligned}$$

As the agent's first-stage IC constraint becomes binding, a P1 menu may still be offered contingent on certain values of σ, but its bonus $b(\sigma)$ is possibly set above $\frac{c}{g(h|1)y(\sigma,h)+g(l|1)y(\sigma,l)}$, the minimal level for inducing the second-stage effort. This is in contrast to a typical moral hazard model with limited liability, in which the agent is not paid any rent except at the objective signal that is most indicative of effort (i.e., $\sigma_{\max}$). The reason for this difference is that in our model, setting $b(\sigma)$ between $\frac{c}{g(h|1)y(\sigma,h)+g(l|1)y(\sigma,l)}$ and $\frac{c}{g(h|0)y(\sigma,h)+g(l|0)y(\sigma,l)}$ can be a cost-effective way to induce first-stage effort. Specifically, suppose that σ is realized and the bonus $b(\sigma)$ is in the aforementioned range. Then the agent's rent is positive if and only if he has exerted first-stage effort, i.e., $e_1 = 1$. This property is valid only if $b(\sigma) \leq \frac{c}{g(h|0)y(\sigma,h)+g(l|0)y(\sigma,l)}$; beyond this level of bonus, the agent can gain a positive rent even by choosing $e_1 = 0$. Finally, $b(\sigma)$ may be set above $\frac{c}{g(h|0)y(\sigma,h)+g(l|0)y(\sigma,l)}$ only upon the realization of an objective signal that minimizes $Z(\sigma)$, which captures the cost-benefit ratio of raising $b(\sigma)$ in relaxing the constraint (15).[21]

The optimal S10 menu, if chosen, is characterized by the same payments as (10) except for $w(\sigma, h)$. Note that the principal's truth-telling constraints, (8) and (9), impose bounds on the set of feasible values of $w(\sigma, h)$. Part (iii) of Proposition 3 states that $w(\sigma, h)$ is set at either one of the bounds. This feature follows from the linear nature of the principal's optimization problem. Intuitively, a positive $w(\sigma, h)$ is costly for the principal but is useful for incentivizing first-stage effort. As a higher σ is more indicative of $e_1 = 1$, it is natural that, for the purpose of motivating first-stage effort, $w(\sigma, h)$ is set at the upper bound if and only if the realized value of σ is high enough.

Proposition 3 also indicates that S11 and S01 menus are never offered even if the first-stage IC constraint binds. In the proof of this proposition, we show that, for any S11 menu offered contingent on σ, we can find a P1 menu that yields the same profit to the principal but increases the LHS of (15) by a larger magnitude. Moreover, a S01 menu is either dominated by a P0 menu for a low value of σ or dominated by a P1 menu for a high value of σ.

Having determined the payment schemes for each type of menu in the optimal contract, we are ready to characterize the region of objective signals in which each type of menu is used. The proposition below shows that the optimal contract has a simple cutoff structure, provided that k_h is sufficiently small, i.e., $s = h$ is sufficiently indicative of the agent's choice of $e_1 = 1$.

Proposition 4 *Suppose that $t = 0$ and that k_h is sufficiently small.*

(i) If ψ is smaller than a threshold $\overline{\psi}$, there exists a unique cutoff $\sigma_1^* \in (\sigma_{\min}, \sigma_l)$ such that a S10 menu is used for all $\sigma \leq \sigma_1^*$, and a P1 menu is used for all $\sigma > \sigma_1^*$.

(ii) If $\sigma_{\max} \to \infty$, $\overline{\psi} \to \infty$.

(iii) The cutoff σ_1^* is lower than σ_l.

Part (i) of Proposition 4 states that if the first-stage moral hazard problem is binding but not severe (i.e., $\psi \leq \overline{\psi}$), then the optimal contract still has a simple cutoff structure. With a binding first-stage moral hazard problem, the choice between a S10 menu and a P1 menu is determined by three factors: (i) the second-stage efficiency, (ii) the truth-telling rent, and (iii) the implied incentive for the first-stage effort solicitation. As analyzed in the previous section, the first two factors both favor the optimal P1

[21] Note that $Z(\sigma)$ may not be a monotone function of σ given an arbitrary second-stage production technology characterized by $y(\sigma, s)$.

menu as σ increases. However, the third factor favors the optimal S10 menu as σ increases. To see this, note that on the one hand, a higher σ lifts the upper bound of $w(\sigma, h)$ in a S10 menu, making such a menu more effective in providing first-stage incentive. On the other hand, a P1 menu with bonus $b(\sigma) = \frac{c}{g(h|0)y(\sigma,h)+g(l|0)y(\sigma,l)}$ gets less effective in motivating first-stage effort as σ increases. As the last factor works in a direction opposite to that of the first two factors, the simple cutoff structure of the optimal contract is retained if the last factor is not of significant concern (e.g., if $\psi \leq \overline{\psi}$).

Part (ii) asserts a sufficient condition under which the simple cutoff structure is preserved for any ψ. To induce $e_1 = 1$, the principal can offer a P1 menu following $\sigma = \sigma_{\max}$ and increase its bonus $b(\sigma)$ until (15) is satisfied. Thus, if there exists an objective signal that is very informative about the first-stage effort, the first-stage moral hazard problem can be tackled by paying a limited amount of rents to the agent. This in turn renders the third factor discussed above less crucial in determining the optimal contract, and the simple cutoff structure is retained.

Part (iii) states that if the optimal contract has a simple cutoff structure, given $t = 0$ and a sufficiently small k_h, the second-stage effort is upwardly distorted following $\sigma \in (\sigma_1^*, \sigma_l)$.[22] The intuition is similar to that for Proposition 2. Moreover, this result also highlights that the subjective signal is compressed in the optimal contract. If the signal s is contractible, then the principal would find it (weakly) optimal to offer a separating menu for every σ. This is because revealing s (and linking payments to s) does not incur additional agency costs[23] but weakly enhances the incentive for the first-stage effort. However, the subjective nature of s renders its credible transmission costly, and imposes limits on its effectiveness in providing first-stage incentive (recall $w(\sigma, h)$ in a S10 menu is bounded from above because of truth-telling constraint (8)).

The proposition below compares the optimal cutoffs with and without a binding first-stage moral hazard problem.

Proposition 5 *Suppose that $t = 0$ and that k_h and $\frac{1}{\sigma_{\max}}$ are sufficiently small. Then the cutoff σ_1^* in Proposition 4 is not lower than the cutoff σ^* in Proposition 1.*

Given the conditions stated in Proposition 5, the introduction of a binding first-stage moral hazard problem expands the use of the subjective signal. This is because at the margin, using a separating menu (and linking a reward to $s = h$) is relatively cost-effective for motivating first-stage effort. Recall in Rajan

[22] As shown in the proof to Proposition 4, an increase in ψ first increases the cutoff σ_1^* and then reduces σ_1^*. Moreover, the maximum value of σ_1^* is always smaller than σ_l.

[23] Given that $t = 0$, there is no limited-liability rent paid due to the second-stage moral hazard problem.

and Reichelstein (2009), a subjective signal is used in contracting only at the worst possible objective signal, *regardless of the severity of the previous-stage moral hazard problem.* However, in our setting, the subjective signal—which transmits information about next-stage productivity—is used more intensively in the optimal contract as the previous-stage moral hazard problem gets more severe.[24]

5 Concluding Remarks

We consider a dynamic moral hazard problem in which the agent exerts effort sequentially to produce a final output, and two signals on his progress, one objective and one subjective, are present in the interim stage. We analyze the interaction of the two signals in the optimal contract design. In particular, we find that the truth-telling constraint on the principal for reporting the subjective signal creates an agency rent. This truth-telling rent is increasing in the realized value of objective signal, thus making the use of subjective signal more costly following favorable objective signal. This results in a unique cutoff in the objective signal realization above which subjective signals are not used in the optimal contract. The simple cutoff structure allows us to conduct neat comparative statics on the optimal contract; it is shown that an increase in the severity of the moral hazard problem expands the use of subjective signals in contracting. Moreover, the presence of the truth-telling rent causes the under-utilization of separating menus and an upward distortion in effort solicitation (both relative to the first-best contract).

There are several promising directions for future research. First, one may consider a more general signal structure that allows for correlation between the objective and subjective signals (conditional on agent's action). As the objective signal contains some information content of the subjective signal, it may help lowering the truth-telling rent required. Second, in many real-world scenarios, the objective signal must be voluntarily disclosed by the agent, rather than automatically generated and publicized. As shown by Ben-Porath, Dekel, and Lipman (2014), voluntary disclosure may have implications on the agent's choice over both effort and project. Finally, it is interesting to consider a setting in which the subjective signal is observed by a supervisor rather than the principal herself. Then the optimal contract must take into account potential collusion between the supervisor and the agent.[25]

[24] This comparative statics result is not driven by the assumption of limited liability. In the literature of moral hazard with limited liability, it is well recognized that the optimal contract stipulates a positive bonus if and only if the objective signal is the most favorable, i.e., $\sigma = \sigma_{\max}$ (see, e.g., Laux 2001). Consequently, if the subjective evaluation purely serves an incentive-provision role, the region of objective signals with which a separating menu is offered would not change with an increase in the effort cost.

[25] See Tirole (1986) for a seminal contribution to the analysis of collusion-proof contract in a three-tier contracting relation.

References

[1] Baiman, S., and J. Demski. 1980. Economically Optimal Performance Evaluation and Control Systems. *Journal of Accounting Research,* 18: pp.184-220.

[2] Baiman, S. and Rajan, M. 1995. The Informational Advantages of Discretionary Bonus Schemes. *The Accounting Review,* 70, pp.557-579.

[3] Baker, G., Gibbons, R. and Murphy, K.J. 1994. Subjective Performance Measures in Optimal Incentive Contracts. *Quarterly Journal of Economics*, 109, pp.1125-1156.

[4] Baldenius, T., Glover, J. and Xue, H. 2016. Relational Contracts with and between Agents. *Journal of Accounting and Economics*, 61(2), pp.369-390.

[5] Beaudry, P. 1994. Why an Informed Principal May Leave Rents to an Agent? *International Economic Review*, 35, pp.821-832.

[6] Ben-Porath, E., Dekel, E., and Lipman, B.L. 2014. Disclosure and Choice, working paper, Northwestern University.

[7] Bolton, P. and Dewatripont, M. 2005. *Contract theory.* MIT press.

[8] Casamatta, C. 2003. Financing and Advising: Optimal Financial Contracts with Venture Capitalists. *Journal of Finance*, 58(5), pp.2059-2086.

[9] Cleveland, J.N., Murphy, K.R. and Williams, R.E. 1989. Multiple Uses of Performance Appraisal: Prevalence and Correlates. *Journal of Applied Psychology*, 74(1), p.130.

[10] Chade, H. and Silvers, R. 2002. Informed Principal, Moral Hazard, and the Value of a More Informative Technology. *Economics Letters*, 74(3), pp.291-300.

[11] Chen, B.R. and Y.S. Chiu. 2013. Interim Performance Evaluation in Contract Design. *Economic Journal,* 123, pp. 665-698.

[12] Ederhof, M., Rajan, M.V. and Reichelstein, S. 2011. Discretion in Managerial Bonus Pools. *Foundations & Trends in Accounting*, 5(4), pp.243-316.

[13] Ederhof, M. 2010. Discretion in Bonus Plans. *The Accounting Review*, 85(6), pp.1921-1949.

[14] Fagart, M.C. and Sinclair-Desgagné, B. 2007. Ranking Contingent Monitoring Systems. *Management Science*, 53(9), pp.1501-1509.

[15] Fong, Y.F. and Li, J. Forthcoming. Relational Contracts, Limited Liability, and Employment Dynamics. *Journal of Economic Theory.*

[16] Fuchs, W. 2007. Contracting with Repeated Moral Hazard and Private Evaluations. *American Economic Review*, 97, pp. 1432-1448.

[17] Fuchs, W. 2015. Subjective Evaluations: Discretionary Bonuses and Feedback Credibility. *American Economic Journal: Microeconomics*, 7, pp.99-108

[18] Inderst, R. 2001. Incentive Schemes as a Signaling Device. *Journal of Economic Behavior & Organization*, 44(4), pp.455-465.

[19] Innes, R.D., 1990. Limited Liability and Incentive Contracting with Ex-ante Action Choices. *Journal of Economic Theory*, 52(1), pp.45-67.

[20] Laffont, J., and D. Martimort. 2001. *The Theory of Incentives: the Principal-agent Model*. Princeton University Press.

[21] Lambert, R. 1985. Variance Investigation in Agency Settings. *Journal of Accounting Research*, 23(3), pp.633-647.

[22] Laux, C., 2001. Limited-liability and Incentive Contracting with Multiple Projects. *RAND Journal of Economics*, 32(3), pp.514-526.

[23] Levin, J. 2003. Relational Incentive Contracts. *American Economic Review*, 93, 835-857.

[24] Lizzeri, A., M. Meyer. and N. Persico. 2002. The Incentive Effects of Interim Performance Evaluations. CARESS working paper #02-09.

[25] MacLeod, W.B. 2003. Optimal Contracting with Subjective Evaluation. *American Economic Review*, 93, pp.216-240.

[26] Manso, G. 2011. Motivating Innovation. *Journal of Finance,* 66, pp.1823-1860.

[27] Myerson, R.B. 1979. Incentive Compatibility and the Bargaining Problem. *Econometrica*, 47, pp.61-73.

[28] Rajan, M.V. and Reichelstein, S. 2006. Subjective Performance Indicators and Discretionary Bonus Pools. *Journal of Accounting Research*, 44, pp.585-618.

[29] Rajan, M.V. and Reichelstein, S. 2009. Objective versus Subjective Indicators of Managerial Performance. *The Accounting Review*, 84(1), pp.209-237.

[30] Ray, K. 2007. Performance Evaluations and Efficient Sorting. *Journal of Accounting Research*, 45(4), pp.839-882.

[31] Silberberg, E. and Suen, W. 2000. *The Structure of Economics*. McGraw-Hill/Irwin, p.168.

[32] Silvers, R., 2012. The Value of Information in a Principal–agent Model with Moral Hazard: The Ex Post Contracting Case. *Games and Economic Behavior*, 74(1), pp.352-365.

[33] Silvers, R. 2013. The Value of Information in a Principal-Agent Model with Moral Hazard: The Ex Ante Contracting Case. working paper, Deakin University, Faculty of Business and Law, School of Accounting, Economics and Finance.

[34] Suvorov, A. and van de Ven, J. 2009. Discretionary Rewards as a Feedback Mechanism. *Games and Economic Behavior*, 67(2), pp.665-681.

[35] Tirole, J. 1986. Hierarchies and Bureaucracies: On the role of Collusion in Organizations. *Journal of Law, Economics, & Organization*, 2(2), pp.181-214.

[36] Young, R. 1986. A Note on Economically Optimal Performance Evaluation and Control Systems: the Optimality of Two-tailed Investigations. *Journal of Accounting Research,* 24, pp.231-240.

[37] Zábojník, J. 2014. Subjective Evaluations with Performance Feedback. *RAND Journal of Economics*, 45, pp.341–369.

Appendix A: Proof to Lemmas and Propositions

Proof of Lemma 1

Consider an optimal contract and fix an objective signal σ. Define $M_h \equiv\{m \in M\colon \beta_{(\sigma,h)}(m) > 0$ and $\beta_{(\sigma,l)}(m) = 0\}$ and $M_l \equiv\{m \in M\colon \beta_{(\sigma,l)}(m) > 0$ and $\beta_{(\sigma,h)}(m) = 0\}$. Every message $m \in M_h$ must yield the same principal's payoff; and likewise, every message $m \in M_l$ must yield the same payoff. It is thus without loss to assume that $|M_h|$ and $|M_l|$ are at most one. Define $M_\mu \equiv\{m \in M\colon \beta_{(\sigma,h)}(m) > 0$ and $\beta_{(\sigma,l)}(m) > 0\}$. It is also without loss to assume $|M_\mu| = 1$. Suppose not. Take $m_0 \in \arg\min_{m\in M_\mu} \Pr(s = h|m)$. Then for $s \in \{l, h\}$, define a new reporting strategy by setting $\beta'_{(\sigma,s)}(m_0) = \sum_{m\in M_\mu} \beta_{(\sigma,s)}(m)$, and $\beta'_{(\sigma,s)}(m) = \beta_{(\sigma,s)}(m)$ for $m \notin M_\mu$. Fixing the agent's strategy, the principal's profit remains unchanged under the new reporting strategy β'. Taking changes in the agent's strategy into account weakly increases the principal's profit. To see this, note that under strategy β', the agent's belief that $s = h$ after learning m_0 increases. Thus, if $e_2 = 1$ is solicited following (σ, m_0) in the original contract, the incentive constraint for second-stage effort would be preserved in the new contract. On the other hand, if $e_2 = 0$ is solicited following (σ, m_0) in the original contract, the principal's payoff conditional on σ weakly increases under the new contract, as a higher level of agent's effort cannot hurt the principal in the optimal contract.

Consider two cases: (i) $|M_l| = 1$; and (ii) $|M_l| = 0$.

In case (i), if $|M_\mu| = 0$, then $|M_h| = 1$ and we are done. Suppose not. Then define a new incentive scheme as follows. Remove M_h altogether, set $M'_l = M_l$, and $M'_h = M_\mu$. The deletion of some messages removes some truth-telling ICs for the principal. Moreover, the IC for the agent to exert effort is respected, as his belief about being $s = h$ goes up upon receiving M'_h. By the same logic as above, in the new scheme, the principal's profit following signal profile (σ, h) weakly increases (whereas that of (σ, l) remains unchanged).

In case (ii), we must have $|M_\mu| = 1$. Now remove M_h and set $\beta'_{(\sigma,h)}(M_\mu) = 1$. The agent's belief that $s = h$ upon receiving M_μ increases, so the IC for exerting effort is still respected. By the same logic as above, in the new scheme, the principal's profit following signal profile (σ, h) weakly increases (whereas that of (σ, l) remains unchanged). Q.E.D.

Proof of Lemma 2

The optimal S11 menu can be found by solving the following problem:

$$\max_{w(\sigma,h),b(\sigma,h),w(\sigma,l),b(\sigma,l)} g(h|1)\left[(t+y(\sigma,h))(1-b(\sigma,h))-w(\sigma,h)\right]$$
$$+g(l|1)\left[(t+y(\sigma,l))(1-b(\sigma,l))-w(\sigma,l)\right],$$

subject to (3) with $s=h$ and $s=l$, as well as (6) and (7).

Truth-telling constraints (6) and (7) can be written as

$$\begin{aligned}(t+y(\sigma,l))(b(\sigma,l)-b(\sigma,h)) &\leq w(\sigma,h)-w(\sigma,l) \\ &\leq (t+y(\sigma,h))(b(\sigma,l)-b(\sigma,h)).\end{aligned} \tag{18}$$

Clearly, inequality (18) implies that $b(\sigma,l)\geq b(\sigma,h)$. Moreover, inequality (7) binds at optimum, because otherwise the principal can simply lower $w(\sigma,h)$. As a result, in the optimal menu, $w(\sigma,l)=0$, and the bonuses are set at the minimum level consistent with condition (3):

$$\begin{aligned}w(\sigma,l) &= 0, b(\sigma,l)=\frac{c}{y(\sigma,l)} \\ w(\sigma,h) &= (t+y(\sigma,l))\left(\frac{1}{y(\sigma,l)}-\frac{1}{y(\sigma,h)}\right)c;\ b(\sigma,h)=\frac{c}{y(\sigma,h)}.\end{aligned} \tag{19}$$

Because in the optimal S11 menu $w(\sigma,h)>0$, the associated profit denoted by $\pi^{S1}(\sigma)$ satisfies

$$\pi^{S1}(\sigma)<g(h|1)\left[(t+y(\sigma,h))\left(1-\frac{c}{y(\sigma,h)}\right)\right]+g(l|1)\left[(t+y(\sigma,l))\left(1-\frac{c}{y(\sigma,l)}\right)\right].$$

By using the above inequality and (5), we further have

$$\begin{aligned}\pi^{S1}(\sigma)-\pi^{P}(\sigma) &< g(h|1)(t+y(\sigma,h))\left[\frac{1}{g(h|1)y(\sigma,h)+g(l|1)y(\sigma,l)}-\frac{1}{y(\sigma,h)}\right]c \\ &\quad +g(l|1)(t+y(\sigma,l))\left[\frac{1}{g(h|1)y(\sigma,h)+g(l|1)y(\sigma,l)}-\frac{1}{y(\sigma,l)}\right]c \\ &= -t\frac{[y(\sigma,h)-y(\sigma,l)]^2}{y(\sigma,h)y(\sigma,l)}\frac{g(h|1)g(l|1)}{g(l|1)y(\sigma,l)+g(h|1)y(\sigma,h)}\leq 0.\end{aligned}$$

Q.E.D.

Proof of Proposition 1

First, we show that there exists a σ_0 such that $\pi^S(\sigma) \geq t$ if and only if $\sigma \geq \sigma_0$. To this end, note that if σ is sufficiently high such that $w(\sigma,h) = \frac{y(\sigma,l)}{y(\sigma,h)}(y(\sigma,h)-c) - t\frac{c}{y(\sigma,h)} > 0$,

$$\begin{aligned}\pi^S(\sigma) - t &= g(h|1)\left[-t\frac{c}{y(\sigma,h)} + y(\sigma,h) - c - w(\sigma,h)\right] \\ &= g(h|1)\left(1 - \frac{y(\sigma,l)}{y(\sigma,h)}\right)(y(\sigma,h)-c) \geq 0.\end{aligned}$$

If $w(\sigma,h) = 0$,

$$\pi^S(\sigma) - t = g(h|1)\left[-t\frac{c}{y(\sigma,h)} + y(\sigma,h) - c\right],$$

which is greater than 0 iff σ is sufficiently high.

Second, we show that $\pi^P(\sigma) - \pi^S(\sigma)$ is increasing in σ. Using (5) and (11), the difference can be expressed as

$$\begin{aligned}\pi^P(\sigma) - \pi^S(\sigma) &= g(h|1)\frac{-cg(l|1)(t+y(\sigma,h))}{(g(h|1)y(\sigma,h)+g(l|1)y(\sigma,l))}\left(1-\frac{y(\sigma,l)}{y(\sigma,h)}\right) + g(h|1)w(\sigma,h) \\ &+g(l|1)\left[(t+y(\sigma,l))\left(1-\frac{c}{g(h|1)y(\sigma,h)+g(l|1)y(\sigma,l)}\right)-t\right]. \end{aligned} \tag{20}$$

It is clear that both bracket terms above are increasing in σ. Moreover, $w(\sigma,h)$ is non-decreasing in σ. Therefore, $\pi^P(\sigma) - \pi^S(\sigma)$ is increasing in σ.

Finally, observe that $\pi^P(\sigma) - \pi^S(\sigma)$ is negative for $\sigma = \sigma_{\min}$ and positive for $\sigma = \sigma_{\max}$. Combined with the second observation above, there exists a cutoff σ^* such that $\pi^P(\sigma) - \pi^S(\sigma) \geq 0$ iff $\sigma \geq \sigma^*$. Q.E.D.

Proof of Corollary 1

It is clear from equation (20) that $\pi^P(\sigma) - \pi^S(\sigma)$ is decreasing in both c and t (note $w(\sigma,h)$ is non-increasing in c and t). Combined with the fact that $\pi^P(\sigma) - \pi^S(\sigma)$ is increasing in σ, an increase in c and/or t would lead to an increase in σ^*. Q.E.D.

Proof of Proposition 2

We have $\sigma^* < \sigma_l$ if and only if $\pi^S(\sigma_l) < \pi^P(\sigma_l)$. As $w(\sigma_l, h) = \max\left\{0, c\left(1 - \frac{c}{y(\sigma_l,h)}\right) - t\frac{c}{y(\sigma_l,h)}\right\}$, there are two cases. First, if $w(\sigma_l, h) = 0$, i.e., $t \geq y(\sigma_l, h) - c$, then

$$\pi^S(\sigma_l) - \pi^P(\sigma_l) = ctg(l|1)\frac{y(\sigma_l, h) + g(h|1)(y(\sigma_l, h) - c)}{y(\sigma_l, h)[c + g(h|1)(y(\sigma_l, h) - c)]} > 0.$$

Thus, we have $\sigma^* > \sigma_l$ in this case. Second, if $w(\sigma_l, h) > 0$, i.e., $t \leq y(\sigma_l, h) - c$, then

$$\pi^S(\sigma_l) - \pi^P(\sigma_l) = \left[-g(h|1)\left(1 - \frac{c}{y(\sigma_l, h)}\right) + t\frac{1}{g(h|1)\, y(\sigma_l, h) + g(l|1)\, c}\right]c,$$

which is is negative iff condition (12) holds. It is straightforward that condition (12) implies $t \leq y(\sigma_l, h) - c$. Q.E.D.

Proof of Lemma 3

Consider an optimal contract Γ and an associated incentive-compatible strategy profile $\tau = (\alpha, \beta)$. Fix an objective signal σ. Define $M_h \equiv \{m \in M : \beta_{(\sigma,h)}(m) > 0 \text{ and } \beta_{(\sigma,l)}(m) = 0\}$, $M_l \equiv \{m \in M : \beta_{(\sigma,l)}(m) > 0 \text{ and } \beta_{(\sigma,h)}(m) = 0\}$, and $M_\mu \equiv \{m \in M : \beta_{(\sigma,h)}(m) > 0 \text{ and } \beta_{(\sigma,l)}(m) > 0\}$.

First, we show the following claim: if $m, m' \in M$ are such that $\beta_{(\sigma,s)}(m), \beta_{(\sigma,s)}(m') > 0$, then it is necessary that both schemes $(w(\sigma, m), b(\sigma, m))$ and $(w(\sigma, m'), b(\sigma, m'))$ induce the same effort e_2. Suppose not, say scheme $(w(\sigma, m), b(\sigma, m))$ induces $e_2 = 1$ whereas $(w(\sigma, m'), b(\sigma, m'))$ induces $e_2 = 0$. The two schemes must yield the same expected profit to the principal, so they must equal to $-w(\sigma, m') \leq 0$. It is without loss to assume that $b(\sigma, m) = 1$ and $w(\sigma, m) = w(\sigma, m')$, as such scheme brings profit $-w(\sigma, m')$ to the principal and induces second-stage effort.[26] Depending on how β mixes between messages m and m', there are 6 cases to consider.

Case 1: $\beta_{(\sigma,s')}(m) = \beta_{(\sigma,s')}(m') = 0$ for $s' \neq s$

If increasing agency rent at (σ, s) helps relax the first-period effort IC, then setting $\beta'_{(\sigma,s)}(m) = \beta_{(\sigma,s)}(m) + \beta_{(\sigma,s)}(m')$ and $\beta'_{(\sigma,s)}(m') = 0$ would leave the principal's payoff unchanged while relaxing the first-period effort IC.

If decreasing agency rent at (σ, s) helps relax the first-period effort IC, then setting $\beta'_{(\sigma,s)}(m') =$

[26] Any other scheme that brings profit $-w(\sigma, m')$ to the principal and induces second-stage effort yields the same second-stage rent to the agent.

$\beta_{(\sigma,s)}(m) + \beta_{(\sigma,s)}(m')$ and $\beta'_{(\sigma,s)}(m) = 0$ would leave the principal's payoff unchanged while relaxing the first-period effort IC.

Case 2: $\beta_{(\sigma,l)}(m) = 0$, and $\beta_{(\sigma,l)}(m'), \beta_{(\sigma,h)}(m') > 0$

If increasing agency rent at (σ, h) helps relax the first-period effort IC, then setting $\beta'_{(\sigma,h)}(m) = \beta_{(\sigma,h)}(m) + \beta_{(\sigma,h)}(m')$ and $\beta'_{(\sigma,h)}(m') = 0$ would leave the principal's payoff unchanged while relaxing the first-period effort IC.

If decreasing agency rent at (σ, h) helps relax the first-period effort IC, then setting $\beta'_{(\sigma,h)}(m') = \beta_{(\sigma,h)}(m) + \beta_{(\sigma,h)}(m')$ and $\beta'_{(\sigma,h)}(m) = 0$ would leave the principal's payoff unchanged while relaxing the first-period effort IC.

Case 3: $\beta_{(\sigma,l)}(m') = 0$, and $\beta_{(\sigma,l)}(m), \beta_{(\sigma,h)}(m) > 0$

If increasing agency rent at (σ, h) helps relax the first-period effort IC, then setting $\beta'_{(\sigma,h)}(m) = \beta_{(\sigma,h)}(m) + \beta_{(\sigma,h)}(m')$ and $\beta'_{(\sigma,h)}(m') = 0$ would leave the principal's payoff unchanged while relaxing the first-period effort IC.

If decreasing agency rent at (σ, h) helps relax the first-period effort IC, then decreasing agency rent at (σ, l) would also help relax the first-period effort IC. Now setting $\beta'_{(\sigma,s)}(m') = \beta_{(\sigma,s)}(m) + \beta_{(\sigma,s)}(m')$ and $\beta'_{(\sigma,s)}(m) = 0$ for $s = h, l$, would leave the principal's payoff unchanged while relaxing the first-period effort IC.

Case 4: $\beta_{(\sigma,h)}(m') = 0$, and $\beta_{(\sigma,l)}(m), \beta_{(\sigma,h)}(m) > 0$

If decreasing agency rent at (σ, l) helps relax the first-period effort IC, then setting $\beta'_{(\sigma,l)}(m') = \beta_{(\sigma,l)}(m) + \beta_{(\sigma,l)}(m')$ and $\beta'_{(\sigma,l)}(m) = 0$ would leave the principal's payoff unchanged while relaxing the first-period effort IC.

If increasing agency rent at (σ, l) helps relax the first-period effort IC, then increasing agency rent at (σ, h) would also help relax the first-period effort IC. There are two possibilities: (i) $\mu_\sigma y \geq c$, and (ii) $\mu_\sigma y < c$. In possibility (i), setting $\beta'_{(\sigma,l)}(m) = \beta_{(\sigma,l)}(m) + \beta_{(\sigma,l)}(m')$ would leave the principal's payoff unchanged while relaxing the first-period effort IC. In possibility (ii), $\mu_\sigma y < c$ implies $\mu_{\sigma l} y < c$. Therefore, the agency rent at (σ, l) is increased by setting $\beta'_{(\sigma,l)}(m') = \beta_{(\sigma,l)}(m) + \beta_{(\sigma,l)}(m')$.

Case 5: $\beta_{(\sigma,h)}(m) = 0$, and $\beta_{(\sigma,l)}(m'), \beta_{(\sigma,h)}(m') > 0$

If increasing agency rent at (σ, l) helps relax the first-period effort IC, then setting $\beta'_{(\sigma,l)}(m) = \beta_{(\sigma,l)}(m) + \beta_{(\sigma,l)}(m')$ and $\beta'_{(\sigma,l)}(m') = 0$ would leave the principal's payoff unchanged while relaxing the first-period effort IC.

If decreasing agency rent at (σ, l) helps relax the first-period effort IC, then setting $\beta'_{(\sigma,l)}(m') = \beta_{(\sigma,l)}(m) + \beta_{(\sigma,l)}(m')$ and $\beta'_{(\sigma,l)}(m) = 0$ would leave the principal's payoff unchanged while relaxing the first-period effort IC.

Case 6: $\beta_{(\sigma,h)}(m), \beta_{(\sigma,l)}(m), \beta_{(\sigma,h)}(m'), \beta_{(\sigma,l)}(m') > 0$

If increasing agency rent at (σ, h) helps relax the first-period effort IC, then setting $\beta'_{(\sigma,h)}(m) = \beta_{(\sigma,h)}(m) + \beta_{(\sigma,h)}(m')$ and $\beta'_{(\sigma,h)}(m') = 0$ would leave the principal's payoff unchanged while relaxing the first-period effort IC.

If decreasing agency rent at (σ, h) helps relax the first-period effort IC, then decreasing agency rent at (σ, l) also helps relax the first-period effort IC. Now set $\beta'_{(\sigma,s)}(m') = \beta_{(\sigma,s)}(m) + \beta_{(\sigma,s)}(m')$ and $\beta'_{(\sigma,s)}(m) = 0$ for $s = h, l$ would leave the principal's payoff unchanged while relaxing the first-period effort IC.

If increasing agency rent at (σ, l) helps relax the first-period effort IC, then increasing agency rent at (σ, h) helps relax the first-period effort IC. There are two possibilities: (i) $\mu_\sigma y \geq c$, and (ii) $\mu_\sigma y < c$. In possibility (i), setting $\beta'_{(\sigma,l)}(m) = \beta_{(\sigma,l)}(m) + \beta_{(\sigma,l)}(m')$ would leave the principal's payoff unchanged while relaxing the first-period effort IC. In possibility (ii), $\mu_\sigma y < c$ implies $\mu_{\sigma l} y < c$. Therefore, the agency rent at (σ, l) is increased by setting $\beta'_{(\sigma,l)}(m') = \beta_{(\sigma,l)}(m) + \beta_{(\sigma,l)}(m')$.

If decreasing agency rent at (σ, l) helps relax the first-period effort IC, then setting $\beta'_{(\sigma,l)}(m') = \beta_{(\sigma,l)}(m) + \beta_{(\sigma,l)}(m')$ and $\beta'_{(\sigma,l)}(m) = 0$ would leave the principal's payoff unchanged while relaxing the first-period effort IC.

This claim directly implies that it is without loss to suppose $|M_h|, |M_l|, |M_\mu| \leq 1$. This is because conditional on signal profile (σ, s), the profits, surpluses and rent for sending each message on the respective support is identical. Putting all the weights on $\arg\min_m \Pr(x_1 = 1|\beta)$ of each respective message space would ensure that the second-stage effort IC is respected. Moreover, the claim implies that if $\beta_{(\sigma,s)}(m), \beta_{(\sigma,s)}(m') > 0$, then transferring weight between m and m' has no impact on principal's profit and agent's rent, provided that the second-stage effort IC is preserved. To ensure effort IC is satisfied, one can replace β with a pure strategy β' such that if $\beta'_{(\sigma,s)}(m'') > 0$ for some $s \in \{h, l\}$ and $m'' \in \{m, m'\}$, the agent is more optimistic (about x_1) with m'' under β' than β. Specifically, consider two cases: (i) $|M_l| = 1$; and (ii) $|M_l| = 0$.

In case (i), if $|M_\mu| = 0$, then $|M_h| = 1$ and we are done. Suppose not. Then define a new incentive scheme as follows. Remove M_h altogether, set $M'_l = M_l$, and $M'_h = M_\mu$. The removal of some messages removes some truth-telling ICs for the principal. Moreover, the IC for the agent to exert effort is respected, as his belief about being $x_1 = 1$ goes up upon receiving M'_h.

In case (ii), we must have $|M_\mu| = 1$. Now remove M_h and set $\beta'_{(\sigma,h)}(M_\mu) = 1$. The agent's belief that $x_1 = 1$ upon receiving M_μ increases, so the IC for exerting effort is still respected. Q.E.D.

Proof of Proposition 3

To ease presentation, we denote by ϕ the Lagrangian multiplier associated with the first-stage IC constraint, (15), in the principal's problem. Define

$$\hat{\sigma} \equiv \begin{cases} k_h\left(\frac{\phi}{\phi-1}\right) & \text{if } \phi > 1, \\ \sigma_{\max} + 1 & \text{if } \phi \leq 1. \end{cases} \tag{21}$$

We first prove a sequence of claims on the payments given that a certain type of menu is offered contingent on σ.

Claim 1 *(i) Suppose that menu P1 is offered contingent on σ. Its payments are characterized by $w(\sigma) = 0$, and*

$$b(\sigma) = \begin{cases} \frac{c}{g(h|1)y(\sigma,h)+g(l|1)y(\sigma,l)} & \text{if } \phi \leq 1, \\ \frac{c}{g(h|0)y(\sigma,h)+g(l|0)y(\sigma,l)} & \text{if } 1 < \phi < Z(\sigma), \end{cases}$$

and $b(\sigma) \geq \frac{c}{g(h|0)y(\sigma,h)+g(l|0)y(\sigma,l)}$ if $\sigma = \arg\min_{\sigma' \in \Omega: Z(\sigma')>0} Z(\sigma')$.

(ii) The Lagrangian multiplier on the first-stage IC constraint is bounded from above by

$$\phi \leq \min_{\sigma' \in \Omega: Z(\sigma')>0} Z(\sigma'). \tag{22}$$

Proof. Given $\rho_{P1}(\sigma) > 0$, the principal chooses the payments in menu P1 to solve:

$$\max \Pi_{P1}(\sigma) + \phi\left[C_{P1}(\sigma) - \frac{\psi}{f(\sigma|1)}\right], \tag{23}$$

subject to (2).

First, it is without loss to set $w(\sigma) = 0$. Suppose not. One can lower $w(\sigma)$ by ε and raise $b(\sigma)$ by $\frac{\varepsilon}{g(h|1)y(\sigma,h)+g(l|1)y(\sigma,l)}$. This keeps all relevant constraints unchanged, but increases the objective given $\phi > 0$.

Second, the coefficient of $b(\sigma)$ in the expression (23) is

$$\begin{cases} f(\sigma|1)\left[g(h|1)\,y(\sigma,h)+g(l|1)\,y(\sigma,l)\right](\phi-1) & \text{if } b(\sigma)\leq \frac{c}{g(h|0)y(\sigma,h)+g(l|0)y(\sigma,l)}, \\ f(\sigma|1)\left[g(h|1)\left(1-\frac{k_h}{\sigma}\right)y(\sigma,h)+g(l|1)\left(1-\frac{k_l}{\sigma}\right)y(\sigma,l)\right](\phi-Z(\sigma)) & \text{if } b(\sigma)> \frac{c}{g(h|0)y(\sigma,h)+g(l|0)y(\sigma,l)}. \end{cases}$$

If $\phi\leq 1$, then the optimal $b(\sigma)$ is set at its minimum level $\frac{c}{g(h|1)y(\sigma,h)+g(l|1)y(\sigma,l)}$. If $1<\phi<Z(\sigma)$, then the optimal $b(\sigma)$ is set at $\frac{c}{g(h|0)y(\sigma,h)+g(l|0)y(\sigma,l)}$. If $1<\phi=Z(\sigma)$, then $b(\sigma)\geq \frac{c}{g(h|0)y(\sigma,h)+g(l|0)y(\sigma,l)}$ can be optimal.

Third, it holds that $\phi\leq \min\limits_{\sigma'\in\Omega:Z(\sigma')>0} Z(\sigma')$. Suppose that there exists a σ such that $Z(\sigma)<\phi$. In this case, the principal will set $\rho_{P1}(\sigma)>0$ and increases $b(\sigma)$ in menu P1 to infinity. The Lagrangian multiplier ϕ can be interpreted as the principal's loss in response to one unit increase in ψ. If increasing $C_{P1}(\sigma)$ by one unit, $\Pi_{P1}(\sigma)$ is at most reduced by $Z(\sigma)$. To minimize the loss in $\Pi_{P1}(\sigma)$, the principal will pick a σ that minimizes $Z(\sigma)$. ■

Claim 2 *Suppose that menu S10 is offered contingent on σ. Its payments are characterized by*

$$\begin{aligned} b(\sigma,h) &= \frac{c}{y(\sigma,h)}, b(\sigma,l)=0, w(\sigma,l)=0, \\ w(\sigma,h) &= \begin{cases} y(\sigma,h)-c, \text{ if } \sigma>\widehat{\sigma}; \\ y(\sigma,l)\left(1-\frac{c}{y(\sigma,h)}\right), \text{ if } \sigma\leq\widehat{\sigma}. \end{cases} \end{aligned}$$

Proof. Given $\rho_{S10}(\sigma)>0$, the principal chooses the payments in menu S10 to solve:

$$\max \Pi_{S10}(\sigma)+\phi\left[C_{S10}(\sigma)-\frac{\psi}{f(\sigma|1)}\right], \tag{24}$$

subject to (3) with $s=h$, (8), and (9).

First, it is optimal to set $b(\sigma,l)=0$.

Second, it is without loss to set $b(\sigma,h)=\frac{c}{y(\sigma,h)}$. Suppose not. One can lower $b(\sigma,h)$ by ε and raise $w(\sigma,h)$ by $y(\sigma,h)\varepsilon$. This relaxes (9), but leaves the objective and other constraints unchanged.

Third, using the observations above,

$$
\begin{aligned}
& \Pi_{S10}(\sigma) + \phi \left[C_{S10}(\sigma) - \frac{\psi}{f(\sigma|1)} \right] \\
= \; & g(h|1)(y(\sigma,h) - c) - \phi \frac{\psi}{f(\sigma|1)} \\
& - \left[1 - \phi \left(1 - \frac{k_h}{\sigma} \right) \right] g(h|1)\, w(\sigma,h) - \left[1 - \phi \left(1 - \frac{k_l}{\sigma} \right) \right] g(l|1)\, w(\sigma,l). \qquad (25)
\end{aligned}
$$

The coefficient in front of $w(\sigma,h)$ is positive if and only if $\sigma \geq \hat{\sigma}$. Thus, we set $w(\sigma,h)$ to meet the upper bound imposed by the constraint (8), if $\sigma > \widehat{\sigma}$; otherwise, we set $w(\sigma,h)$ to meet the lower bound imposed by the constraint (9).

Fourth, substituting $w(\sigma,h)$ in (25) by using

$$
w(\sigma,h) = \begin{cases} y(\sigma,h) - c + w(\sigma,l), \text{ if } \sigma > \widehat{\sigma}, \\ y(\sigma,l)\left(1 - \frac{c}{y(\sigma,h)}\right) + w(\sigma,l), \text{ if } \sigma \leq \widehat{\sigma}, \end{cases}
$$

the coefficient of $w(\sigma,l)$ in the expression (24) is

$$
-f(\sigma|1)\left[1 - \phi \left(1 - \frac{1}{\sigma} \right) \right].
$$

If $\left(1 - \frac{1}{\sigma}\right) \leq 0$, the above term is negative. If $\left(1 - \frac{1}{\sigma}\right) > 0$,

$$
\begin{aligned}
& -1 + \phi \left(1 - \frac{1}{\sigma} \right) \leq -1 + Z(\sigma) \left(1 - \frac{1}{\sigma} \right) \\
= & -\left(\frac{k_l - k_h}{\sigma} \right) \frac{[y(\sigma,h) - y(\sigma,l)]\, g(l|1)\, g(h|1)}{g(h|1)\left(1 - \frac{k_h}{\sigma}\right) y(\sigma,h) + g(l|1)\left(1 - \frac{k_l}{\sigma}\right) y(\sigma,l)} < 0.
\end{aligned}
$$

Therefore, it is optimal to set $w(\sigma,l) = 0$ in menu S10. ■

Claim 3 *Suppose that menu S11 is offered contingent on σ. Its payments are characterized by*

$$
\begin{aligned}
b(\sigma,h) &= \frac{c}{y(\sigma,h)}, b(\sigma,l) \geq \frac{c}{y(\sigma,l)}, w(\sigma,l) = 0, \\
w(\sigma,h) &= \begin{cases} y(\sigma,h)\left(b(\sigma,l) - \frac{c}{y(\sigma,h)}\right), \text{ if } \sigma > \widehat{\sigma}; \\ y(\sigma,l)\left(b(\sigma,l) - \frac{c}{y(\sigma,h)}\right), \text{ if } \sigma \leq \widehat{\sigma}. \end{cases}
\end{aligned}
$$

Moreover, if $\sigma \leq \widehat{\sigma}$, $b(\sigma,l) = \frac{c}{y(\sigma,l)}$.

Proof. Given $\rho_{S11}(\sigma) > 0$, the principal chooses the payments in menu S11 to solve:

$$\max \Pi_{S11}(\sigma) + \phi \left[C_{S11}(\sigma) - \frac{\psi}{f(\sigma|1)} \right], \tag{26}$$

subject to (3) with $s = h$ and $s = l$, (6), and (7).

First, it is without loss to set $b(\sigma, h) = \frac{c}{y(\sigma,h)}$. Suppose not. One can lower $b(\sigma, h)$ by ε and raise $w(\sigma, h)$ by $y(\sigma, h)\varepsilon$. This relaxes (7), but leaves the objective and other constraints unchanged.

Second, it is without loss of set $w(\sigma, l) = 0$. Suppose not. One can raise $b(\sigma, l)$ by ε and lower $w(\sigma, l)$ by $y(\sigma, l)\varepsilon$. This relaxes (6), but leaves the objective and other constraints unchanged.

Third, we set $w(\sigma, h)$ to meet the upper bound imposed by the constraint (6), if (26) is increasing in $w(\sigma, h)$, which is equivalent to $\sigma > \widehat{\sigma}$; otherwise, we set $w(\sigma, h)$ to meet the lower bound imposed by the constraint (7).

Fourth, if $\sigma \leq \hat{\sigma}$, substituting $w(\sigma, h)$ in (26) by using $w(\sigma, h) = y(\sigma, l)\left(b(\sigma, l) - \frac{c}{y(\sigma,h)}\right)$, the coefficient of $b(\sigma, l)$ in (24) is

$$f(\sigma|1)\, y(\sigma, l) \left[-1 + \phi \left(1 - \frac{1}{\sigma} \right) \right] < 0. \tag{27}$$

Thus, it is optimal to set $b(\sigma, l) = \frac{c}{y(\sigma,h)}$ in menu S11. ■

Claim 4 *Suppose that menu S01 is offered contingent on σ. Its payments are characterized by*

$$\begin{aligned} b(\sigma, h) &= 0, w(\sigma, l) = 0, b(\sigma, l) \geq \max \left\{ \frac{c}{y(\sigma, l)}, 1 \right\}, \\ w(\sigma, h) &= \begin{cases} y(\sigma, h)(b(\sigma, l) - 1), & \text{if } \sigma > \widehat{\sigma}; \\ y(\sigma, l)(b(\sigma, l) - 1), & \text{if } \sigma \leq \widehat{\sigma}. \end{cases} \end{aligned}$$

Moreover, $b(\sigma, l) = \max\left\{\frac{c}{y(\sigma,l)}, 1\right\}$ *except for* $\sigma = \arg\min_{\sigma' \in \Omega : Z(\sigma') > 0} Z(\sigma')$.

Proof. Given $\rho_{S01}(\sigma) > 0$, the principal chooses the payments in menu S01 to solve:

$$\max \Pi_{S01}(\sigma) + \phi \left[C_{S01}(\sigma) - \frac{\psi}{f(\sigma|1)} \right], \tag{28}$$

subject to (3) with $s = l$, and (16) and (17).

First, it is optimal to set $w(\sigma, l) = b(\sigma, h) = 0$.

Second, combining (16) and (17) implies that

$$y(\sigma, h)(1 - b(\sigma, l)) \leq y(\sigma, l)(1 - b(\sigma, l)). \tag{29}$$

Because $y(\sigma, h) \geq y(\sigma, l)$, the inequality above implies that $1 - b(\sigma, l) \leq 0$. Thus, if $\sigma \geq \sigma_l$, then $b(\sigma, l) \geq 1$; if $\sigma < \sigma_l$, then $b(\sigma, l) \geq \frac{c}{y(\sigma,l)}$.

Third, we set $w(\sigma, h)$ to meet the upper bound imposed by the constraint (16) if (28) is increasing in $w(\sigma, h)$ or $\sigma > \widehat{\sigma}$; otherwise, we set $w(\sigma, h)$ to meet the lower bound imposed by the constraint (17).

Fourth, we substitute $w(\sigma, h)$ in (28) by using

$$w(\sigma, h) = \begin{cases} y(\sigma, h)(b(\sigma, l) - 1), \text{ if } \sigma > \widehat{\sigma}; \\ y(\sigma, l)(b(\sigma, l) - 1), \text{ if } \sigma \leq \widehat{\sigma}. \end{cases}$$

If $\sigma < \widehat{\sigma}$, the coefficient in front of $b(\sigma, l)$ in (28) is

$$f(\sigma|1)\, y(\sigma, l) \left[-1 + \phi\left(1 - \frac{1}{\sigma}\right)\right] < 0.$$

Thus, it is optimal to set $b(\sigma, l) = \max\left\{\frac{c}{y(\sigma,l)}, 1\right\}$. If $\sigma > \hat{\sigma}$, the coefficient in front of $b(\sigma, l)$ in (28) is

$$f(\sigma|1)\left[g(h|1)\left(1 - \frac{k_h}{\sigma}\right) y(\sigma, h) + g(l|1)\left(1 - \frac{k_l}{\sigma}\right) y(\sigma, l)\right](\phi - Z(\sigma)).$$

Thus, it is optimal to set $b(\sigma, l) = \max\left\{\frac{c}{y(\sigma,l)}, 1\right\}$ if $Z(\sigma) > \phi$, and set $b(\sigma, l) \geq \max\left\{\frac{c}{y(\sigma,l)}, 1\right\}$ if $Z(\sigma) = \phi$. ■

Define the following functions.

$$V_i(\sigma) \equiv \Pi_i(\sigma) + C_i(\sigma)\,\phi \text{ for } i \in \{\text{P0, P1, S10, S01, S11}\}.$$

If $V_i(\sigma) \geq \max\{V_j(\sigma), 0\}$ for any $j \neq i$, then $\rho_i(\sigma) > 0$; if $V_i(\sigma) > \max\{V_j(\sigma), 0\}$ for any $j \neq i$, then $\rho_i(\sigma) = 1$; in other cases, $\rho_i(\sigma) = 0$. Thus, to determine $\rho_i(\sigma)$, it is essential to compare $V_i(\sigma)$ for $i \in \{\text{P0, P1, S10, S01, S11}\}$ given σ.

The next two claims state that menus S01 and S11 are never used in the optimal contract.

Claim 5 *In the optimal contract, menu S01 is not used for any σ.*

Proof. (i) Consider $\sigma \leq \widehat{\sigma}$. Using Claim 4, we calculate $V_{S01}(\sigma)$ for menu S01. The superscript B denotes the case of $\sigma \leq \widehat{\sigma}$.

$$\begin{aligned} V_{S01}^{B}(\sigma) &= \Pi_{S01}^{B}(\sigma) + C_{S01}^{B}(\sigma)\phi \\ &= \left[-1 + \left(1 - \frac{1}{\sigma}\right)\phi\right] y(\sigma,l) \max\left\{\frac{c}{y(\sigma,l)}, 1\right\} + y(\sigma,l) \\ &\quad -\phi\left[g(h|1)\left(1 - \frac{k_h}{\sigma}\right) y(\sigma,l) + g(l|1)\left(1 - \frac{k_l}{\sigma}\right) c\right]. \end{aligned}$$

If $y(\sigma,l) < c$, then

$$V_{S01}^{B}(\sigma) = [y(\sigma,l) - c]\left[1 - g(h|1)\left(1 - \frac{k_h}{\sigma}\right)\phi\right] \leq 0.$$

The inequality above follows because

$$\phi\left(1 - \frac{k_h}{\sigma}\right) \leq \phi\left(1 - \frac{k_h}{\hat{\sigma}}\right) = \phi\left(1 - \frac{k_h}{k_h\left(\frac{\phi}{\phi-1}\right)}\right) = 1.$$

If $y(\sigma,l) \geq c$, then

$$V_{S01}^{B}(\sigma) = \phi g(l|1)\left(1 - \frac{k_l}{\sigma}\right)[y(\sigma,l) - c].$$

Obviously, if $1 - \frac{k_l}{\sigma} < 0$, $V_{S01}^{B}(\sigma) < 0$; if $1 - \frac{k_l}{\sigma} \geq 0$,

$$V_{S01}^{B}(\sigma) \leq g(h|1)\left(1 - \frac{k_h}{\sigma}\right)[y(\sigma,h) - c]\phi + g(l|1)\left(1 - \frac{k_l}{\sigma}\right)[y(\sigma,l) - c]\phi = V_{P1}(\sigma)|_{b(\sigma)=1},$$

which implies that such a S01 menu is dominated by a P1 menu with $b(\sigma) = 1$.

(ii) Consider $\sigma > \hat{\sigma}$. The superscript A denotes the case of $\sigma > \hat{\sigma}$.

$$\begin{aligned} V_{S01}^{A}(\sigma) &= \Pi_{S01}^{A}(\sigma) + C_{S01}^{A}(\sigma)\phi \\ &= \left[-g(h|1)\,y(\sigma,h) - g(l|1)\,y(\sigma,l) + \phi g(h|1)\left(1 - \frac{k_h}{\sigma}\right) y(\sigma,h)\right]\left(\max\left\{\frac{c}{y(\sigma,l)}, 1\right\} - 1\right) \\ &\quad +\phi g(l|1)\left(1 - \frac{k_l}{\sigma}\right)\left[y(\sigma,l)\max\left\{\frac{c}{y(\sigma,l)}, 1\right\} - c\right] \end{aligned}$$

If $y(\sigma,l) \geq c$, then

$$V_{S01}^{A}(\sigma) = \phi g(l|1)\left(1 - \frac{k_l}{\sigma}\right)[y(\sigma,l) - c].$$

It has been shown above that such a menu is dominated either by a P0 or P1 menu.

If $y(\sigma, l) < c$, then

$$V_{S01}^{A}(\sigma) = \left[-g(h|1)\, y(\sigma, h) - g(l|1)\, y(\sigma, l) + \phi g(h|1)\left(1 - \frac{k_h}{\sigma}\right) y(\sigma, h)\right]\left(\frac{c}{y(\sigma, l)} - 1\right) \tag{30}$$

Given the same σ, if we construct a P1 menu with $b(\sigma) = \frac{c}{y(\sigma,l)}$, the associated $V_{P1}(\sigma)$ is

$$\begin{aligned} V_{P1}(\sigma)|_{b(\sigma)=\frac{c}{y(\sigma,l)}} &= [g(h|1)\, y(\sigma, h) + g(l|1)\, y(\sigma, l)]\left(1 - \frac{c}{y(\sigma, l)}\right) \\ &\quad +\phi\left(\left[g(h|1)\left(1 - \frac{k_h}{\sigma}\right) y(\sigma, h) + g(l|1)\left(1 - \frac{k_l}{\sigma}\right) y(\sigma, l)\right]\frac{c}{y(\sigma, l)} - \left(1 + \frac{1}{\sigma}\right) c\right) \end{aligned}$$

We further find that

$$V_{S01}^{A}(\sigma) - V_{P1}(\sigma)|_{b(\sigma)=\frac{c}{y(\sigma,l)}} = -\phi g(h|1)\left(1 - \frac{k_h}{\sigma}\right)[y(\sigma, h) - c] \leq 0.$$

The inequality above follows because $\sigma > \hat{\sigma} \geq k_h$, and $y(\sigma_{\min}, h) \geq c$.

In sum, menu S01 is never optimal for any σ. ■

Claim 6 *In the optimal contract, menu S11 is not used for any σ.*

Proof. (i) Consider $\sigma \leq \hat{\sigma}$. Using Claim 3, we calculate $V_{S11}(\sigma)$ for menu S11.

$$\begin{aligned} V_{S11}(\sigma) &= g(h|1)[y(\sigma, h) - c] + g(l|1)[y(\sigma, l) - c] - \left[1 - \phi\left(1 - \frac{k_h}{\sigma}\right)\right] g(h|1)\left(1 - \frac{y(\sigma, l)}{y(\sigma, h)}\right) c \\ &< g(h|1)[y(\sigma, h) - c] + g(l|1)[y(\sigma, l) - c] = V_{P1}(\sigma). \end{aligned}$$

The inequality above holds because $\left[1 - \phi\left(1 - \frac{k_h}{\sigma}\right)\right] \geq 0$ ($\Leftrightarrow \sigma \leq \hat{\sigma}$).

(ii) Consider $\sigma > \hat{\sigma}$. If offering a S11 menu contingent on σ, the principal obtains a profit,

$$\Pi_{S11}(\sigma) = g(h|1)\, y(\sigma, h)(1 - b(\sigma, l)) + g(l|1)\, y(\sigma, l)(1 - b(\sigma, l));$$

using this menu raises the LHS of (15) by

$$\begin{aligned} f(\sigma|1)\, C_{S11}(\sigma) &= f(\sigma|1)\, g(h|1)\left(1 - \frac{k_h}{\sigma}\right)[y(\sigma, h)\, b(\sigma, l) - c] \\ &\quad +f(\sigma|1)\, g(l|1)\left(1 - \frac{k_l}{\sigma}\right)[y(\sigma, l)\, b(\sigma, l) - c]. \end{aligned}$$

Given the same σ, if the principal offers a P1 menu with $b(\sigma)$ equal to $b(\sigma, l)$ used in the above S11 menu, she obtains the same profit, and relaxes (15) by the same amount. This suggests that for any $\sigma \in [\widehat{\sigma}, \sigma_{\max}]$, a S11 menu can be replaced by a P1 menu. ■

This concludes the proof of Proposition 3. Q.E.D.

Proof of Proposition 4

Using Claim 2, we calculate $V_{S10}(\sigma)$ for menu S10.

$$V_{S10}(\sigma) = \begin{cases} V_{S10}^{A}(\sigma) \equiv \phi g(h|1)\left(1 - \frac{k_h}{\sigma}\right)[y(\sigma, h) - c] & \text{if } \sigma > \hat{\sigma}; \\ V_{S10}^{B}(\sigma) \equiv g(h|1)\left[1 - \left(1 - \phi\left(1 - \frac{k_h}{\sigma}\right)\right)\frac{y(\sigma,l)}{y(\sigma,h)}\right](y(\sigma, h) - c) & \text{if } \sigma \leq \hat{\sigma}. \end{cases}$$

Straightforward algebra gives:

$$V_{S10}(\sigma) > 0 \Leftrightarrow k_h < \sigma_{\min}.$$

Thus, if k_h is sufficiently small, menu S10 dominates menu P0 for all σ.

Using Claim 1, we calculate $V_{P1}(\sigma)$ for menu P1. If $\phi \leq 1$,

$$V_{P1}(\sigma) = V_{P1}^{A}(\sigma) \equiv [g(h|1)\, y(\sigma, h) + g(l|1)\, y(\sigma, l)] - c;$$

if $\phi > 1$,

$$\begin{aligned} V_{P1}(\sigma) &= V_{P1}^{B}(\sigma) \equiv [g(h|1)\, y(\sigma, h) + g(l|1)\, y(\sigma, l)]\left(1 - \frac{c}{g(h|0)\, y(\sigma, h) + g(l|0)\, y(\sigma, l)}\right) \\ &\quad + \phi\left(\frac{g(h|1)(1 - k_h)\, y(\sigma, h) + g(l|1)(1 - k_l)\, y(\sigma, l)}{g(h|0)\, y(\sigma, h) + g(l|0)\, y(\sigma, l)}\right) c. \end{aligned}$$

In what follows, we compare $V_{P1}(\sigma)$ with $V_{S10}(\sigma)$ in two cases: (i) $\phi \leq 1$ and (ii) $\phi > 1$.

Claim 7 *Suppose that $\phi \leq 1$. There exists a cutoff σ_1^* such that menu S10 is chosen for $\sigma \leq \sigma_1^*$ and menu P1 is chosen otherwise. The cutoff σ_1^* increases with ϕ, but is lower than σ_l.*

Proof. Because $\phi \leq 1$, $V_{S10}(\sigma) = V_{S10}^{B}(\sigma)$ and $V_{P1}(\sigma) = V_{P1}^{A}(\sigma)$. The difference is given by

$$V_{P1}^{A}(\sigma) - V_{S10}^{B}(\sigma) = g(l|1)[y(\sigma, l) - c] + g(h|1)\underbrace{\left[1 - \phi\left(1 - \frac{k_h}{\sigma}\right)\right]}_{(+)}\frac{y(\sigma, l)}{y(\sigma, h)}(y(\sigma, h) - c).$$

We derive the following derivative:

$$
\begin{aligned}
& \frac{\partial \left(V_{P1}^{A}(\sigma) - V_{S10}^{B}(\sigma)\right)}{\partial \sigma} \\
= \; & g(l|1) \frac{\partial y(\sigma, l)}{\partial \sigma} + g(h|1) \left[1 - \phi \left(1 - \frac{k_h}{\sigma}\right)\right] \frac{\partial}{\partial \sigma} \left[\frac{y(\sigma, l)}{y(\sigma, h)} (y(\sigma, h) - c)\right] \\
& - g(h|1) \frac{y(\sigma, l)}{y(\sigma, h)} (y(\sigma, h) - c) \frac{\phi k_h}{\sigma^2} \\
\to \; & g(l|1) \frac{\partial y(\sigma, l)}{\partial \sigma} + g(h|1)(1 - \phi) \frac{\partial}{\partial \sigma} \left[\frac{y(\sigma, l)}{y(\sigma, h)} (y(\sigma, h) - c)\right] \text{ as } k_h \to 0.
\end{aligned}
$$

The limit of the above derivative is strictly positive. It suggests that there exists a $\underline{k}_h > 0$ such that for all $k_h < \underline{k}_h$, $V_{P1}^{A}(\sigma) - V_{S10}^{B}(\sigma)$ is increasing in σ.

We further derive that

$$
\frac{\partial \left(V_{P1}^{A}(\sigma) - V_{S10}^{B}(\sigma)\right)}{\partial \phi} = -g(h|1) \left(1 - \frac{k_h}{\sigma}\right) \frac{y(\sigma, l)}{y(\sigma, h)} (y(\sigma, h) - c).
$$

Given that k_h is sufficiently small, the above derivative is strictly negative because $y(\sigma, h) \geq y(\sigma_{\min}, h) > c$. This implies that σ_1^* is increasing in ϕ.

Finally, it is true that $\sigma_1^* \leq \sigma_l$ for $\phi \leq 1$. This is because when $\phi = 1$ and $\sigma = \sigma_l$,

$$
V_{P1}^{A}(\sigma) - V_{S10}^{B}(\sigma) = g(h|1) \frac{k_h}{\sigma} \frac{y(\sigma, l)}{y(\sigma, h)} (y(\sigma, h) - c) > 0.
$$

■

Claim 8 *Suppose that $\phi > 1$. When ϕ is sufficiently close to 1, there exists a cutoff σ_1^* such that menu S10 is chosen for $\sigma \leq \sigma_1^*$ and menu P1 is chosen otherwise. Moreover, the cutoff σ_1^*, if existing, decreases with ϕ and is lower than σ_l.*

Proof. Because $\phi > 1$, $V_{P1}(\sigma) = V_{P1}^{B}(\sigma)$. Consider two cases separately: (i) $V_{S10}(\sigma) = V_{S10}^{B}(\sigma)$ if $\sigma \leq \hat{\sigma}$, and (ii) $V_{S10}(\sigma) = V_{S10}^{A}(\sigma)$ if $\sigma > \hat{\sigma}$.

(i) Consider $\sigma \leq \hat{\sigma}$. The difference is given by

$$\begin{aligned}
V_{P1}^{B}(\sigma) - V_{S10}^{B}(\sigma) &= [g(h|1)\,y(\sigma,h) + g(l|1)\,y(\sigma,l)]\left(1 - \frac{c}{g(h|0)\,y(\sigma,h) + g(l|0)\,y(\sigma,l)}\right) \\
&\quad + \phi\left(\frac{g(h|1)(1-k_h)\,y(\sigma,h) + g(l|1)(1-k_l)\,y(\sigma,l)}{g(h|0)\,y(\sigma,h) + g(l|0)\,y(\sigma,l)}\right)c \\
&\quad - g(h|1)\left[1 - \left(1 - \phi\left(1 - \frac{k_h}{\sigma}\right)\right)\frac{y(\sigma,l)}{y(\sigma,h)}\right](y(\sigma,h) - c) \\
&= \frac{1}{k_l - k_h}\left[(k_l - 1)\,y(\sigma,h) + (1-k_h)\,y(\sigma,l)\right] \\
&\quad - \frac{k_l - 1}{k_l - k_h}\left[1 - \left(1 - \phi\left(1 - \frac{k_h}{\sigma}\right)\right)\frac{y(\sigma,l)}{y(\sigma,h)}\right](y(\sigma,h) - c) \\
&\quad - \frac{(k_l - 1)[1 - \phi(1-k_h)] + [1 + \phi(k_l - 1)](1-k_h)\frac{y(\sigma,l)}{y(\sigma,h)}}{k_h(k_l - 1) + k_l(1-k_h)\frac{y(\sigma,l)}{y(\sigma,h)}}c.
\end{aligned}$$

We find that

$$\begin{aligned}
\frac{\partial\left(V_{P1}^{B}(\sigma) - V_{S10}^{B}(\sigma)\right)}{\partial\phi} &= -\frac{k_l - 1}{k_l - k_h}\left(1 - \frac{k_h}{\sigma}\right)\frac{y(\sigma,l)}{y(\sigma,h)}(y(\sigma,h) - c) \\
&\quad - \frac{(k_l - 1)(1-k_h)}{k_h(k_l - 1) + k_l(1-k_h)\frac{y(\sigma,l)}{y(\sigma,h)}}\left(-1 + \frac{y(\sigma,l)}{y(\sigma,h)}\right)c \\
&\to -\frac{k_l - 1}{k_l}\left[y(\sigma,l) - \frac{y(\sigma,l)}{y(\sigma,h)}c - \frac{y(\sigma,h)}{y(\sigma,l)}c + c\right] \text{ as } k_h \to 0. \qquad (31)
\end{aligned}$$

Moreover, as $k_h \to 0$ and $\phi \to 1$, we find that $V_{P1}^{B}(\sigma) - V_{S10}^{B}(\sigma) \to \frac{1}{k_l}(y(\sigma,l) - c)$, and

$$\frac{\partial\left(V_{P1}^{B}(\sigma) - V_{S10}^{B}(\sigma)\right)}{\partial\sigma} \to \frac{1}{k_l}\frac{\partial y(\sigma,l)}{\partial\sigma} > 0. \qquad (32)$$

(ii) Consider $\sigma > \hat{\sigma}$. The difference is given by

$$\begin{aligned}
V_{P1}^{B}(\sigma) - V_{S10}^{A}(\sigma) &= [g(h|1)\,y(\sigma,h) + g(l|1)\,y(\sigma,l)]\left(1 - \frac{c}{g(h|0)\,y(\sigma,h) + g(l|0)\,y(\sigma,l)}\right) \\
&\quad + \phi\left(\frac{g(h|1)(1-k_h)\,y(\sigma,h) + g(l|1)(1-k_l)\,y(\sigma,l)}{g(h|0)\,y(\sigma,h) + g(l|0)\,y(\sigma,l)}\right)c \\
&\quad - \phi g(h|1)\left(1 - \frac{k_h}{\sigma}\right)[y(\sigma,h) - c] \\
&= \frac{1}{k_l - k_h}\left[(k_l - 1)\,y(\sigma,h) + (1-k_h)\,y(\sigma,l) - \phi(k_l - 1)\left(1 - \frac{k_h}{\sigma}\right)(y(\sigma,h) - c)\right] \\
&\quad - \frac{(k_l - 1)[1 - \phi(1-k_h)] + [1 + \phi(k_l - 1)](1-k_h)\frac{y(\sigma,l)}{y(\sigma,h)}}{k_h(k_l - 1) + k_l(1-k_h)\frac{y(\sigma,l)}{y(\sigma,h)}}c.
\end{aligned}$$

We find that

$$\begin{aligned}\frac{\partial\left(V_{P1}^{B}(\sigma)-V_{S10}^{A}(\sigma)\right)}{\partial\phi} &= -\frac{1}{k_l-k_h}(k_l-1)\left(1-\frac{k_h}{\sigma}\right)(y(\sigma,h)-c) \\ &\quad -(1-k_h)\frac{-1+\frac{y(\sigma,l)}{y(\sigma,h)}}{k_h+k_l\frac{(1-k_h)}{(k_l-1)}\frac{y(\sigma,l)}{y(\sigma,h)}}c \\ &\to -\frac{1}{k_l}(k_l-1)\,y(\sigma,h)\left(1-\frac{1}{y(\sigma,l)}c\right) \text{ as } k_h\to 0. \qquad (33)\end{aligned}$$

Moreover, as $k_h \to 0$ and $\phi \to 1$, we find that $V_{P1}^{B}(\sigma)-V_{S10}^{A}(\sigma) \to \frac{1}{k_l}(y(\sigma,l)-c)$, and

$$\frac{\partial\left(V_{P1}^{B}(\sigma)-V_{S10}^{A}(\sigma)\right)}{\partial\sigma} \to \frac{1}{k_l}\frac{\partial y(\sigma,l)}{\partial\sigma} > 0. \qquad (34)$$

To sum up, (32) and (34) indicate that if $k_h \to 0$ and $\phi \to 1$, there exists a cutoff σ_1^* as defined in the proposition. Moreover, (31) and (33) indicate that if $k_h \to 0$ and $\sigma \in [\sigma_{\min}, \sigma_l]$, $V_{P1}(\sigma) - V_{S10}(\sigma)$ increases with ϕ; that means, the cutoff σ_1^*, if belonging to the region $[\sigma_{\min}, \sigma_l]$, decreases with ϕ. ■

According to Silberberg and Suen (2000, p.168), $\frac{\partial\phi}{\partial\psi} \geq 0$. Moreover, (22) implies that

$$\begin{aligned}\phi &\leq \frac{1+\frac{1-k_h}{k_l-1}\frac{y(\sigma_{\max},l)}{y(\sigma_{\max},h)}}{1-\frac{k_h}{\sigma_{\max}}+\frac{1-k_h}{k_l-1}\left(1-\frac{k_l}{\sigma_{\max}}\right)\frac{y(\sigma_{\max},l)}{y(\sigma_{\max},h)}} \\ &\leq \frac{1+\frac{1-k_h}{k_l-1}}{1-\frac{k_h}{\sigma_{\max}}+\frac{1-k_h}{k_l-1}\left(1-\frac{k_l}{\sigma_{\max}}\right)} \to 1 \text{ as } \sigma_{\max}\to\infty.\end{aligned}$$

To summarize the above analysis, we find that (i) there exists a threshold $\overline{\psi}$ such that if $\psi \leq \overline{\psi}$, the cutoff σ_1^* defined in the proposition exists; (ii) the cutoff σ_1^*, if existing, first increases with ψ and then decreases with ψ, but in any case, σ_1^* is lower than σ_l; (iii) if $\sigma_{\max} \to \infty$, $\overline{\psi} \to \infty$. Q.E.D.

Proof of Proposition 5

First note that with $t = 0$, the cutoff σ^* in the optimal contract with non-binding first-stage moral hazard is given by the solution to:

$$\triangle(\sigma) \equiv \pi^P(\sigma) - \pi^S(\sigma) = g(l|1)[y(\sigma,l)-c] + g(h|1)\frac{y(\sigma,l)}{y(\sigma,h)}(y(\sigma,h)-c) = 0.$$

Consider two cases separately: (i) $\phi \leq 1$ and (ii) $\phi > 1$.

(i) If $\phi \leq 1$, then σ_1^* is determined by $V_{P1}^A(\sigma) = V_{S10}^B(\sigma)$, i.e.,

$$\begin{aligned} & g(l|1)[y(\sigma,l)-c] + g(h|1)\left[1-\phi\left(1-\frac{k_h}{\sigma}\right)\right]\frac{y(\sigma,l)}{y(\sigma,h)}(y(\sigma,h)-c) = 0 \\ \Leftrightarrow \quad & \triangle(\sigma) - \left[\phi g(h|1)\left(1-\frac{k_h}{\sigma}\right)\frac{y(\sigma,l)}{y(\sigma,h)}(y(\sigma,h)-c)\right] = 0. \end{aligned}$$

For k_h sufficiently small, the bracketed term is positive for all σ. The root to the equation above is therefore strictly less than that of $\triangle(\sigma) = 0$. Thus, $\sigma_1^* > \sigma^*$.

(ii) If $\phi > 1$, σ_1^* is determined by either $V_{P1}^B(\sigma) = V_{S10}^A(\sigma)$ (given that $\sigma_1^* > \hat{\sigma}$) or $V_{P1}^B(\sigma) = V_{S10}^B(\sigma)$ (given that $\sigma_1^* \leq \hat{\sigma}$). Comparing relevant functions, we have

$$\begin{aligned} V_{P1}^B(\sigma) &= \pi^P(\sigma) + (\phi-1)\left(\frac{g(h|1)(1-k_h)y(\sigma,h) + g(l|1)(1-k_l)y(\sigma,l)}{g(h|0)y(\sigma,h) + g(l|0)y(\sigma,l)}\right)c; \\ V_{S10}^A(\sigma) &= \pi^S(\sigma) + g(h|1)\left[\phi\left(1-\frac{k_h}{\sigma}\right) - 1 + \frac{y(\sigma,l)}{y(\sigma,h)}\right](y(\sigma,h)-c); \\ V_{S10}^B(\sigma) &= \pi^S(\sigma) + \phi g(h|1)\left(1-\frac{k_h}{\sigma}\right)\frac{y(\sigma,l)}{y(\sigma,h)}(y(\sigma,h)-c). \end{aligned}$$

Thus,

$$\begin{aligned} & V_{P1}^B(\sigma) - V_{S10}^A(\sigma) - \triangle(\sigma) \\ = \; & (\phi-1)\left(\frac{g(h|1)(1-k_h)y(\sigma,h) + g(l|1)(1-k_l)y(\sigma,l)}{g(h|0)y(\sigma,h) + g(l|0)y(\sigma,l)}\right)c \\ & -g(h|1)\left(\phi\left(1-\frac{k_h}{\sigma}\right) - 1 + \frac{y(\sigma,l)}{y(\sigma,h)}\right)(y(\sigma,h)-c). \end{aligned}$$

The expression is negative for ϕ close to 1 and k_h sufficiently small.

$$\begin{aligned} & V_{P1}^B(\sigma) - V_{S10}^B(\sigma) - \triangle(\sigma) \\ = \; & (\phi-1)\left(\frac{g(h|1)(1-k_h)y(\sigma,h) + g(l|1)(1-k_l)y(\sigma,l)}{g(h|0)y(\sigma,h) + g(l|0)y(\sigma,l)}\right)c \\ & -\phi g(h|1)\left(1-\frac{k_h}{\sigma}\right)\frac{y(\sigma,l)}{y(\sigma,h)}(y(\sigma,h)-c). \end{aligned}$$

The expression is negative for ϕ close to 1 and k_h sufficiently small.

In sum, if ϕ is close to 1 and k_h is sufficiently small, then $V_{P1}(\sigma) - V_{S10}(\sigma)$ is strictly increasing but everywhere less than $\triangle(\sigma)$. Thus, its root σ_1^* is strictly higher than σ^*. Q.E.D.

Appendix B: Continuous effort

In the model outlined in Section 2, the agent's effort choice is binary. In this appendix, we show that our result is robust to alternative assumptions that allow for continuous effort choice by the agent. Suppose in Stage 2, the agent chooses a continuous effort $e_2 \in [0, \bar{e}] \subset \mathbb{R}$. The effort cost to the agent is $\frac{1}{2}e_2^2$. At the end of Stage 2, the final outcome $x \in \{0,1\}$ realizes according to distribution function (1). The marginal product of effort e_2, $y(\sigma, s)$, still satisfies Assumptions **A1-A5**. Moreover, we assume that the agent chooses $e_1 = 1$ in Stage 1 and the IC constraint for inducing $e_1 = 1$ is not binding.

Before Stage 1, the principal proposes a contract, denoted by $\Gamma \equiv \{w(\sigma, m), b(\sigma, m)\}_{\sigma \in \Omega, m \in M}$. Here, M is a compact message space of the principal. Given that the objective signal $\sigma \in \Omega$ is observed and a message $m \in M$ is disclosed by the principal at the end of Stage 1, $w(\sigma, m)$ is the wage that must be paid, while $b(\sigma, m)$ is the bonus paid if and only if $x = 1$.

We first characterize the agent's second-stage effort decision. His effort decision problem is

$$\max_{e_2 \in [0,\bar{e}]} w(\sigma, m) + (t + E[y(\sigma, s)|m]e_2)b(\sigma, m) - \frac{1}{2}e_2^2.$$

The solution of the problem is

$$e_2 = E[y(\sigma, s)|m]b(\sigma, m). \tag{35}$$

It is apparent that the effort e_2 exerted increases in $E[y(\sigma, s)|m]$. As a result, the argument in the proof of Lemma 1 applies to this setting and it is without loss to focus on the contract with a binary message space for any objective signal σ.

Pooling menu First consider the pooling menu $(w(\sigma), b(\sigma))$, under which no payment varies with the reported m given the realization of σ. Using (35), the agent's effort choice is

$$e_2 = [g(h|1)y(\sigma, h) + (1 - g(h|1))y(\sigma, l)]b(\sigma).$$

The principal's profit conditional on using a pooling menu $(w(\sigma), b(\sigma))$ at signal σ is

$$\left(t + [g(h|1)y(\sigma, h) + (1 - g(h|1))y(\sigma, l)]^2 b(\sigma)\right)(1 - b(\sigma)) - w(\sigma).$$

It is clear that the optimal base wage is $w(\sigma) = 0$. The optimal bonus $b(\sigma)$ can be obtained by the first-order condition:

$$b(\sigma) = \frac{1}{2} - \frac{t}{2\left[g(h|1)\,y(\sigma,h) + (1-g(h|1))\,y(\sigma,l)\right]^2}.$$

The principal's profit is thus

$$\pi^P(\sigma) \equiv \frac{1}{4}\frac{\left(\left[g(h|1)\,y(\sigma,h) + (1-g(h|1))\,y(\sigma,l)\right]^2 + t\right)^2}{\left[g(h|1)\,y(\sigma,h) + (1-g(h|1))\,y(\sigma,l)\right]^2}.$$

Separating menu Next consider separating menu, under which the agent can fully infer the subjective signal s. Therefore, the agent exerts effort $e_2 = y(\sigma,s)\,b(\sigma,s)$. The principal's truth-telling constraints can be written as

$$\begin{aligned} & (t + y(\sigma,h)\,y(\sigma,h)\,b(\sigma,h))\,(1-b(\sigma,h)) - w(\sigma,h) \\ \geq\ & (t + y(\sigma,h)\,y(\sigma,l)\,b(\sigma,l))\,(1-b(\sigma,l)) - w(\sigma,l), \end{aligned} \tag{36}$$

$$\begin{aligned} & (t + y(\sigma,l)\,y(\sigma,l)\,b(\sigma,l))\,(1-b(\sigma,l)) - w(\sigma,l) \\ \geq\ & (t + y(\sigma,l)\,y(\sigma,h)\,b(\sigma,h))\,(1-b(\sigma,h)) - w(\sigma,h). \end{aligned} \tag{37}$$

The principal's problem is therefore choosing $\{w(\sigma,s), b(\sigma,s)\}$ to maximize

$$\begin{aligned} & g(h|1)\left[\left(t + (y(\sigma,h))^2\,b(\sigma,h)\right)(1-b(\sigma,h)) - w(\sigma,h)\right] \\ & + (1-g(h|1))\left[\left(t + (y(\sigma,l))^2\,b(\sigma,l)\right)(1-b(\sigma,l)) - w(\sigma,l)\right], \end{aligned}$$

subject to constraints (36) and (37). Note that the constraints imply

$$y(\sigma,h)\,b(\sigma,h)\,(1-b(\sigma,h)) \geq y(\sigma,l)\,b(\sigma,l)\,(1-b(\sigma,l)).$$

Whenever this inequality holds, it is clear that at optimum, constraint (37) binds and $w(\sigma, l) = 0$. Setting $w(\sigma, l) = 0$ and substituting (37) into the principal's objective, we get

$$\begin{aligned} & g(h|1)\left[(y(\sigma,h) - y(\sigma,l))\, y(\sigma,h)\, b(\sigma,h)(1 - b(\sigma,h)) - \left(t + y(\sigma,l)^2 b(\sigma,l)\right)(1 - b(\sigma,l))\right] \\ & + (1 - g(h|1))\left[\left(t + y(\sigma,l)^2 b(\sigma,l)\right)(1 - b(\sigma,l))\right]. \end{aligned}$$

By their respective first-order conditions, it is thus optimal to set

$$b(\sigma,h) = \frac{1}{2} \text{ and } b(\sigma,l) = \frac{1}{2} - \frac{t}{2(y(\sigma,l))^2}.$$

The corresponding base wage is

$$w(\sigma,h) = \frac{1}{2}\left(t + \frac{1}{2} y(\sigma,l)\, y(\sigma,h)\right) - \frac{1}{4}\left(1 + \frac{t}{(y(\sigma,l))^2}\right)\left(t + (y(\sigma,l))^2\right).$$

The principal's profit is thus

$$\pi^S(\sigma) \equiv \frac{1}{4} g(h|1)(y(\sigma,h) - y(\sigma,l))\, y(\sigma,h) + \frac{1}{4}\left(t + (y(\sigma,l))^2\right)\left(1 + \frac{t}{(y(\sigma,l))^2}\right).$$

The principal's optimal contract It remains to compare $\pi^P(\sigma)$ with $\pi^S(\sigma)$. Straightforward algebra shows that

$$\begin{aligned} & \pi^P(\sigma) \geq \pi^S(\sigma) \\ \Leftrightarrow\ & \frac{y(\sigma,l)}{y(\sigma,h)} - (1 - g(h|1))\left(1 - \frac{y(\sigma,l)}{y(\sigma,h)}\right) - t^2 \frac{(2 - g(h|1))\frac{1}{y(\sigma,h)} + g(h|1)\frac{1}{y(\sigma,l)}}{y(\sigma,l)\left(g(h|1)\, y(\sigma,h) + (1 - g(h|1))\, y(\sigma,l)\right)^2} \geq 0. \end{aligned}$$

It is clear that under the assumption that $\frac{y(\sigma,l)}{y(\sigma,h)}$ is weakly increasing in σ, the LHS of the inequality above is increasing in σ. Thus, the following result is immediate.

Summary 1 *There exists a cutoff σ^* such that the pooling menu is optimal if $\sigma \geq \sigma^*$ and the separating menu is optimal if $\sigma < \sigma^*$.*

To gain some intuition of the finding above, suppose for simplicity that $t = 0$. Because of no limited-liability rent, the principal's payoff can be written as the difference between (i) the total surplus and (ii) the truth-telling rent. For a pooling menu, the first term is $\frac{1}{4}\left[g(h|1)\, y(\sigma,h) + (1 - g(h|1))\, y(\sigma,l)\right]^2$, whereas the second term is zero. For a separating menu, the first term is $\frac{1}{4}\left[g\left(h|1\right)y\left(\sigma,h\right)^{2}+\left(1-g\left(h|1\right)\right)y\left(\sigma,l\right)^{2}\right]$, whereas the second term is $\frac{1}{4}g\left(h|1\right)y\left(\sigma,l\right)\left[y\left(\sigma,h\right)-y\left(\sigma,l\right)\right]$. The benefit of a separating menu relative to a pooling menu arises from a larger total surplus, and the magnitude of this benefit is proportional to $\left(1-g\left(h|1\right)\right)\left(y\left(\sigma,h\right)-y\left(\sigma,l\right)\right)$. The cost of a separating menu relative to a pooling menu is the truth-telling rent, and the magnitude of this cost is proportional to $y\left(\sigma,l\right)$. It is possible that as σ increases, the relative benefit of a separating menu diminishes, while its relative cost increases. In other cases, Assumption **A4** (i.e., $\frac{y(\sigma,l)}{y(\sigma,h)}$ is weakly increasing in σ) guarantees that as σ increases, the relative benefit of a separating menu diminishes at a faster rate than its relative cost if they both diminish, or the former benefit increases at a slower rate than the latter cost if they both increase. Consequently, a separating menu is dominated by a pooling menu for all σ exceeding a certain cutoff.

In this setting, the efficiency is given by $t+y\left(\sigma,s\right)e_{2}-\frac{1}{2}e_{2}^{2}$, so the efficient level of effort is $e_{2}^{*}\left(\sigma,s\right)=y\left(\sigma,s\right)$. Thus, if $\sigma<\sigma^{*}$, a separating menu is used and the effort in the optimal contract is downwardly distorted. On the other hand, if $\sigma\geq\sigma^{*}$, a pooling menu is used and the effort for $s=l$ in the optimal contract is upwardly distorted provided that

$$\left[g\left(h|1\right)y\left(\sigma,h\right)+\left(1-g\left(h|1\right)\right)y\left(\sigma,l\right)\right]\left[g\left(h|1\right)\left(y\left(\sigma,h\right)-y\left(\sigma,l\right)\right)-y\left(\sigma,l\right)\right]>t.$$

If $t=0$, the inequality above simplifies to $\frac{y(\sigma,l)}{y(\sigma,h)}<\frac{g(h|1)}{1+g(h|1)}$. Therefore, we have the following result.

Summary 2 *Suppose that $t=0$ and $\frac{y(\sigma^{*},l)}{y(\sigma^{*},h)}<\frac{g(h|1)}{1+g(h|1)}$. Denote σ that solves $\frac{y(\sigma,l)}{y(\sigma,h)}=\frac{g(h|1)}{1+g(h|1)}$ by $\sigma^{\#}$. In the optimal contract, there exists an upward distortion in effort e_{2} if $\sigma\in\left(\sigma^{*},\sigma^{\#}\right)$ and $s=l$.*

Finally, in the absence of the truth-telling constraints, the principal's payoff as a function of s is convex. Thus, she finds it optimal to adopt the separating menu for all σ. As a result, an under-utilization of subjective signal arises in this setting.

Appendix C: The subjective signal has multiple levels

The subjective signal s is assumed to have a finite support of size n, $s\in S\equiv\{s_{1},s_{2},...,s_{n}\}$. The probability by which $s=s_{k}$ is realized given first-stage effort e_{1} is $g\left(s_{k}|\,e_{1}\right)$. To simplify notations, define $g_{k}\equiv g\left(s_{k}|\,1\right)$. Assume that $y\left(\sigma,s_{i}\right)\geq y\left(\sigma,s_{j}\right)$ if $i\geq j$. Moreover, we assume that the agent chooses $e_{1}=1$ in Stage 1 or the IC constraint for the agent's first-stage effort is not binding.

Given σ, define E as the set of subjective signals with which $e_2 = 1$ is solicited. The following observations are true. First, by an argument similar to Lemma 2, it is optimal to set $w(\sigma, s_i) = w(\sigma, s_j)$ and $b(\sigma, s_i) = b(\sigma, s_j)$ for any $s_i, s_j \in E$. Second, because of the truth-telling constraints of the principal and the assumption that $y(\sigma, s)$ is increasing in s, the set E takes the form $E = \{s_k, s_{k+1}, ..., s_n\}$ for some $k = 0, 1, ..., n$. Third, for any $s_i \notin E$, $w(\sigma, s_i) = b(\sigma, s_i) = 0$.

Using the observations above, for each objective signal σ, the menu can be labelled by k, the minimum subjective signal with which $e_2 = 1$ is solicited. For $k = 1$, i.e., effort is solicited for all s, the menu is pooling:

$$w(\sigma) = 0, \text{ and } b(\sigma) = \frac{c}{\sum_{i=1}^{n} g_i y(\sigma, s_i)}.$$

The corresponding profit is

$$\begin{aligned} \pi_1(\sigma) &= \left(t + \sum_{i=1}^{n} g_i y(\sigma, s_i)\right)\left(1 - \frac{c}{\sum_{i=1}^{n} g_i y(\sigma, s_i)}\right) \\ &= t - c + \sum_{i=1}^{n} g_i y(\sigma, s_i) - \frac{tc}{\sum_{i=1}^{n} g_i y(\sigma, s_i)}. \end{aligned} \tag{38}$$

For $k = 2, 3, ..., n$, the menu is separating:

$$\begin{aligned} w(\sigma, s_i) &= b(\sigma, s_i) = 0 \text{ for all } i = 1, 2, ..., k-1; \\ b(\sigma, s_i) &\geq c\frac{\left(\sum_{j=k}^{n} g_j\right)}{\sum_{j=k}^{n} g_j y(\sigma, s_j)} \text{ for all } i = k, k+1, ..., n. \end{aligned}$$

To pin down the minimum wage $w(\sigma, s_i)$ for $i \geq k$, note that all truth-telling incentive constraints would be satisfied provided that the one corresponding to signal s_{k-1} is satisfied. Thus, the only binding constraint is

$$\begin{aligned} & t \geq (t + y(\sigma, s_{k-1}))(1 - b(\sigma, s_k)) - w(\sigma, s_k) \\ \Leftrightarrow \quad & w(\sigma, s_k) \geq y(\sigma, s_{k-1}) - (t + y(\sigma, s_{k-1}))\, b(\sigma, s_k). \end{aligned}$$

Note that because $y(\sigma, s_i) \geq y(\sigma, s_{k-1})$ for all $i \geq k$, it is optimal to set, for all $i \geq k$,

$$b(\sigma, s_i) = c\frac{\sum_{j=k}^{n} g_j}{\sum_{j=k}^{n} g_j y(\sigma, s_j)} \text{ and } w(\sigma, s_i) = y(\sigma, s_{k-1}) - c(t + y(\sigma, s_{k-1}))\frac{\sum_{j=k}^{n} g_j}{\sum_{j=k}^{n} g_j y(\sigma, s_j)}. \tag{39}$$

The profit of menu $k > 1$ at objective signal σ is thus

$$\pi_k(\sigma) \equiv t + \sum_{i=k}^{n} g_i \left[y(\sigma, s_i) - (t + y(\sigma, s_i)) b(\sigma, s_k) - w(\sigma, s_k) \right], \tag{40}$$

where $b(\sigma, s_k)$ and $w(\sigma, s_k)$ are given by (39).

To simplify the subsequent analysis, we assume that the marginal product of second-stage effort is multiplicatively separable in σ and s, i.e., $y(\sigma, s_i) = h(\sigma) s_i$, where h is a non-decreasing differentiable function of σ. The profit of menu $k > 1$ is thus given by

$$\pi_k(\sigma) = t + h(\sigma) \sum_{i=k}^{n} g_i (s_i - s_{k-1}) - c \left(\sum_{i=k}^{n} g_i \right) \left(1 - \frac{s_{k-1} \sum_{i=k}^{n} g_i}{\sum_{i=k}^{n} g_i s_i} \right).$$

With equation (38), the profit of menu $k = 1$ is given by

$$\pi_1(\sigma) = t - c + h(\sigma) \sum_{i=1}^{n} g_i s_i - \frac{tc}{h(\sigma) \sum_{i=1}^{n} g_i s_i}.$$

First, observe that π_k is increasing in σ. Moreover, the derivative of π_k with respect to σ is non-increasing in k. For $k \geq 2$,

$$\begin{aligned} \frac{\partial \pi_k(\sigma)}{\partial \sigma} - \frac{\partial \pi_{k+1}(\sigma)}{\partial \sigma} &= h'(\sigma) \left(\sum_{i=k}^{n} g_i (s_i - s_{k-1}) - \sum_{i=k+1}^{n} g_i (s_i - s_k) \right) \\ &= h'(\sigma) \left(g_k (s_k - s_{k-1}) + \sum_{i=k+1}^{n} g_i (s_i - s_{k-1}) - \sum_{i=k+1}^{n} g_i (s_i - s_k) \right) \\ &= h'(\sigma) (s_k - s_{k-1}) \sum_{i=k}^{n} g_i \geq 0. \end{aligned}$$

For $k = 1$,

$$\begin{aligned} \frac{\partial \pi_1(\sigma)}{\partial \sigma} - \frac{\partial \pi_2(\sigma)}{\partial \sigma} &= h'(\sigma) \sum_{i=1}^{n} g_i s_i + \frac{h'(\sigma) tc}{\left(h(\sigma) \sum_{i=1}^{n} g_i s_i \right)^2} - h'(\sigma) \sum_{i=2}^{n} g_i (s_i - s_1) \\ &= h'(\sigma) \left(s_1 \sum_{i=1}^{n} g_i + \frac{tc}{\left(h(\sigma) \sum_{i=1}^{n} g_i s_i \right)^2} \right) > 0. \end{aligned}$$

The above analysis leads to the following result.

Summary 3 *There exists a sequence of increasing cutoffs σ_k^* such that menu k is used for $\sigma \in [\sigma_k^*, \sigma_{k-1}^*)$.*

Next, we show that solicitation of second-stage effort can be upwardly distorted. To this end, note that at signal profile (σ, s_i), $e_2 = 1$ is socially efficient if and only if $h(\sigma) s_i \geq c$, or equivalently, $\sigma \geq \sigma_i^{eff}$, where

$$h\left(\sigma_i^{eff}\right) = \frac{c}{s_i}.$$

An upward distortion in effort e_2 arises if for some $k = 2, ..., n$,

$$\pi_k\left(\sigma_k^{eff}\right) > \pi_{k+1}\left(\sigma_k^{eff}\right).$$

The inequality above can be equivalently written as

$$\frac{s_{k-1}}{s_k}\left(\sum_{i=k}^{n} g_i\right)\left(\sum_{i=k+1}^{n} g_i s_i\right) < \left(\sum_{i=k+1}^{n} g_i\right)\left(\sum_{i=k}^{n} g_i s_i\right). \tag{41}$$

On the other hand, for $k = 1$,

$$\pi_1\left(\sigma_1^{eff}\right) > \pi_2\left(\sigma_1^{eff}\right) \Leftrightarrow t < c\left(1-g_1\right)\frac{1}{s_1}\left[\sum_{i=1}^{n} g_i s_i - g_1\left(1-g_1\right)\frac{s_1^2}{\sum_{i=2}^{n} g_i s_i} - \left(1-g_1\right)s_1\right]. \tag{42}$$

Summary 4 *An upward distortion in effort e_2 can arise around σ_1^{eff} if t is sufficiently small. Moreover, an upward distortion or a downward distortion in effort e_2 may arise around σ_i^{eff} depending on whether inequality (41) holds or not.*

Finally, whenever inequality (42) holds, the subjective signal is under-utilized for objective signals in the interval $\left(\sigma_1^*, \sigma_1^{eff}\right)$.

Appendix D: An alternative model setup in which $y(\sigma, s)$ is endogenous

In Section 3, we assume that the agent's first-stage effort $e_1 \in \{0, 1\}$ stochastically determines the realization of two signals, σ and s, which, in turn, determines the marginal product of second-stage effort $e_2 \in \{0, 1\}$ by a function $y(\sigma, s)$. We can consider an alternative setup: the first-stage effort e_1 stochastically determines the quality of an interim product, $a \in \{H, L\}$, which in turn determines the productivity in the second stage. Specifically, given e_1, the probability that $a = H$ is realized is denoted by p_{e_1}, and we assume that $p_1 > p_0$. The final outcome is determined stochastically by the interim product's quality, a, and the agent's second-stage effort, e_2, according to the following distribution:

$$\Pr\left(x=1|e_2,a\right)=t+\lambda_a e_2,\ \text{for}\ \lambda_a\in\left\{\lambda_L,\lambda_H\right\}.$$

We assume that $0\equiv\lambda_L<\lambda_H\equiv\lambda$, that is, the second-stage productivity is higher conditional on a high-quality interim product.

The interim product's quality a is assumed to be unobservable to any party. However, before the agent chooses e_2, two signals, σ and s, which are independently distributed conditional on a, realize. The objective signal $\sigma\in\Omega$ is distributed according to conditional probability function $f\left(\sigma|\,a\right)$, while the subjective signal, $s\in\{h,l\}$, is distributed according to conditional probability $g\left(s|\,a\right)$. We further impose the monotone likelihood ratio conditions, that is, a higher value of σ or $s=h$ is weakly more indicative of $a=H$.

A1′ $R\left(\sigma\right)\equiv\frac{f(\sigma|L)}{f(\sigma|H)}$ is non-increasing in σ.

A2′ $k\left(h\right)\equiv\frac{g(\,h|L)}{g(\,h|H)}\leq\frac{g(\,l|L)}{g(\,l|H)}\equiv k\left(l\right)$.

Suppose that $e_1=1$ is always solicited in the equilibrium. Given that σ and s are realized, the marginal product of second-stage effort is denoted by $y\left(\sigma,s\right)$ and calculated as follows:

$$y\left(\sigma,s\right)=\frac{p_1 f\left(\sigma|H\right)g\left(s|\,H\right)}{p_1 f\left(\sigma|H\right)g\left(s|\,H\right)+\left(1-p_1\right)f\left(\sigma|L\right)g\left(s|\,L\right)}\lambda=\frac{1}{1+\frac{1-p_1}{p_1}R\left(\sigma\right)k\left(s\right)}\lambda. \tag{43}$$

Moreover, we find that

$$\frac{y\left(\sigma,l\right)}{y\left(\sigma,h\right)}=\frac{1+\frac{1-p_1}{p_1}R\left(\sigma\right)k\left(h\right)}{1+\frac{1-p_1}{p_1}R\left(\sigma\right)k\left(l\right)}.$$

Given Assumptions **A1′**, **A2′**, it is apparent that $y\left(\sigma'',s\right)>y\left(\sigma',s\right)$ for all $s\in\{h,l\}$, and $\sigma'',\sigma'\in\Omega$ such that $\sigma''>\sigma'$; and $y\left(\sigma,h\right)>y\left(\sigma,l\right)$ for all $\sigma\in\Omega$; $\frac{y(\sigma,l)}{y(\sigma,h)}$ is non-decreasing in σ. In other words, Assumptions **A1′** and **A2′** in this alternative model imply Assumptions **A2** and **A4** in the model presented in Section 2, which are crucial for our analysis. Thus, in this alternative model, using (43) to replace $y\left(\sigma,s\right)$, the analysis of incentive contracts is identical to Sections 3 and 4. Moreover, similar results to Propositions 1-5 can be obtained.