The Loser's Curse in the Search for Advice

Pak Hung Au*

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Abstract

An agent searches sequentially for advice from experts concerning the payoff of taking an operation. Consulting each expert incurs a positive search cost. There are infinitely many experts, each has access to an identically and conditionally independent signal structure about the payoff, and each makes a recommendation after observing the signal realization. The interests of the experts and the agent are partially aligned. We show that for sufficiently small search cost, a loser's curse effect is present, hampering the quality of information gathered. We identify conditions that ensure perfect information aggregation with vanishing search cost. The more surprising finding is that both the agent's payoff and social welfare can be strictly lower than the alternative scenario in which the agent can consult only a single expert.

Keywords Search, Expert Advice, Information Transmission, Information Aggregation

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1 Introduction

A patient with some medical symptoms is unsure whether surgery is an appropriate treatment. For advice, he consults a doctor, who diagnoses the case and makes a recommendation. Suppose the interests of the doctor and the patient are aligned, so the patient's only concern is that the doctor's diagnosis is not accurate (rather than the fact that the doctor may have incentives for lying). He may consult another doctor for a second opinion. If the recommendations of the two doctors match, then he is

*Nanyang Technological University, email: phau@ntu.edu.sg. I would like to thank the audience of the 2016 Asian Meeting of the Econometric Society and Workshop on Game Theory and Applications held at the Chinese University of Hong Kong. All errors are my own.

more confident about the appropriate course of treatment. Nonetheless, he may continue the process of consulting other doctors for more accurate information. If he is eventually confident enough that the surgery is necessary, then he undergoes the surgery with one of the doctors that recommended the surgery to him. It is natural to expect that by consulting more doctors, the patient could gather better information about the appropriate treatment. A similar scenario arises in the process of a customer looking for a repair service, an entrepreneur looking for financial investment of a venture capitalist, and a claimant looking for legal advice and service of a lawyer.

In this paper, we analyze a model in which an (male) agent sequentially consults (female) experts for advice on whether to undergo an operation or not. His payoff of having the operation is uncertain, taking a positive value if the operation is suitable for him (an event denoted by the state $\omega = 1$), and a negative value if the operation is unsuitable for him (an event denoted by the state $\omega = 0$). There are infinitely many potential experts for the agent to consult, and each expert has access to an identical and conditionally independent signal structure for learning about the agent's state ω. For each visit and consultation, the agent has to incur a positive search cost. A consulted expert makes a recommendation after privately learning her own signal realization, but not those of previously consulted experts. If she recommends the operation, the agent can decide whether to undergo the operation with her, or seek more advice from other experts. If she recommends against the operation, then the agent cannot have the operation with her (but he may still consult another expert). Eventually, the agent may undergo the operation with an expert that recommends him to do so, or quit the process of consulting experts without taking the operation.

The payoffs of the agent and the experts depend on the state and whether the operation is performed. We assume the interests of the experts and the agent are partially aligned. More specifically, if the agent undergoes the operation with an expert, then the sign of their payoffs are equal. On the other hand, if the agent does not undergo the operation with her, then she gets a payoff normalized to zero. For simplicity, the payoff structure is exogenously fixed, so we have abstracted away from considerations such as the pricing of the operation and bargaining over the division of liability of failed operation (i.e., the operation is carried out with the agent's state being $\omega = 0$).

Our objective is to investigate the equilibrium outcome and welfare consequence of allowing the agent to sequentially search for experts' advice. In particular, we would like to compare the agent's payoff and the social welfare under two scenarios: (i) a benchmark setting in which there is only one expert available for consultation; and (ii) a setting in which there are infinitely many experts available for consultation at an infinitesimal search cost. At first sight, it seems that the agent would prefer the second scenario, as he would be able to learn the state with almost no cost. Our analysis shows that whether this conjecture holds or not crucially depends on the experts' signal structure. We find that there are informative signal structures under which the agent would not be able to learn the true state in scenario (ii), despite having almost costless access to infinitely many experts. More strikingly, with some payoff and informative signal structures, both the agent's payoff and the social welfare are strictly lower in scenario (ii) than in scenario (i).

The key driving force behind our results is a *loser's curse* effect: in an equilibrium in which the experts' advice is informative, each expert understands that the agent decides to undergo the operation if and only if he has received sufficiently favorable information from other experts, and that her recommendation affects her payoff if and only if she is pivotal. In an equilibrium with a low search cost, the loser's curse consideration leads each expert to recommend the operation more often than she would, were she base her recommendation only on her privately observed signal. This in turn hurts the agent because it worsens the quality of information transmitted by each expert.

An implication of the loser's curse is that the agent's equilibrium payoff may go down with a decrease in his search cost. The intuition is as follows. Suppose an expert suffers less than the agent from a failed operation and thus is inherently biased towards recommending the operation. As the agent's search cost gets smaller, the experts anticipate that he can afford to consult more experts and will decide to undergo the operation only after he becomes more confident. Each expert is therefore willing to further lower her own standard for recommending the operation, making a recommendation even less informative. Consequently, the agent may become worse off, as he needs to consult a larger number of experts and hence incur a higher total search cost to reach the same level of confidence before deciding to undergo the operation.

We illustrate the effect discussed in the previous paragraph by investigating the properties of the limiting equilibrium with vanishing search cost. It is found that as the search cost vanishes, the loser's curse effect becomes extremely severe: every expert almost always discards her private signal and recommends the operation. Moreover, if the expert's signal structure does not contain a signal that fully reveals $\omega = 0$, and if each expert suffers less than the agent in the case of a failed operation, there are informative signal structures under which both the agent's payoff and the social welfare are strictly lower than those in the single-expert benchmark.

The intuition is as follows. Suppose the experts are willing to adopt a partially informative recommendation rule. In the absence of a signal that fully reveals $\omega = 0$, the agent can never be very confident that $\omega = 1$. Otherwise, the experts would know that being pivotal is extremely good news and would always recommend the operation, thus making her recommendation completely uninformative. The agent's posterior belief when he decides to undergo the operation is therefore bounded away from one, and this observation provides an upper bound on the agent's limit equilibrium payoff. Furthermore, we find that this upper bound decreases with the experts' liability. Intuitively, if experts suffer little from a failed operation, they would inherently adopt a low standard for recommending the operation. This worsens the informativeness of their recommendations, making the agent's search for advice more ineffective. Somewhat strikingly, we find that if the experts' liability is sufficiently small, then the only equilibrium outcome involves experts adopting a completely uninformative recommendation rule, even though partial information transmission in the single-expert benchmark is possible.

The loser's curse has a significant impact on the effectiveness of collecting information dispersedly held by experts. As each expert observes a conditionally independent signal about the state ω, if the agent can consult a large enough number of experts whose recommendations are sufficiently informative about their signals, he is able to learn the true state ω and hence take the ex-post correct decision concerning the operation almost surely —– information is perfectly aggregated. The argument in the previous paragraph suggests that the existence of a signal that fully reveals $\omega = 0$ is a necessary condition for perfect information aggregation to arise in the limit with vanishing search cost. It turns out the condition is also almost sufficient,[1] and the reason is as follows. As noted above, the loser's curse makes each expert almost always recommend the operation in the limit. A recommendation against the operation thus implies that the expert must have seen a signal that fully (or almost fully) reveals that $\omega = 0$. As the search cost is vanishingly small, the agent can afford to sample a large number of recommendations, and is bounded to receive quite a number of recommendations against the operation if indeed $\omega = 0$. Therefore, the agent can learn the true state almost surely.

Our analysis suggests that the loser's curse can potentially be a significant source of inefficiency in settings involving search for advice. By mitigating the loser's curse, the agent's welfare as well as social welfare may be improved by policies that mandate a higher consultation fee, or enforce contracts that commit the agent not to seek second opinions.

1.1 Related Literature

This paper is related to the following four lines of literature.

Large election Feddersen and Pesendorfer (1997) analyze two-candidate elections in which

[1] An extra condition is needed to ensure the uniqueness of equilibrium outcome. See Corollary 2.

voters receive conditionally independent signals about a state variable that affects the utility of all voters. They show that as the size of the electorate goes to infinity, the election is almost always very close, and information is perfectly aggregated (in the sense that the election outcome would not change were all private signals become public). Similarly, we consider an agent deciding between two options (whether to have an operation or not) and soliciting recommendations (analogous to votes) from partially informed experts (analogous to voters). We are also interested in the scenario in which the agent's search cost vanishes (analogous to having infinitely many voters). Our model can therefore be viewed as a sequential search version of aggregating information held dispersedly in some population.

An expert in our search setting faces strategic consideration similar to a voter in an election as they both evaluate their payoffs conditional on being pivotal. Despite these apparent similarities, we obtain different results. First, we find that when the agent concludes the search and picks an option, the collection of solicited recommendations would clearly favor the chosen option (and is thus unlike having a close election outcome). Moreover, information is perfectly aggregated if and only if the experts' signal space contains a perfectly-revealing signal, whereas the existence of such signal is not needed for large elections. The difference in results arises because the agent's stopping rule in our search setting is endogenous, whereas the rule (the size of electorate and the fraction of votes a candidate needs to win) is exogenously fixed in an election. The endogeneity of the agent's stopping rule implies that only when he has received sufficient information would he stop searching; this explains the first difference above. More importantly, the endogeneity of the stopping rule implies that being pivotal is good news to the expert performing the operation: the agent must have collected sufficiently favorable information from other experts. In contrast, when the election rule is exogenously fixed, being pivotal is a neutral piece of information: other voters' signals are fairly close. As a result, the loser's curse occurs only in our model (but not in elections), and it makes information aggregation less effective. Consequently, a more stringent condition is needed for perfect information aggregation in the limit.

Other notable studies on the information aggregation properties of elections include Dekel and Piccione (2000) and Feddersen and Pesendorfer (1996). Dekel and Piccione (2000) show that any symmetric equilibrium of a simultaneous voting game remains to be an equilibrium if voters cast their votes sequentially. As a result, the (limit) perfect information aggregation result remains valid in sequential voting games. Feddersen and Pesendorfer (1996) identify the swing voter's curse in elections: less-informed, indifferent voters may strictly prefer to abstain rather than vote for any candidate. They show that even though a substantial fraction of the electorate would abstain, information is still perfectly aggregated in the limit.

Common-value auction Our model has some of the flavor of a common-value auction, as the state of the agent is common to all experts, and the agent eventually picks one expert (if any) to carry out the operation. However, our model does not involve bidding, and each expert only makes a binary recommendation. The relation of our findings on information aggregation and the welfare effect of the loser's curse to the counterparts in the auction literature is discussed below.

Studies in common-value auctions have identified conditions under which the price paid by the winning bidder converges, in the limit as the number of bidders goes to infinity, to the value of the asset. Milgrom (1979) identifies necessary and sufficient conditions for full information aggregation in the limit for single-asset auctions. Pesendorfer and Swinkels (1997) analyze information aggregation in a common-value auction with a large supply. They show that the winner's curse and the loser's curse balance each other out, leading to perfect information aggregation. In the search setting considered here, it is reasonable to fix the supply at one, as each agent's problem is independent. Kremer (2002) shows that the competitiveness of a large auction forces the limiting price to approach the expected asset value conditional on the pivotal bidder's information. Moreover, information is fully aggregated in the limit if and only if the limiting price is informative about the asset's value. In our model of advice-search, we are interested in the condition under which, after sequentially consulting many experts, the agent takes the ex-post correct decision on the operation almost surely. Note that the information aggregation question we consider is quite different from the one considered in the auction literature: whereas the auction literature asks when the price would reflect the underlying state (asset value) in the limit, we ask when the advice of all consulted experts would collectively reflect the underlying state (whether the operation is appropriate).[2]

Although the condition for full information aggregation we obtain is similar to that identified in Milgrom (1979), the underlying driving force is quite different. Whereas the result of Milgrom (1979) is driven primarily by competition among bidders, the experts' loser's curse and the agent's endogenous stopping rule are key for our result.[3] Furthermore, when the condition in Milgrom (1979) fails, the information aggregation properties in the two settings are drastically different: whereas the limiting price is completely uninformative in common-value auctions (as shown in Kremer (2002)), the agent's

[2] Note that if an observer of an auction has access to not only the winning price, but also all bids submitted, then she can always recover the asset value when the number of bidders grows to infinity. To see this, note that in either the first-price or second-price common-value auction, the unique symmetric equilibrium bidding function is strictly monotone, regardless of the number of bidders (see, for instance, Krishna (2009)). Thus, the observer can always back out the signal observed by each bidder.

[3] We discuss this distinction in greater detail in Section 4.3.

eventual decision on the operation in the limit is still partially correlated to the state in our model of advice-search. Intuitively, the difference arises because the competition among experts does not affect their recommendation, so they are still willing to adopt a sufficiently informative recommendation strategy, even when they expect a large number of experts to be consulted.[4] On the other hand, with a large number of bidders, the competition for the common-valued asset becomes extremely intense, forcing the price to approach the asset's expected value conditional on the pivotal bidder's information. When the condition in Milgrom (1979) fails, the pivotal bidder's information is completely uninformative.

Our finding that the agent may suffer from a decrease in his search cost because of an exacerbation of the experts' loser's curse is related to the finding in the auction literature that the auctioneer's expected revenue may decrease in the number of bidders because of the bidders' winner's curse. Bulow and Klemperer (2002) illustrate this possibility with two cases: (i) symmetric bidders with an increasing hazard rate, and (ii) asymmetric bidders with a decreasing hazard rate.[5] In contrast, we consider symmetric experts without any assumption on the hazard rate. Moreover, the underlying economic reason for our finding is fundamentally different from theirs. Whereas the payoff of the auctioneer is the expected price paid by the winning bidder, the agent in our model cares about the quality of information gathered through consulting experts. In a common-value auction, while the winner's curse makes bidders shade their bid, it does not affect the information content of the equilibrium bidding function.[6] On the other hand, in our setting, the loser's curse has a direct impact on the information content of each expert's equilibrium recommendation strategy. We show that the collective information content of all experts' advice may go down as the loser's curse exacerbates with decreasing search cost. Finally, in a common-value auction without reserve price, whenever the auctioneer suffers from an increase in the number of bidders, the joint payoff of bidders would increase by exactly the same amount, as the price is merely a transfer between the winning bidder and the auctioneer. Consequently, there is no net effect on the social welfare. In contrast, in our model, a lower search cost may hurt not only the agent's payoff, but also the experts' joint payoff. As a result, the social welfare can go down with a lower search cost. This different implication for social welfare can be traced to the nature of externality brought about by the loser's curse consideration.[7]

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[4] In fact, the recommendation strategy has to be sufficiently informative to induce equilibrium search by the agent.

[5] Hong and Shum (2002) show empirically that an increase in the number of bidders hurts the auctioneer's payoff in their sample of procurement auctions.

[6] Recall that in either the first-price or second-price common-value auction, the unique symmetric equilibrium bidding function is strictly monotone.

[7] We will discuss this point in more detail in Section 4.2.

Search with adverse selection Our model features an agent sequentially consulting multiple experts, who hold private information concerning a common state of nature. Therefore, it is related to a body of work that studies the interaction of search activity and information asymmetry, such as Lauermann and Wolinsky (2016), Inderst (2005) and Guerrieri, Shimer, and Wright (2010). In Lauermann and Wolinsky (2016), a buyer is endowed with private information about the cost of the transaction (incurred by the seller), and he sequentially samples sellers who observe conditionally independent signals about his cost. The buyer has incentives to search for a seller who observes a favorable signal and hence is willing to accept a low price. Lauermann and Wolinsky (2016) identify a strong winner's curse, which they call the sampling curse, in this search setting. The sampling curse implies that perfect information aggregation (in the limit as search cost vanishes) requires stronger conditions than that of Milgrom (1979). Our setting is similar to theirs, as the agent's (buyer's) type distribution, from the expert's (seller's) perspective, is endogenously determined by his search behavior, and this affects the experts' (sellers') best response. However, the reason for search is quite different in the two settings: the agent in our model consults experts to learn about an underlying state, whereas the buyer in their setting searches for a favorable deal. Being sampled is necessarily bad news for a seller in their setting, as a high-cost buyer tends to search longer. Experts in our setting may find it either good news or bad news, depending on the expected duration of search for each state ω.

Credence goods provision In the credence goods market, the expert becomes more informed about the type and/or quality of the service the customer needs after performing a diagnosis. The expert then recommends and provides, subject to the customer's approval, the recommended service (see Dulleck and Kerschbamer (2006) for a survey). Inefficiency in the competitive market for credence goods has been studied in Wolinsky (1993), Pesendorfer and Wolinsky (2003), Wolinsky (2005), and Alger and Salanié (2006). Wolinsky (1993) shows that competition can mitigate experts' incentives to prescribe overtreatment (i.e., providing unnecessarily expensive treatment) if there are firms that specialize in providing low-cost repair. Alger and Salanié (2006) show that price competition in the low-cost repair induces overtreatment. In contrast, our model does not feature any price competition, and overtreatment is not a problem here. Pesendorfer and Wolinsky (2003) analyze a setting in which experts need to exert effort to learn about the treatment needed, and show that price competition leads to inefficient effort exertion. Wolinsky (2005) analyzes a setting in which experts exert effort to devise an appropriate plan for the customer. Inefficiency arises because the customer does not internalize the effort cost of the experts sampled. Our model, on the other hand, does not have any moral hazard.

The outline of the paper is as follows. The model is set up in Section 2. In Section 3, we analyze the benchmark case in which there is only one available expert. In the main analysis of the model in Section 4, we first establish equilibrium existence and provide some characterizations. We then illustrate a potential inefficiency caused by the agent's search behavior and the loser's curse consideration by looking at the limit equilibrium with a vanishingly small search cost in Section 4.2. Finally, in Section 4.3, we identify conditions under which dispersed information is perfectly aggregated in the limit of vanishing search cost. Section 5 discusses a few alternative settings. Lengthy proofs are relegated to the appendix.

2 Model

An (male) agent can either undergo an operation (denoted by $a = 1$) or not (denoted by $a = 0$). His payoff of undergoing the operation depends on a binary state of the world $\omega \in \{0, 1\}$. If the state is $\omega = 1$ ($\omega = 0$), then the operation is suitable (unsuitable) for the agent. His prior belief about the state is denoted by $\pi \equiv \Pr(\omega = 1) \in (0, 1)$. The operation must be carried out by an (female) expert. There are infinitely many ex-ante identical experts. In each of the infinitely many periods, the agent can visit one expert. Upon a visit, each expert conducts a test which generates an informative signal about the state ω. After privately observing the signal, she then makes a recommendation to the agent. The experts have a common payoff function, as well as common information acquisition technology, which will be discussed below. The agent has free access to one expert, and he always consults this first expert. For each additional visit and consultation of other experts, the agent has to incur a search cost of $c \in (0, 1)$. Each of the infinitely many experts is drawn (without recall) with equal probability in every period.

Each expert can run a costless test to obtain a signal $s \in [\underline{s}, \bar{s}] \subset [0, \infty]$ about ω. The signal of each expert is distributed identically and independently (conditional on the state ω). Specifically, the signal is generated according to conditional density function $f(s|\omega)$, with the corresponding conditional distribution function $F(s|\omega)$. It is without loss to label the signals as their likelihood ratios, i.e., $s \equiv \frac{f(s|1)}{f(s|0)}$. With this labelling, a high signal is more indicative of $\omega = 1$. Moreover, the signal structure is informative if and only if $\underline{s} \in [0, 1)$ and $\bar{s} \in (1, \infty]$. In addition to admitting a conditional density function, we further assume that $f(s|\omega)$ has full support on $[\underline{s}, \bar{s}]$, for $\omega = 0, 1$.

The signal realization of the test is unverifiable and observed privately by the consulted expert. Moreover, we assume that the signal realization is so complicated that it is infeasible to communicate its full content to the agent. Instead, each expert makes a binary recommendation of having the operation or not. Denote the set of recommendations by $\{Y, N\}$, where Y stands for a non-binding recommendation for the operation, and N stands for a recommendation against the operation. If the expert recommends against the operation (i.e. recommendation N), it means she refuses to carry out the operation for the agent, who must then part with the expert. On the other hand, if the expert suggests having the operation (i.e. recommendation Y), it means she is willing to perform the operation for the agent, who can choose whether or not to have the operation with her. In other words, a recommendation N is a rejection by the expert, whereas a recommendation Y means the expert provides an option for the agent to undergo the operation with her. If the expert makes recommendation Y and the agent agrees to have the operation with her, then they collect their respective payoffs described below, and the game is over. If the agent chooses not to have the operation with her, he then parts with the expert. He can then either consult another expert, or stop the search for advice altogether.

The respective payoffs received by the agent and the expert for different scenarios are tabulated below:

State		Action $a = 1$	Action $a = 0$
	$\omega = 1$	$1, 1$	$0, 0$
	$\omega = 0$	$-L, -l$	$0, 0$

Here, L and l are positive numbers. The agent receives a positive payoff normalized to one if the state is $\omega = 1$ and the operation is carried out. He suffers a loss L if the state is $\omega = 0$ and the operation is carried out. His payoff of not undergoing the operation is state-independent and normalized to zero. To see that the payoff specification above is a normalization, suppose the agent's payoff $u_{\omega a}$ depends on both action a and state ω, with the property that $u_{11} > u_{10}$ and $u_{00} > u_{01}$. Given an arbitrary belief $p \in [0, 1]$ that $\omega = 1$, the agent prefers having the operation if and only if $pu_{11} + (1-p)\, u_{01} \geq p\,(u_{10}) + (1-p)\, u_{00}$, which can be re-arranged as $p\,(u_{11} - u_{10}) + (1-p)\,(u_{01} - u_{00}) \geq 0$, or simply $p \geq \left(1 + \frac{u_{11}-u_{10}}{u_{00}-u_{01}}\right)^{-1}$. Therefore, setting $u'_{10} = u'_{00} = 0$, $u'_{11} = u_{11} - u_{10}$ and $u'_{01} = u_{01} - u_{00}$ would leave the agent's behavior unaffected. Moreover, the agent's behavior depends only on the ratio $\frac{u'_{11}}{u'_{01}}$, so it is without loss to further normalize $u'_{11} = 1$.

The expert who performs the operation for the agent has a partially-aligned payoff function: a positive payoff normalized to one if the operation is indeed suitable for the agent, and a negative payoff equals $-l$ if the operation is not suitable. Her payoff is normalized to zero if she does not carry out the operation for the agent. It is clear that if the state ω is publicly known, the agent and all experts would agree on the operation decision.

We assume the agent cannot communicate his history of recommendations received from previously consulted experts. One justification for this assumption is that if the message about this history is cheap talk, there is always an equilibrium in which such communication is babbling. Furthermore, we assume the search process is without recall. This is an innocuous assumption as all experts are identical replica of each other. If there is a small positive cost of returning to a previously visited expert, the agent strictly prefers to carry out the operation with the current expert. Finally, we assume that the experts do not know the number of previous experts the agent has consulted. This assumption is natural if the agent becomes aware of his problem at a random time, which is not observable by the experts. Further discussion on these assumptions can be found in Section 5.

To summarize, the timeline of the game is as follows. Events unfold in each period in the following order:

1. The agent decides whether to consult an expert (that he has not visited before) or not. If he chooses not to, nothing else happens in this period. If he decides to consult an expert, he has to incur a search cost c (his first consultation is free).

2. An expert is randomly chosen to be consulted. The expert consulted privately observes a signal s about the state ω. She then makes a recommendation for or against the operation, denoted as recommendations Y and N respectively. If she recommends N, then she refuses to carry out the operation for the agent, and nothing else happens in this period.

3. If the expert recommends Y, then the agent chooses whether or not to have the operation with her. If he does, they collect their respective state-dependent payoffs. Otherwise, the expert collects a zero payoff and her role in the game is over.

In our model, a pure strategy of an expert is the set of signals, denoted by A, under which she recommends the operation (i.e., choosing Y). A history of the agent after consulting n experts is a sequence of recommendations $\{Y, N\}^n$. Denote the set of all possible recommendation histories by $H = \{\varnothing\} \cup (\cup_{n \in \mathbb{N}} \{Y, N\}^n)$. At the beginning of a period, the agent decides whether to visit an expert that he has not consulted before. At the end of a period, if he has consulted an expert and the expert recommends Y, he then decides whether to have the operation with the current expert or not. A behavioral strategy of the agent, denoted by $b = (b_0, b_1)$, consists of two components, both of which are mappings from H to $[0, 1]$. First, $b_0(h)$ is the probability that the agent chooses not to consult any expert for that period, provided that the current history is h. Second, $b_1(h)$ is the probability that the agent decides to have the operation at the end of a period after being recommended Y by the current expert.[8] Denote by Λ the set of all behavioral strategies of the agent.

The solution concept is weak perfect Bayesian equilibrium. Moreover, as all experts face an identical problem, we restrict attention to equilibria in which all experts play an identical pure strategy. Furthermore, we impose the following restriction on the agent's off-the-equilibrium-path belief. If $A = \varnothing$, then the agent's posterior belief after learning a recommendation Y cannot be lower than that after learning a recommendation N (which coincides with his belief prior to getting any recommendation). In other words, the agent cannot treat a recommendation Y as a worse news than a recommendation N.

Finally, we say an equilibrium is **informative** if both (i) the experts' recommendation varies with signal, i.e., $A \neq \varnothing$ and $A \neq [\underline{s}, \bar{s}]$, and (ii) the agent's strategy satisfies $b_1(h, Y) > 0$ for some history $h \in H$ on the equilibrium path. If an equilibrium fails either one of the conditions above, then it is uninformative. Intuitively, in an informative equilibrium, the equilibrium outcome of whether the operation is eventually carried out varies stochastically with the state ω. In an uninformative equilibrium, the outcome is either that the operation is always carried out irrespective of the state, or it is never carried out irrespective of the state. While informative and uninformative equilibria may coexist for some parameters, our subsequent analysis would put more emphasis on informative equilibria (whenever they exist) because they have more interesting welfare and information properties.

3 Benchmark Model: Single Expert

For comparison of results later, we consider a benchmark model in which the agent can consult only one expert. The specific objective is to compute the maximum payoff of the agent and the expert among all permissible signal structures.

We first identify necessary and sufficient conditions for an informative equilibrium. These conditions require the expert to make her recommendation dependent on the observed signal, as well as require the agent to follow her recommendation. Specifically, after learning signal s, the expert updates her belief on ω according to Bayes' rule:

$$\Pr(\omega = 1|s) = \frac{\pi f(s|1)}{\pi f(s|1) + (1-\pi) f(s|0)} = \frac{1}{1 + \frac{1-\pi}{\pi}\frac{1}{s}}.$$

The expert finds it optimal to recommend the operation if

$$\Pr(\omega = 1|s) - l \Pr(\omega = 0|s) \geq 0 \Leftrightarrow s \geq \frac{1-\pi}{\pi} l \equiv \tilde{s}. \tag{1}$$

[8] In our notation, for $i = 0, 1$, $b_i(h) = 1$ stands for stopping the search, and $b_i(h) = 0$ stands for continuing the search.

Therefore, her recommendation is partially informative if and only if $\tilde{s} \in (\underline{s}, \bar{s})$, or equivalently, $\pi \in \left(\frac{l}{\bar{s}+l}, \frac{l}{\underline{s}+l}\right)$. As this cutoff $\tilde{s}$ is unique, there is at most one informative equilibrium.

Upon receiving a positive recommendation from the expert, the agent's payoff of taking the operation is

$$\Pr(\omega = 1 | s \geq \tilde{s}) + (-L)\Pr(\omega = 0 | s \geq \tilde{s}) = -L + \frac{1}{1 + \frac{1-\pi}{\pi}\frac{1-F(\tilde{s}|0)}{1-F(\tilde{s}|1)}}(1 + L).$$

The agent follows the recommendation if the payoff above is nonnegative, or equivalently,

$$\frac{\pi}{1-\pi}\frac{1 - F(\tilde{s}|1)}{1 - F(\tilde{s}|0)} \geq L. \tag{2}$$

Consequently, an informative equilibrium exists if and only if both $\pi \in \left(\frac{l}{\bar{s}+l}, \frac{l}{\underline{s}+l}\right)$ and inequality (2) holds.

Next, consider uninformative equilibria. An uninformative equilibrium in which the agent always takes the operation requires that $\tilde{s} \leq \underline{s}$ and $-L + \pi(1 + L) \geq 0$, so that the expert always recommends the operation and the agent always follows. These conditions can be rearranged into $\pi \geq \max\left\{\frac{l}{l+\underline{s}}, \frac{L}{1+L}\right\}$. It is clear that the equilibrium is unique when this inequality holds. An uninformative equilibrium in which the agent never takes the operation exists if either $\tilde{s} \geq \bar{s}$ (equivalently, $\pi \leq \frac{l}{\bar{s}+l}$) or $\pi \leq \frac{L}{1+L}$. In the former case, although the agent may follow a recommendation Y, the expert does not find it profitable to carry out the operation even with the most favorable signal. In the latter case, the agent always rejects recommendation Y under the belief that such a recommendation is uninformative.[9] Conversely, if $\pi > \max\left\{\frac{l}{l+\bar{s}}, \frac{L}{1+L}\right\}$, there is no uninformative equilibrium with $s^* = \bar{s}$. This is because each expert could profitably deviate by recommending Y after observing a signal sufficiently close to $\bar{s}$.[10]

While an informative equilibrium and an uninformative equilibrium in which the agent never takes the operation may coexist, the former strictly Pareto dominates the latter. This follows immediately from the observation that either the expert or the agent can veto the operation, so their respective expected payoffs must be positive whenever the operation is carried out.[11]

[9] The belief that recommendation Y is uninformative means that the posterior after getting the recommendation remains at π. This is the most pessimistic belief that satisfies our off–the-equilibrium-path belief requirement set out at the end of Section 2.

[10] Recall that according to the requirement on off-the-equilibrium-path belief introduced in Section 2, the agent must treat a recommendation Y at least as favorably as a recommendation N. As $\pi > \frac{L}{1+L}$, the agent necessarily accepts a recommendation Y. This gives a strictly positive profit to the expert as $\pi > \frac{l}{l+\bar{s}}$.

[11] Another reason why the informative equilibrium is more reasonable is that if an expert believes with positive probability (regardless of how small it is) that the agent may follow her recommendation Y, she would find it strictly optimal

Denote by $U_1(F)$ the agent's highest equilibrium payoff given a signal structure F of the expert. The discussion above yields

$$U_1(F) = \begin{cases} \pi\left(1 - F\left(\frac{1-\pi}{\pi}l|1\right)\right) - L(1-\pi)\left(1 - F\left(\frac{1-\pi}{\pi}l|0\right)\right) & \text{if (2) holds and } \pi \in \left(\frac{l}{l+\bar{s}}, \frac{l}{l+\underline{s}}\right); \\ -L + \pi(1+L) & \text{if } \pi > \max\left\{\frac{l}{l+\underline{s}}, \frac{L}{1+L}\right\}; \\ 0 & \text{otherwise.} \end{cases} \quad (3)$$

Similarly, the highest equilibrium payoff of the expert, denoted by $T_1(F)$, is given by

$$T_1(F) = \begin{cases} \pi\left(1 - F\left(\frac{1-\pi}{\pi}l|1\right)\right) - l(1-\pi)\left(1 - F\left(\frac{1-\pi}{\pi}l|0\right)\right) & \text{if (2) holds and } \pi \in \left(\frac{l}{l+\bar{s}}, \frac{l}{l+\underline{s}}\right); \\ -l + \pi(1+l) & \text{if } \pi > \max\left\{\frac{l}{l+\underline{s}}, \frac{L}{1+L}\right\}; \\ 0 & \text{otherwise.} \end{cases} \quad (4)$$

Bounds on Payoffs

In this subsection, we derive tight upper bounds on $U_1(F)$ and $T_1(F)$ **among all permissible signal structures,**[12] **fixing the lower bound of the signal space** $\underline{s} \in [0, 1)$**.** The main purpose of this derivation is to compare the welfare between the single-expert benchmark case analyzed above, and that of the case in which the agent can sequentially seek advice from multiple experts (to be considered in Section 4 below). As the most interesting results in this comparison concern the case $l < L$, i.e., the agent suffers more from a failed operation than the expert, we will focus on this case for the rest of this subsection.[13]

Suppose for the time being that a discrete signal structure is permitted, i.e., the signal space can be finite. Denote by $F_{\underline{s}}$ the most informative signal structure, given a fixed lower bound of the signal space $\underline{s}$. It has a binary support: $\{\underline{s}, \bar{s}\}$, where $\bar{s}$ fully reveals $\omega = 1$. Under $F_{\underline{s}}$, the probabilities of $\underline{s}$ and $\bar{s}$ conditional on ω are respectively $\Pr(\underline{s}|\omega = 0) = 1$, and $\Pr(\underline{s}|\omega = 1) = \underline{s} = 1 - \Pr(\bar{s}|\omega = 1)$.[14]

to condition her recommendation on her observed signal.

[12] Recall that a signal structure is permissible if there is a conditional density function with a connected support.

[13] The other case of $l > L$ can be analyzed using the same method described below. It is omitted, as the welfare comparison conducted later does not involve this case.

[14] That is, $F_{\underline{s}}(\underline{s}|0) = 1$, and

$$F_{\underline{s}}(s|1) = \begin{cases} 0 & \text{if } s < \underline{s} \\ \underline{s} & \text{if } s \in [\underline{s}, \bar{s}] \\ 1 & \text{if } s = \bar{s} \end{cases}.$$

Lemma 1 *Fix the lower bound of the expert's signal space at* $\underline{s} \in [0,1)$. *Suppose discrete signal structures are permitted. If* $l < L$, *then* $U_1(F)$ *and* $T_1(F)$ *are both maximized at* $F = F_{\underline{s}}$. *If an informative equilibrium exists, which requires* $\pi < \frac{l}{l+\underline{s}}$, *then*

$$U_1(F_{\underline{s}}) = T_1(F_{\underline{s}}) = \pi(1-\underline{s}).$$

Lemma 1 is quite intuitive. As the interests of the agent and the expert are partially aligned, it is not surprising that the agent would prefer an expert endowed with a more informative signal structure. Note that if $\underline{s} \geq \tilde{s}$, or equivalently, $\pi \geq \frac{l}{l+\underline{s}}$, there is no informative equilibrium, and the payoffs $U_1(F)$ and $T_1(F)$ are independent of the expert's signal structure. In this case, the lemma holds trivially.

The signal structure F that maximizes $U_1(F)$ can be computed by considering the following auxiliary game. Take the same setting as the benchmark model, but allow the agent to choose F, and mandate the expert to truthfully report the learned signal along with her recommendation. As the agent in this auxiliary game can never be worse off than his counterpart in the benchmark model, an upper bound to $U_1(F)$ can be found by solving the optimal signal structure of this auxiliary game, which turns out to be $F_{\underline{s}}$. The fact that this upper bound is achievable follows from the simplicity of $F_{\underline{s}}$: the agent does not need to know the realized signal to implement the optimal decision rule. The signal structure that maximizes $T_1(F)$ can be computed similarly by considering the expert's optimal choice of F.

It is not difficult to see that $F_{\underline{s}}$ can be approximated arbitrarily well using signal structures with a connected support $[\underline{s}, \infty]$ and well-defined conditional density function. Consequently, $U_1(F_{\underline{s}})$ and $T_1(F_{\underline{s}})$ indeed provide tight upper bounds for equilibrium payoffs of the single-expert benchmark.

Proposition 1 *Fix a* $\underline{s} \in [0,1)$ *and denote by* $\Sigma_{\underline{s}}$ *the set of permissible signal structures with a lower bound of the signal space* $\underline{s}$. *Suppose* $l < L$ *and* $\pi < \frac{l}{l+\underline{s}}$. *Then*

$$\sup_{F \in \Sigma_{\underline{s}}} U_1(F) = U_1(F_{\underline{s}}) \text{ and } \sup_{F \in \Sigma_{\underline{s}}} T_1(F) = T_1(F_{\underline{s}}).$$

4 Multiple Experts

We analyze the main model, in which the agents can sequentially consult multiple experts who respond to the agent's search strategy optimally. We show that an equilibrium exists and provide some characterizations in Section 4.1. In Section 4.2 and Section 4.3, we consider, respectively, the issues of welfare and information aggregation in the limiting case of vanishing search cost.

4.1 Equilibrium Existence and Characterization

Consider first the agent's search strategy. Formally, a behavioral strategy of the agent $b = (b_0, b_1) \in \Lambda$ is a mapping from the set of recommendation histories H to the probability of stopping the search at the beginning of a period (captured by b_0), as well as the probability of following a recommendation Y at the end of a period (captured by b_1). Recall that a pure strategy of an expert is a set of signals A under which she recommends Y, and we focus on equilibrium in which the experts play an identical strategy. The agent's posterior belief (that $\omega = 1$) associated with a history $h \in H$ depends only on the experts' strategy A, and can be computed with a simple application of Bayes' rule. Therefore, we can write $p(h; A)$ to stand for the induced posterior belief. The following lemma states that the agent's best response to any common strategy of the experts has a Markov structure, with the state being the agent's current belief. The Markov property of the agent's strategy follows from the simple observation that each expert's recommendation strategy is invariant in her position in the agent's search order.

Lemma 2 *For any strategy A of the experts, the optimal search strategy of the agent is characterized by a unique pair of posterior beliefs $p_0(A), p_1(A) \in [0, 1]$ and has the following structure.*

(i) At the beginning of a period (other than the first period), the agent quits searching if $p(h; A) < p_0(A)$, and consults an expert if $p(h; A) > p_0(A)$. At $p(h; A) = p_0(A)$, the agent is indifferent between quitting and consulting an expert.

(ii) Suppose the current expert recommends Y. The agent takes the operation with her if $p(h; A) > p_1(A)$, and does not take it if $p(h; A) < p_1(A)$. At $p(h; A) = p_1(A)$, the agent is indifferent between taking and not taking.

As each expert adopts an identical recommendation strategy, the agent faces a standard Markov decision problem and the cutoff property of his optimal search strategy follows naturally. For a formal proof of Lemma 2, see Ross (1983). As is standard in dynamic programming, the unique cutoffs $p_0(A)$ and $p_1(A)$ can be found by computing the agent's continuation value as a function of his current belief. The details are relegated to the appendix.

If $p_0(A) < p_1(A)$ and $\pi \in (p_0(A), p_1(A))$, then the agent would typically solicit a number of advice, stopping the search whenever $p(h; A)$ falls below $p_0(A)$ or rises above $p_1(A)$. As it is assumed that the agent always consults the first expert (which is costless to him), it is possible that an expert may meet an agent with a prior belief less than $p_0(A)$, but this can only arise in the first period and $\pi < p_0(A)$. Moreover, an agent may hold a belief exceeding $p_1(A)$ prior to learning the current expert's recommendation. This possibility can only arise if $\pi > p_1(A)$ and the agent has not received any recommendation Y before consulting the current expert.[15]

To fully specify an optimal strategy for the agent, it remains to describe his behavior when his belief hits exactly $p_0(A)$ or $p_1(A)$. Lemma 2 implies that the agent's best response takes the following form:

$$b_0(h) \begin{cases} = 0 & \text{if } p(h;A) \in (p_0(A), 1] \\ \in [0,1] & \text{if } p(h;A) = p_0(A) \\ = 1 & \text{if } p(h;A) \in [0, p_0(A)) \end{cases} ; \text{ and}$$

$$b_1(h) \begin{cases} = 0 & \text{if } p(h;A) \in [0, p_1(A)) \\ \in [0,1] & \text{if } p(h;A) = p_1(A) \\ = 1 & \text{if } p(h;A) \in (p_1(A), 1] \end{cases} . \quad (5)$$

Next, consider the problem of the experts. The key observation is that an expert's recommendation matters to her payoff if and only if she is pivotal. Specifically, a recommendation N gives her a sure payoff of zero. A recommendation Y gives her a non-zero payoff if and only if the agent follows her recommendation and takes the operation with her. Therefore, when deciding her recommendation, the expert should compare her payoffs conditional on the pivotal event that the agent would follow her recommendation Y and take the operation with her. Denote this pivotal event by $piv(b)$, and suppose it is a positive-probability event in equilibrium. Then, facing a strategy profile (A, b) of other players, an individual expert makes a recommendation Y if and only if her signal $s \in [\underline{s}, \bar{s}]$ is such that

$$\Pr(\omega = 1 | s, piv(b), A, b) \geq \frac{l}{1+l}. \quad (6)$$

The conditional probability in inequality (6) is increasing in s, because holding other things constant, a higher signal s (which, conditional on ω, is independent of other events in the game) indicates a higher likelihood of $\omega = 1$. Consequently, each expert necessarily adopts a cutoff strategy: recommend the operation if and only if $s \geq s^*$, for some $s^* \in [\underline{s}, \bar{s}]$. Without loss of generality, we will simply use the cutoff s^*, instead of the recommendation set A, to stand for an expert's strategy.

The conditional probability $\Pr(\omega = 1 | s, piv(b), s^*, b)$ can be expressed more explicitly in terms of the strategy profile (s^*, b). To this end, observe first that each strategy profile (s^*, b) generates a distribution

[15] Another possibility of the optimal search strategy features $p_0(A) > p_1(A)$, which may arise if the search cost is sufficiently high and/or the experts' recommendations are sufficiently uninformative. If $\pi < p_0(A)$, then the agent will not search beyond the first expert. If π exceeds $p_0(A)$ sufficiently, then the agent may search beyond the first expert and stop either when $p(h;A)$ falls below $p_0(A)$ or when some expert recommends Y before $p(h;A)$ falls below $p_0(A)$.

over the agent's histories of received recommendations. It thus determines the distribution of the agent's belief prior to meeting an expert, conditional on the pivotal event that the agent is going to accept the expert's recommendation Y. Specifically, denote by $q_\omega(h; s^*, b)$ the ex-ante probability that history $h \in H$ arises, given a strategy profile (s^*, b) and the state being $\omega \in \{0, 1\}$. Using Bayes' rule and the conditional independence of s, the conditional probability $\Pr(\omega = 1|s, piv(b), s^*, b)$ can be expressed as

$$\Pr(\omega = 1|s, piv(b), s^*, b) = \left(1 + \frac{1}{s}\frac{1-\pi}{\pi}\frac{\sum_{h\in H} q_0(h; s^*, b)\, b_1(h, Y)}{\sum_{h\in H} q_1(h; s^*, b)\, b_1(h, Y)}\right)^{-1}. \tag{7}$$

Recall that in an informative equilibrium, $b_1(h, Y) > 0$ for some $h \in H$ on the equilibrium path, or equivalently, $piv(b)$ is a positive-probability event. In this case, the likelihood ratio $\frac{\sum_{h\in H} q_0(h;s^*,b)b_1(h,Y)}{\sum_{h\in H} q_1(h;s^*,b)b_1(h,Y)}$, associated with the pivotal event $piv(b)$, is well-defined (i.e., it takes a value in $[0, \infty]$). Therefore, the conditional probability $\Pr(\omega = 1|s, piv(b), s^*, b)$ is also well-defined in an informative equilibrium.

Using the observations above, in an informative equilibrium such that $s^* \in (\underline{s}, \bar{s})$, each individual expert plays a best response to the agent's strategy b and other experts' strategy s^* if and only if

$$\left(1 + \frac{1}{s^*}\frac{1-\pi}{\pi}\frac{\sum_{h\in H} q_0(h; s^*, b)\, b_1(h, Y)}{\sum_{h\in H} q_1(h; s^*, b)\, b_1(h, Y)}\right)^{-1} = \frac{l}{1+l}. \tag{8}$$

In sum, a strategy profile (s^*, b) constitutes an informative equilibrium if and only if

1. experts' best response: equation (8) holds;
2. agent's best response: b satisfies equation (5) with $A = s^*$;
3. informative outcome: $s^* \in (\underline{s}, \bar{s})$ and $piv(b)$ is a positive-probability event.

Below, we discuss the implication of the pivotal nature of the experts' problem by considering equation (8). When an individual expert compares her respective expected payoff of recommending Y and N, she should do her calculations conditional on being pivotal. If the agent is expected to engage in an active search for advice, then being pivotal is good news to an expert because the agent has collected sufficiently favorable information from other experts that warrant implementing the operation with a single extra recommendation Y. In other words, in the event that an expert is pivotal, the agent's belief prior to consulting her must be sufficiently close to $p_1(s^*)$ (or above). Specifically, denote by $\tilde{p}_1(s^*)$ the minimum belief for the consulted expert to be pivotal. That is, with a beginning-of-period belief $\tilde{p}_1(s^*)$, the agent's posterior would jump up to $p_1(s^*)$ after receiving a recommendation Y from the current expert. Thus, $p_1(s^*)$ and $\tilde{p}_1(s^*)$ are related by

$$p_1(s^*) = \frac{1}{1 + \frac{1-\tilde{p}_1(s^*)}{\tilde{p}_1(s^*)}\frac{1-F(s^*|0)}{1-F(s^*|1)}}. \tag{9}$$

Using the definition of $\tilde{p}_1(s^*)$, the likelihood ratio associated with the event $piv(b)$ in equation (8) can be bounded from above as follows:

$$\frac{1-\pi}{\pi}\frac{\sum_{h\in H} q_0(h;s^*,b)\, b_1(h,Y)}{\sum_{h\in H} q_1(h;s^*,b)\, b_1(h,Y)} \leq \frac{1-\pi}{\pi} \sup_{\{h\in H: b_1(h,Y)>0\}} \frac{q_0(h;s^*,b)}{q_1(h;s^*,b)} \leq \frac{1-\tilde{p}_1(s^*)}{\tilde{p}_1(s^*)}.$$

The equilibrium condition on the experts' strategy (8) then implies:

$$\left(1+\frac{1}{s^*}\left(\frac{1-\tilde{p}_1(s^*)}{\tilde{p}_1(s^*)}\right)\right)^{-1} \leq \frac{l}{1+l}. \tag{10}$$

Therefore, when deciding her recommendation, it is as if the expert replaces the prior belief π with some belief higher than $\tilde{p}_1(s^*)$. If $\tilde{p}_1(s^*) > \pi$, then being pivotal is necessarily a good news in equilibrium. A failure to take this into account would make an individual expert suffer the **loser's curse**: by adopting an excessively high cutoff, she may end up recommending N, even though the expected payoff of carrying out the operation (which would happen were she recommend Y) is positive. In the next subsection, we will show that $\tilde{p}_1(s^*) > \pi$ arises in equilibrium if the search cost is sufficiently small and π is not too high.

In addition to the informative equilibria discussed above, uninformative equilibria may also exist. In an uninformative equilibrium, the agent and the experts play their respective best response to each other, and their beliefs are updated using Bayes' rule whenever possible. However, the equilibrium outcome is not informative of the state ω in that either (i) $s^* \in \{\underline{s}, \bar{s}\}$ or (ii) $s^* \in (\underline{s}, \bar{s})$ but $b_1(h,Y) = 0$ for all on-the-equilibrium-path history h. As the experts' recommendations are uninformative, the agent would not search beyond the first expert, i.e., $b_0(h) = 1$ for all $h \neq \varnothing$. In an uninformative equilibrium, each consulted expert believes (correctly) that she is the first one the agent consults.

The analysis of uninformative equilibria is similar to that in the single-expert benchmark. An uninformative equilibrium in which $s^* = \underline{s}$ and the agent always accepts the first recommendation Y requires $\pi \geq \max\left\{\frac{L}{1+L}, \frac{l}{l+\underline{s}}\right\}$. The condition ensures a nonnegative payoff to the first expert for carrying out the operation regardless of signals, and to the agent for accepting the recommendation. On the other hand, an uninformative equilibrium in which $s^* \in \{\bar{s}, \underline{s}\}$ and the operation is never carried out requires that either $\pi \leq \frac{l}{l+\bar{s}}$ or $\pi \leq \frac{L}{1+L}$ holds. If $\pi \leq \frac{l}{l+\bar{s}}$, the first expert is willing to recommend N regardless of signals. If $\pi \leq \frac{L}{1+L}$, it is a mutual best response for the agent to always reject the first expert's recommendation Y, and for the first expert to ignore her signal in making recommendation. An uninformative equilibrium with $s^* \in (\underline{s}, \bar{s})$ but $b_1(h,Y) = 0$ for all on-the-equilibrium-path history h requires that $\frac{\pi}{1-\pi}\frac{1-F(s^*|1)}{1-F(s^*|0)} < L$, which ensures that the agent always rejects recommendation Y sent by the first expert. Moreover, as $\frac{1-F(s^*|1)}{1-F(s^*|0)} > 1$, the condition implies $\pi \leq \frac{L}{1+L}$. Therefore, whenever such an equilibrium exists, so does an uninformative equilibrium with $s^* = \bar{s}$. The following lemma summarizes the discussion on uninformative equilibria above.

Lemma 3 *(i) There exists an uninformative equilibrium in which $s^* = \underline{s}$ and the agent follows the first expert's recommendation Y if and only if $\pi \geq \max\left\{\frac{L}{1+L}, \frac{l}{l+\underline{s}}\right\}$.*

(ii) There exists an uninformative equilibrium in which the operation is never carried out if and only if $\pi \leq \max\left\{\frac{L}{1+L}, \frac{l}{l+\bar{s}}\right\}$.

According to Lemma 3, among the uninformative equilibria, the outcome in terms of whether the operation is carried out is uniquely determined. The proposition below establishes the existence of informative equilibrium for cases not covered by Lemma 3.

Proposition 2 *An equilibrium exists. Moreover, if $\pi \in \left(\max\left\{\frac{l}{l+\bar{s}}, \frac{L}{1+L}\right\}, \frac{l}{l+\underline{s}}\right)$, then only informative equilibria exist.*

An informative equilibrium may still exist, even if $\pi \notin \left(\max\left\{\frac{l}{l+\bar{s}}, \frac{L}{1+L}\right\}, \frac{l}{l+\underline{s}}\right)$, but its existence is not guaranteed by Proposition 2. In the next subsection, we show that the equilibrium is essentially unique in the limit when the search cost c is vanishingly small. However, we cannot establish equilibrium uniqueness for the general case of a positive c. Thus, if $\pi \in \left(\max\left\{\frac{l}{l+\bar{s}}, \frac{L}{1+L}\right\}, \frac{l}{l+\underline{s}}\right)$, there may be multiple informative equilibria. If $\pi \notin \left(\max\left\{\frac{l}{l+\bar{s}}, \frac{L}{1+L}\right\}, \frac{l}{l+\underline{s}}\right)$, informative and uninformative equilibria may coexist.

4.2 Welfare Loss with Vanishing Search Cost

In this subsection, we illustrate the welfare loss resulting from the experts' loser's curse. To this end, we consider a scenario in which the agent's search cost c is vanishingly small. Specifically, take an arbitrary sequence $\{c_n\}$ such that $c_n > 0$ for all $n \in \mathbb{N}$, and $\lim_{n\to\infty} c_n = 0$. This gives a sequence of equilibria, each of which belongs to the game with the corresponding search cost. In the absence of the loser's curse (for instance, if all experts naively believe that they are the only expert being consulted) so that the experts adopt the cutoff rule in (1), it is straightforward that the agent is able to learn the true state ω with probability arbitrarily close to one as $c_n \to 0$.[16] In this case, there is essentially full efficiency, as the payoff of the agent and expert are both arbitrarily close to the highest possible level, which is equal to π. Therefore, in this hypothetical setting where the loser's curse is absent, the agent (and the social

[16] This is a consequence of the law of large numbers.

planner) would strictly prefer a setting with a negligible search cost, rather than a high search cost under which the agent can afford to consult only one expert. However, the conclusion can be very different once the loser's curse is taken into account. We illustrate this below by computing upper bounds on the equilibrium payoffs of the agent and experts with vanishing search cost, and compare them with those identified in Proposition 1 for the single-expert benchmark. To facilitate this comparison, we focus on the case $l < L$ and $\pi < \frac{l}{l+\underline{s}}$, as in Proposition 1. We believe that these are the relevant parameter configurations in most applications, including the case of a patient looking for adivce on surgery and a claimant looking for legal advice.

As discussed in the previous subsection, the loser's curse would affect each experts' cutoff, and consequently, the quality of information transmission in an informative equilibrium. It turns out this effect becomes extremely severe in the limit as the search cost vanishes. Specifically, Lemma 4 below states that when the search cost is sufficiently small, the experts' recommendation in every informative equilibrium becomes almost completely uninformative. Denote by $s^*(c_n)$ the experts' cutoff in an informative equilibrium of the game with search cost c_n.[17]

Lemma 4 *Suppose $l < L$ and an informative equilibrium exists for each c_n. Then $\lim_{n\to\infty} s^*(c_n) = \underline{s}$.*

The intuition of the lemma is as follows. Suppose $\lim_{n\to\infty} s^*(c_n) > \underline{s}$, so that the experts recommend N for a positive measure of unfavorable signals. In this case, even though the search cost is vanishingly small, the expert's advice remains somewhat informative. Consequently, the agent would sample advice until he is almost sure that $\omega = 1$ before taking the operation. This in turn implies that regardless of the private signal s, as long as the agent takes the operation following a recommendation Y, the expert's belief that the agent has $\omega = 1$ is very close to one. This contradicts that $\lim_{n\to\infty} s^*(c_n) > \underline{s}$.

Lemma 4 implies that if c_n is sufficiently small, the agent must engage in active searching in an informative equilibrium.[18] However, his search strategy is nontrivial: on one hand, sampling an additional advice is almost costless; on the other hand, the advice is almost completely uninformative. The lemma below characterizes the agent's search strategy in the limit. Recall that the agent's equilibrium strategy is characterized by two cutoffs, $p_0(s^*(c))$ and $p_1(s^*(c))$, where $p_1(s^*(c))$ is the cutoff belief at which the agent decides to undergo the operation.

[17] If the game has more than one informative equilibrium, the function $s^*(c_n)$ selects an arbitrary equilibrium cutoff.

[18] To see this, suppose there exists a subsequence $\{n_k\}$ such that in the corresponding informative equilibria, the agent stops at the first expert and $s^*(c_{n_k}) \in (\underline{s}, \bar{s})$. Then it is necessary that $s^*(c_{n_k}) = \tilde{s} = \frac{1-\pi}{\pi} l$ (recall (1)). However, this contradicts Lemma 4.

Lemma 5 *(i) Suppose $l < L$, $\pi < \frac{l}{l+\underline{s}}$ and $\underline{s} \in \left[0, \frac{l}{L}\right]$. If an informative equilibrium exists for each c_n, then* $\lim_{n\to\infty} p_1\left(s^*\left(c_n\right)\right) = \frac{l}{l+\underline{s}}$.

(ii) Suppose $l < L$ and $\underline{s} \in \left(\frac{l}{L}, 1\right)$. Then, for any sufficiently small c_n, there is no informative equilibrium.

The intuition for part (i) of the lemma is as follows. Suppose π is so low that the agent does not stop at the first expert, i.e., $\pi < \tilde{p}_1\left(s^*\left(c_n\right)\right)$ for all c_n. In this case, being a pivotal expert implies that the agent approaches her with a prior belief in the region $\left[\tilde{p}_1\left(s^*\left(c_n\right)\right), p_1\left(s^*\left(c_n\right)\right)\right]$ (with the exact distribution of prior belief determined by the equilibrium strategy profile). Thus,

$$\left(1 + \frac{1}{s^*\left(c_n\right)}\left(\frac{1-\tilde{p}_1\left(s^*\left(c_n\right)\right)}{\tilde{p}_1\left(s^*\left(c_n\right)\right)}\right)\right)^{-1} \leq \frac{l}{1+l} \leq \left(1 + \frac{1}{s^*\left(c_n\right)}\left(\frac{1-p_1\left(s^*\left(c_n\right)\right)}{p_1\left(s^*\left(c_n\right)\right)}\right)\right)^{-1}. \tag{11}$$

As $s^*\left(c_n\right) \to \underline{s}$, the increase in the agent's posterior by a recommendation Y of the pivotal expert vanishes in the limit. That is, $p_1\left(s^*\left(c_n\right)\right) - \tilde{p}_1\left(s^*\left(c_n\right)\right) \to 0$. Now, taking limit on both sides of (11) yields $\lim_{n\to\infty} p_1\left(s^*\left(c_n\right)\right) = \frac{l}{l+\underline{s}}$.

The intuition for part (ii) is as follows. Recall that the agent is willing to take the operation only if his posterior belief p is such that $-L + p\left(1+L\right) \geq 0$, i.e., $p \geq \frac{L}{1+L}$. The assumption $\underline{s} > \frac{l}{L}$ implies that $\frac{l}{l+\underline{s}} < \frac{L}{1+L}$. Consequently, in an informative equilibrium, an individual expert would find the pivotal event to be overwhelmingly good news that justifies a recommendation Y even if she receives signal $\underline{s}$. This is because her expected payoff conditional on being pivotal is no less than

$$-l + \left(1 + \frac{1}{\underline{s}}\frac{1-\frac{L}{1+L}}{\frac{L}{1+L}}\right)^{-1}\left(1+l\right) = \frac{L}{L\underline{s}+1}\left(\underline{s} - \frac{l}{L}\right) > 0.$$

This contradicts the requirement that information is transmitted through recommendation in an informative equilibrium. In this case, only uninformative equilibria exist. Recall that in an uninformative equilibrium, the agent does not search beyond the first expert. Whether the operation is carried out depends on the sign of $-L + \pi\left(1+L\right)$. If it is positive, then the experts adopt $s^* = \underline{s}$ and the agent accepts the first expert's recommendation Y. If it is negative, either the experts always recommend N, or the agent always rejects a recommendation Y.

We can use Lemma 5 to derive an upper bound on the equilibrium payoff of the agent and the experts. Denote by $U\left(c\right)$ the agent's (ex-ante) expected payoff in an equilibrium of the game in which the search cost is c. To be specific, $U\left(c\right)$ is given by the agent's expected payoff from the decision of whether to undergo the operation or not, minus the expected total search cost incurred. Also, denote by $T\left(c\right)$ the ex-ante expected payoff of all experts in the corresponding game.[19]

[19] If the game has more than one equilibrium, the functions $U\left(c\right)$ and $T\left(c\right)$ select the payoffs of an arbitrary equilibrium.

Consider first the condition for part (i) of Lemma 5: $\underline{s} \in \left[0, \frac{l}{L}\right]$ and $\pi < \frac{l}{l+\underline{s}}$. The key observation is that in an informative equilibrium, when the agent's belief reaches $p_1 \left(s^* \left(c_n\right)\right)$, it is optimal for him to take the operation, which gives him an expected payoff $-L + (1+L) p_1 \left(s^* \left(c_n\right)\right)$. Moreover, as the agent's updated belief in the search process is a martingale, the probability that his belief reaching $p_1 \left(s^* \left(c_n\right)\right)$ is at most $\frac{\pi}{p_1(s^*(c_n))}$.[20] Consequently, the agent's ex-ante payoff $U \left(c_n\right)$ is bounded from above by

$$\frac{\pi}{p_1 \left(s^* \left(c_n\right)\right)} \left(-L + (1+L) p_1 \left(s^* \left(c_n\right)\right)\right).$$

which converges to $\pi \left(1 - \frac{L}{l} \underline{s}\right)$ as $s^* \left(c_n\right) \to \underline{s}$. It is straightforward to verify that the agent's payoff in an uninformative equilibrium is strictly less than this upper bound.

An upper bound on the experts' joint payoff can be calculated analogously. Conditional on an expert carrying out the operation, her belief is at most

$$\Pr \left(\omega = 1 | p = p_1 \left(s^* \left(c_n\right)\right), s \geq s^* \left(c_n\right)\right) = \left(1 + \frac{1 - p_1 \left(s^* \left(c_n\right)\right)}{p_1 \left(s^* \left(c_n\right)\right)} \frac{1 - F \left(s^* \left(c_n\right) |0\right)}{1 - F \left(s^* \left(c_n\right) |1\right)}\right)^{-1}.$$

As the ex-ante probability of the operation is at most $\frac{\pi}{p_1(s^*(c_n))}$, the experts' ex-ante joint payoff $T \left(c_n\right)$ is no more than

$$\frac{\pi}{p_1 \left(s^* \left(c_n\right)\right)} \left(-l + (1+l) \left(1 + \frac{1 - p_1 \left(s^* \left(c_n\right)\right)}{p_1 \left(s^* \left(c_n\right)\right)} \frac{1 - F \left(s^* \left(c_n\right) |0\right)}{1 - F \left(s^* \left(c_n\right) |1\right)}\right)^{-1}\right),$$

which converges to $\pi \left(1 - \underline{s}\right)$ as $s^* \left(c_n\right) \to \underline{s}$. It is straightforward to verify that the experts' payoff in an uninformative equilibrium is strictly less than this upper bound.

Next, for the case $\underline{s} \in \left(\frac{l}{L}, 1\right)$, part (ii) of Lemma 5 implies that there is no informative equilibrium in the limit. As a result, the limit payoffs of the agent and experts are simply given by those of uninformative equilibria. The discussion above leads us to the following proposition.

Proposition 3 *(i) Suppose $l < L$, $\pi < \frac{l}{l+\underline{s}}$ and $\underline{s} \in \left[0, \frac{l}{L}\right]$. Then*

$$\limsup_{n \to \infty} U \left(c_n\right) \leq \pi \left(1 - \frac{L}{l} \underline{s}\right), \text{ and } \limsup_{n \to \infty} T \left(c_n\right) \leq \pi \left(1 - \underline{s}\right).$$

(ii) Suppose $l < L$ and $\underline{s} \in \left(\frac{l}{L}, 1\right)$. Then, for c_n sufficiently small,

$$U \left(c_n\right) = \max \left\{-L + \pi (1+L), 0\right\}, \text{ and } T \left(c_n\right) = \begin{cases} -l + \pi (1+l) & \text{if } \pi \geq \frac{L}{1+L} \\ 0 & \text{if } \pi < \frac{L}{1+L} \end{cases}.$$

[20] A tighter upper bound is $\frac{\pi - p_0(s^*(c_n))}{p_1(s^*(c_n)) - p_0(s^*(c_n))}$, which is weakly smaller than $\frac{\pi}{p_1(s^*(c_n))}$.

Note that, except for the lower bound $\underline{s}$ of the signal space, Proposition 3 makes no assumption on the experts' signal structure. The most interesting implication of Proposition 3 arises when the experts' signal structure F is close to $F_{\underline{s}}$. Recall from the benchmark model analyzed in Section 3 that if the agent has access to only one expert, his expected payoff, denoted by $U_1(F)$,[21] is approximately $U_1(F_{\underline{s}}) = \pi(1-\underline{s})$ (see Proposition 1). On the other hand, Proposition 3 states that with an infinitesimal search cost and access to infinitely many experts, his limit payoff is no more than $\pi\left(1-\frac{L}{l}\underline{s}\right)$, which is strictly below $\pi(1-\underline{s})$.

The most striking case occurs if $\underline{s} \in \left(\frac{l}{L}, 1\right)$ and $\pi < \frac{l}{l+\underline{s}}$: there is no informative equilibrium as search cost vanishes, and the agent's equilibrium payoff is zero. This case provides a stark illustration of the effect of the loser's curse. In the single-expert benchmark, the expert takes the prior belief of the agent at π and recommends against the operation if s is so low that

$$\Pr(\omega = 1|s,\pi) = \frac{1}{1+\frac{1-\pi}{\pi}\frac{1}{s}} < \frac{l}{1+l},$$

a condition that holds for some interior value of s, provided that $\pi < \frac{l}{l+\underline{s}}$. Consequently, partial information transmission is possible in the single-expert benchmark. However, when multiple experts are available at an infinitesimal search cost, the loser's curse consideration in an informative equilibrium requires each expert to make her recommendation conditional on being pivotal. This event occurs if the agent's belief prior to learning her recommendation is close to (or larger than) $p_1(s^*)$, which is no less than $\frac{L}{1+L}$. If L is large relative to l, the pivotal event can be so favorable for the expert that she finds it optimal to recommend Y regardless of signals, rendering her recommendation completely uninformative. This in turn hurts the agent, who is unable to solicit any informative advice in equilibrium.

Compared with the single-expert case, the possibility of consulting infinitely many experts at infinitesimal search cost can also hurt the experts' joint payoff. This occurs when $\pi < \frac{l}{l+\underline{s}}$ and the condition in part (ii) of Lemma 3 holds, i.e., $l < L$ and $\underline{s} > \frac{l}{L}$. With these parameters, the expert's payoff in the single-expert benchmark is approximately $\pi(1-\underline{s})$ if the signal structure is close to $F_{\underline{s}}$ (recall Proposition 1). On the other hand, with an infinitesimal search cost and access to infinitely many experts, there is no informative equilibrium and the experts' joint payoff is zero, regardless of their signal structure.[22] The following corollary summarizes the comparison.

[21] Recall the definition of $U_1(F)$ in equation (3).

[22] For the case $l < L$, $\pi < \frac{l}{l+\underline{s}}$, and $\underline{s} \leq \frac{l}{L}$, the limit upper bound we derive on experts' joint payoff coincides with that in the single-expert benchmark.

Corollary 1 *Suppose $l < L$ and $\pi < \frac{l}{l+\underline{s}}$. If the experts' signal structure F is close to $F_{\underline{s}}$, then*

$$\limsup_{n\to\infty} U(c_n) < U_1(F) \text{ ; and } \limsup_{n\to\infty} T(c_n) \leq T_1(F).$$

The intuition of the result above is as follows. Under the condition in the corollary, if $\underline{s} > 0$, then $\lim_{n\to\infty} p_1(s^*(c_n)) = \frac{l}{\underline{s}+l} < 1$. Therefore, with positive probability, the agent mistakenly takes the operation when $\omega = 0$. Consequently, his equilibrium payoff is necessarily bounded away from the first-best level, which is equal to π. Furthermore, as $l < L$, experts suffer relatively little from making a mistake, and are willing to adopt a relatively low cutoff in making recommendations. Compounded with the loser's curse, the experts' recommendations become uninformative very quickly as the search cost vanishes.

The result reported in Corollary 1 is related to the finding in Bulow and Klemperer (2003) that in a common-value auction, the auctioneer's expected revenue may decrease with a larger number of bidders. The driving forces are respectively the loser's curse and winner's curse. In a common-value auction, the winner's curse make all bidders shade their bids; whereas in our advice-searching model, the loser's curse makes all experts adopt a less informative recommendation rule. Whereas the winner's (loser's) curse unambiguously hurts the auctioneer (agent), their effect on bidders/experts is in the opposite direction. Specifically, by taking the winner's curse into account, a bidder would lower her bid, thus imposing a positive externality on other bidders. The bidders' joint payoff increases due to the winner's curse. On the other hand, an expert taking the loser's curse into account imposes a negative externality on other experts. The reason for this is that when facing the loser's curse, each expert lowers her cutoff in recommendation, making her recommendation less informative (in expectation). This makes it difficult for the agent to gather precise information about his case, which in turn harms the expert who eventually carries out the operation, as the probability of a failed operation increases. Consequently, the loser's curse may cause the experts' joint payoff, as well as the social welfare, to go down.

Corollary 1 implies that a market for advice may be inherently inefficient. Suppose we augment our model with a preceding stage in which experts compete by choosing their publicly-posted consultation fees. The consultation fee would enter into our model as a markup on the agent's search cost. Moreover, as the fee is independent of the recommendation and operation outcome, it would not affect the experts' incentives in choosing their cutoffs. Then by a standard argument for Bertrand competition, the consultation fees of all experts would be driven to zero, as the signals are assumed to be costless to the experts. If the agent's intrinsic search cost is very small, Corollary 1 implies that this outcome is inefficient. In the extreme case, the market may completely break down if $\underline{s} \in \left(\frac{l}{L}, 1\right)$.

Suppose a social planner, whose objective is maximizing social welfare, can decide a consultation fee and force all experts to charge that fee. Corollary 1 shows that the efficient level of consultation fee can be strictly positive.[23] Similarly, suppose a trade organization (say, that of lawyers and physicians) can decide the consultation fee its members must charge. The analysis above implies that if the trade organization sets a higher consultation fee, it is possible that not only would its members benefit, but also the agent seeking their advice and service. Computing the optimal or socially efficient consultation fee is a challenging problem that is left for future research; our analysis highlights that such computation must take into account the effect of loser's curse in information transmission.

4.3 Information Aggregation

This subsection investigates the effectiveness of the collection of information dispersedly held by experts. We focus on the case $l < L$ as in the subsection above. Given any common cutoff $s^* \in (\underline{s}, \bar{s})$ adopted by experts, if the agent has free access to all recommendations, he can learn the true state ω, as these recommendations are conditionally independent and identically distributed. Strategic recommendation by the experts makes the agent's problem less trivial, because as shown in Lemma 4, the experts' equilibrium recommendation becomes completely uninformative in the limit of vanishing search cost. In the subsequent analysis, we are interested in identifying conditions under which perfect information aggregation occurs as search cost vanishes, i.e., the agent perfectly learns the true state ω by consulting experts sequentially. If information is almost perfectly aggregated, the agent takes the ex-post correct action with probability close to one. That is, $p_0(s^*)$ and $p_1(s^*)$ are close to 0 and 1 respectively. As the limiting value of $p_1(s^*)$ has been determined in Lemma 5, we are left with determining the limiting value of $p_0(s^*)$.

Lemma 6 *Suppose for each* c_n *an informative equilibrium exists such that* $\lim_{n\to\infty} s^*(c_n) = \underline{s}$.

(i) If $\underline{s} = 0$*, then* $\lim_{n\to\infty} p_0(s^*(c_n)) = 0$.

(ii) If $l < L$ *and* $\underline{s} > 0$*, then* $\liminf_{n\to\infty} p_0(s^*(c_n)) > 0$.

In the proof of part (i) of Lemma 6, we compute the limiting payoff of the following simple and necessarily suboptimal search strategy: sample a fixed number of experts and have the operation in the end if and only if all of them recommend Y. It is shown that by choosing the fixed number of experts appropriately, the agent can attain an ex-ante payoff arbitrarily close to the highest possible level, which equals π. This means that in the limit, the agent necessarily learns the true state with a negligible total

[23] Wolinsky (2005) has a related finding in a moral hazard setting.

search cost. Intuitively, as each expert adopts a cutoff arbitrarily close to $\underline{s} = 0$, it is highly likely that the agent will receive a recommendation N if and only if the state ω is 0. Consequently, the strategy under consideration is highly effective in learning the state. As a result, the agent almost always finds it profitable to seek additional advice, even with a very low belief that the operation is suitable. In other words, $p_0(s^*(c_n))$ approaches 0 in the limit.

In contrast, under the condition of part (ii) of Lemma 6, the strategy discussed in the previous paragraph can no longer guarantee that the agent can learn the true state with a high probability. The reason is that even if the experts adopt a cutoff close to $\underline{s} > 0$, there is a strictly positive probability that they recommend N when $\omega = 1$.[24] As a result, the agent that follows the strategy will receive a number of recommendations N, even when $\omega = 1$. This makes the inference problem associated with this strategy difficult. In the proof of the lemma, we find that when the agent's belief is sufficiently low, it is impossible to find a strategy for learning the state that justifies the search costs involved.

Given $l < L$, Lemma 5 and 6 imply that $\underline{s} = 0$ is necessary for ensuring that $\lim_{n\to\infty} p_1(s^*(c_n)) = 1$ and $\lim_{n\to\infty} p_0(s^*(c_n)) = 0$ for any sequence of informative equilibria. Moreover, by Lemma 3, uninformative equilibria are ruled out whenever $\pi > \max\left\{\frac{l}{l+\bar{s}}, \frac{L}{1+L}\right\} = \frac{L}{1+L}$. We have thus identified the necessary and sufficient condition for perfect information aggregation in the limit.

Corollary 2 *Suppose $l < L$. Perfect information aggregation arises as the unique limit equilibrium outcome if and only if $\underline{s} = 0$ and $\pi > \frac{L}{1+L}$.*

The condition for perfect information aggregation identified in the corollary above depends crucially on the lower bound of the signal space $\underline{s}$. This echoes the condition for perfect information aggregation identified by Milgrom (1979) in the context of a sealed-bid first-price auction for an object of common value. To illustrate the finding of Milgrom (1979), suppose the object's common value is either $V = 0$ or $V = 1$ and each bidder receives an identically and conditionally independently distributed private signal $s \in S$ concerning V. Milgrom finds that perfect information aggregation (in the limit as the number of bidders grow to infinity) arises if and only if the bidders' signal structure allow them to distinguish the event $\{V = 1\}$ from the event $\{V = 0\}$. That is,[25]

$$\inf_{s\in S} \frac{\Pr(s|V=0)}{\Pr(s|V=1)} = 0.$$

[24] The probability is no less than $f(\underline{s}|1) = \underline{s}f(\underline{s}|0) > 0$.

[25] Note that this definition is asymmetric: it is possible that $\{V = 1\}$ can be distinguished from $\{V = 0\}$ with a signal structure, but not the reverse.

If the signal space S is closed, the condition above requires it to have a signal that is possible under $V = 1$ but impossible under $V = 0$. In the notation of our model, a signal that is possible under $\omega = 0$ but impossible under $\omega = 1$ is $\underline{s} = 0$.

Though the condition for perfect information aggregation identified in Corollary 2 is similar to that in Milgrom (1979),[26] the driving force is quite different. In a common-value auction, competition among bidders plays a key role in shaping the information content of the winning bid. If there exists a signal that is possible only under $V = 1$, then many bidders in a large auction will observe it if $V = 1$. Each of these bidders understands that she faces intense competition from other bidders who observe the same signal, so is willing to bid 1. This explains the sufficiency of Milgrom's condition. Its necessity is straightforward: if the condition fails, then $\sup_{s \in S} E[V|s] < 1$, so the bid of 1 is never justifiable (even in the absence of winner's curse consideration).[27] It is clear that the result of Milgrom (1979) is unrelated to the winner's curse in bid formation. On the other hand, the result in Corollary 2 is driven by the loser's curse and the agent's optimal search behavior. If $\underline{s} > 0$, the agent cannot have $p_1 = 1$ in equilibrium. Otherwise, the experts always recommend the operation, as being pivotal implies $\omega = 1$ with almost certainty. This explains the necessity in Corollary 2. In contrast, if $\underline{s} = 0$, then even if the agent's strategy is such that p_1 is close to 1, an expert that observes a signal s extremely close to 0 would still be willing to recommend against the operation. Moreover, when $\omega = 0$, many experts would observe such a low signal and recommend N. Thus, by sampling a large number of experts, the agent must get some N recommendations and learn that $\omega = 0$. This explains the sufficiency in Corollary 2. It is clear that Corollary 2 is unrelated to competition among experts. In fact, competition with other experts does not play any role in shaping the equilibrium recommendation strategy. Specifically, when making their recommendations, each expert does not take into account the probability of serving the agent.

It is also interesting to contrast the equilibrium outcome of our setting to that of common-value auctions when $\underline{s} > 0$ (and correspondingly, when Milgrom's condition fails). Kremer (2002) shows that in a common-value auction, if $\sup_{s \in S} E[V|s] < 1$, the winning bid in the limit is $E[V]$ and hence is uncorrelated with V. This result is again due to the intense competition among bidders in the limit, which drives the winning bid towards the expected value of V conditional on the winning bidder's information. If the most favorable signal does not preclude $V = 0$, then the event that someone, among

[26] To ensure the uniqueness of equilibrium outcome in perfect information aggregation, we need $\pi > \frac{L}{1+L}$ in our search setting. If the condition fails, there exists an uninformative equilibrium in which the experts recommend N regardless of signals. On the other hand, Milgrom (1979) does not have an analogous requirement.

[27] Section 2 of Milgrom (1979) discusses the intuition of his finding.

a large number of bidders, observing this most favorable signal is almost completely uninformative. On the other hand, in our search setting, even if $\underline{s} > 0$, the agent's eventual decision on whether or not to take the operation could still partially reflect the dispersed information held by the experts. In particular, with $\pi \in \left(\frac{L}{1+L}, \frac{l}{l+\underline{s}}\right)$, Proposition 2 states that an informative equilibrium exists for all $c_n > 0$. Moreover, as $\lim_{n\to\infty} p_1(s^*(c_n)) = \frac{l}{l+\underline{s}}$ and $\lim_{n\to\infty} p_0(s^*(c_n)) < \frac{L}{1+L}$, the agent's eventual decision on whether or not to carry out the operation is positively correlated with state ω in the limit.[28]

In the rest of this subsection, we explain the welfare loss and the failure of information aggregation when $\underline{s} > 0$ can be interpreted as a commitment problem of the agent. Suppose the agent commits to the following decision rule. Fixing a $q \in (0,1)$, he consults $n+1 \in \mathbb{N}$ experts and undergoes the operation (with a randomly chosen expert who recommends him to do so) whenever more than qn experts recommend Y. Thus, he is adopting a voting mechanism, and if c is sufficiently small, the cost of collecting a large number of votes is still very low. Feddersen and Pesendorfer (1997) show that in a similar setup, as n goes to infinity, information is perfectly aggregated for all informative signal structures of voters. Below we provide a heuristic argument on why information is always perfectly aggregated if the agent can commit to a decision rule. Suppose a symmetric voting equilibrium exists in which each of the $n+1$ expert adopts a common cutoff rule $s_n^* \in (\underline{s}, \bar{s})$. Such an equilibrium cutoff would be characterized by the following equation

$$\Pr(\omega = 1 | piv, s_n^*) = \frac{l}{1+l}, \tag{12}$$

where piv is the event that an expert's recommendation is pivotal. Studying equation (12) yields the following findings:

Claim 1 *(i) For n sufficiently large, a solution $s_n^* \in (\underline{s}, \bar{s})$ to equation (12) exists.*

(ii) $s^* \equiv \lim_{n\to\infty} s_n^*$ exists and is unique. Moreover,

$$1 - F(s^*|0) < q < 1 - F(s^*|1).$$

Given part (ii) of the claim above, it follows from the central limit theorem that as n goes to infinity, it is almost surely that the fraction of recommendations Y exceeds q if and only if $\omega = 1$. As a result, information is perfectly aggregated in the limit as long as experts' signal structure is informative.[29]

In this voting game, although experts are strategic and decide their recommendations conditional on being pivotal, the loser's curse is absent because being pivotal is no longer good news. In fact,

[28] Straightforward calculation shows that the covariance is at least $\frac{\pi-(1-\pi)L}{1-\frac{\underline{s}}{l}L} > 0$.

[29] That is, $\underline{s} < 1 < \bar{s}$.

in the limit with infinitely many experts, being pivotal is completely neutral: it can be shown that $\lim_{n\to\infty} \Pr(\omega = 1|piv) = \pi$. Consequently, experts are willing to adopt a strictly informative recommendation rule in the limit.[30]

The discussion above illustrates that the extent to which information is aggregated, and thus the agent and social welfare, could be improved if the agent is able to commit to a decision rule. It also highlights that the source of inefficiency in our model is the agent's sequentially optimal search behavior, which generates the experts' loser's curse.

5 Discussion

For simplicity and tractability, our model of searching for advice has abstracted away from some realistic considerations. Below we briefly discuss a few possible variations and extensions.

Conservative experts Throughout our analysis on welfare and information aggregation, we have focused on the case $l < L$, i.e., experts suffering less from a failed operation than the agent. Although we believe that this is the relevant case in most applications, it is also natural to ask what would happen if $l \geq L$. First, the existence results concerning informative and uninformative equilibria in Section 4.1 are still valid. Second, the upper bounds on players' payoffs for the limiting case of infinitesimal search cost identified in Section 4.2 are still relevant. As long as $\pi \in \left(\max\left\{\frac{l}{l+\bar{s}}, \frac{L}{1+L}\right\}, \frac{l}{l+\underline{s}}\right)$, we can still guarantee the existence of a sequence of informative equilibria with $s^*(c_n) \to \underline{s}$. If we select such a sequence, the bounds identified in part (i) of Proposition 3 still apply. The new issue that arises in the case $l \geq L$ is that we cannot rule out the possibility that there exists a sequence of informative equilibria with experts' cutoff $s^*(c_n) \to \bar{s}$. If such a sequence exists, it can be shown that $\limsup_{n\to\infty} p_1(s^*(c_n)) \leq \frac{l}{1+l} < \frac{l}{\underline{s}+l} < 1$. Following an argument similar to that in Section 4.2, the agent's limit payoff is bounded from above by $\pi\left(1 - \frac{L}{l}\right)$, which is below the upper bound associated with a sequence of equilibria such that $s^*(c_n) \to \underline{s}$. A similar conclusion holds for the experts' limit payoff. Consequently, allowing for the possibility that the limiting experts' cutoff is $\bar{s}$ does not lift the upper bounds on the limit payoffs.

Third, with $l \geq L$, perfect information aggregation in the limit still requires $\underline{s} = 0$, as in Section 4.3. However, the possibility of a sequence of informative equilibria with $s^*(c_n) \to \bar{s}$ means that even if conditions in Corollary 2 hold, the limiting equilibrium outcome may not be unique. The counterpart of Corollary 2 for the case $l \geq L$ is as follows.

[30]The limiting cutoff is characterized by equation (29) in the appendix.

Corollary 3 *Suppose $l \geq L$. Perfect information aggregation arises as a limit equilibrium outcome if $\underline{s} = 0$ and $\pi > \max\left\{\frac{l}{l+\bar{s}}, \frac{L}{1+L}\right\}$. Perfect information aggregation is not a limit equilibrium outcome if $\underline{s} > 0$.*

Details of derivations for findings discussed above are relegated to the appendix.

Observability of agent's history The key ingredient of the loser's curse is that each expert believes there is a positive probability that she is pivotal in the agent's final decision. In our model, this is achieved by assuming that the experts do not know, and cannot learn, the history of the agent. Any modification of our model that removes the positive probability of being pivotal would also eliminate the potential loser's curse effect. For instance, if the experts can observe the agent's history of received recommendations fully or partially (e.g., his time on the market), then experts consulted in the early stage of the search process are certain that they are not pivotal, so adopting any recommendation rule is optimal (including one that is most favorable and/or informative to the agent).[31] However, a realistic concern about non-pivotal experts is that they do not have strict incentives to perform an informative diagnoses. For instance, experts may need to exert an unobservable and costly effort in running tests and diagnosis in order to obtain the signal. Thus, an expert is willing to exert effort only if he believes that there is a sufficiently high probability that he is pivotal. Consequently, regardless of how small the effort cost is, the equilibrium number of experts consulted must be small, and perfect information aggregation is impossible.[32] Therefore, if the non-performance of non-pivotal experts is a significant concern, the agent has incentives to hide his history. In applications such as a patient seeking doctors' advice and a client looking for lawyers' advice, it is quite natural that the experts do not have much knowledge about the agent's history of received advice.

Search with recall In the main model, we assume the agent undergoes the operation, if he ever chooses to, with the last expert he visits. An alternative assumption is to allow recall. For instance, the agent randomly chooses, with equal probability, any expert that gave him a recommendation Y. All of our results are unaffected by allowing recall. This is because the availability of the recall option would not affect the agent's optimal stopping decisions (which depend only on his prior belief π and the cutoff adopted by experts). Moreover, as $b_1(h) > 0$ only if the last recommendation received is Y, the set of pivotal events with recall $\{h \in H : b_1(h, Y, h') > 0 \text{ for some } h' \in H\}$ coincides with the set of pivotal events without recall $\{h \in H : b_1(h, Y) > 0\}$. Consequently, equation (8) still characterizes the

[31] The same is true for the case in which the agent can credibly commit to not having the operation with some experts.

[32] See Pesendorfer and Wolinsky (2003) for a related model that highlights the inefficiency resulting from the experts' moral hazard problem in diagnosis.

expert's best response in a search setting with recall.

Importantly, with recall, every consulted expert is pivotal with positive probability. Therefore, even if the agent's history of received recommendation is observable to the experts, the loser's curse is still present: in the event that the agent returns to her for an operation, he must have received sufficiently favorable information from other experts. In a setting with both agent recall and observability of recommendation histories, an expert's cutoff could, in general, depend on the agent's history. This is because the (pivotal) event that the agent comes back to her for an operation, denoted by $piv(b,h)$, is in general dependent on the agent's current history h. Specifically, the conditional probability of the state being $\omega = 1$ becomes:

$$\Pr(\omega = 1 | s, piv(b,h), s^*, b) = \left(1 + \frac{1}{s}\frac{1-\pi}{\pi}\frac{\sum_{h' \in H} q_0(h, Y, h'; s^*, b)\, b_1(h, Y, h')}{\sum_{h' \in H} q_1(h, Y, h'; s^*, b)\, b_1(h, Y, h')}\right)^{-1}.$$

Thus, the loser's curse is still present if the likelihood ratio $\frac{\sum_{h' \in H} q_0(h,Y,h';s^*,b) b_1(h,Y,h')}{\sum_{h' \in H} q_1(h,Y,h';s^*,b) b_1(h,Y,h')}$ is smaller than 1, a condition that is likely to hold if the agent takes the operation only if he has received a sufficiently large number of recommendations Y.[33]

Communication of agent's history In the main model, we assume the agent cannot communicate his history to the experts. An alternative assumption is that the agent can send a cheap-talk message about his current belief.[34] We argue below that the endogenous conflict of interest induced by the loser's curse make information transmission via cheap talk very difficult, if not impossible. Suppose that non-pivotal experts do not perform informative diagnosis (e.g., because of the moral hazard problem discussed above). As shown in Section 4.2, if search cost is small, each expert adopts an extremely uninformative recommendation strategy with a very low cutoff. In this case, the agent would prefer the adoption of a higher cutoff, which in turn requires the consulted expert to hold a "pessimistic conditional belief", i.e., conditional on the agent following her positive recommendation, the agent's belief is close to $\tilde{p}_1(\cdot)$ (as defined in (9)). As a result, the agent would always send a message that induces the most pessimistic conditional belief, rendering babbling the only equilibrium outcome. A similar logic holds if non-pivotal experts perform informative diagnosis and adopt a recommendation most preferred

[33] As there is a lot more freedom in specifying experts' off-the-equilibrium-path belief in this setting, equilibria with flavor distinct from those we analyzed would emerge. Nonetheless, it is conjectured that our informative equilibrium (or its perturbation) would continue to exist in this setting when the search cost is sufficiently small. Intuitively, with experts' almost always adopting a cutoff very close to $\underline{s}$, the agent's updating on the state in the pivotal consultation is minimal. Consequently, the distribution of the agent's prior belief in the pivotal consultation is almost degenerate, and almost invariant with the agent's current history. If this is the case, $\Pr(\omega = 1 | s, piv(b,h), s^*, b)$ is almost history independent.

[34] Note that his current belief is a sufficient statistic for his current history.

by the agent. In this case, the agent would always claim that the current consultation is non-pivotal in his decision.

Transfers We have abstracted away from pricing and liability in our model. If each expert can charge an outcome-independent operation fee and the agent can learn the fee only after visiting each expert, then each expert would charge a monopoly fee[35], with which the agent's surplus is fully extracted. Consequently, there is no search in equilibrium, a result analogous to the Diamond paradox. On the other hand, if experts' operation fees are publicly posted before the agent begins his search, then Bertrand competition would force the fee to zero, provided that the agent's search cost is sufficiently small. Loosely speaking, if the search cost is sufficiently low, Lemma 4 shows that the experts' cutoff is too low for effective information collection. Charging a positive operation fee would only push this cutoff even lower, thus unambiguously making visiting such expert less desirable than those charging a zero operation fee. With a zero operation fee, we are then back to the model analyzed above.

A natural question to consider, once we allow for transfers between the agent and expert, is whether the expert can signal her observed signal s through offering different contracts. For signaling through transfers to work, it is necessary that the contract can be made contingent on the operation outcome. Consider a fee-and-compensation contract (ϕ, ψ) under which the expert collects a fee ϕ for carrying out the operation and pays a compensation ψ if the operation fails. If this class of contracts is feasible, then the expert can potentially signal a high s by offering a contract with high values of ϕ and ψ. However, if the agent learns of the offered contract only after visiting an expert, then the optimal contract to each expert does not involve signaling at all. By offering a contract $(\phi, \psi) = (1, 1 + L)$ regardless of the observation of s, the consulted expert effectively "buys" the problem from the agent. If every expert offers this contract in equilibrium, the agent is left with zero surplus and does not search beyond the first (free) expert. Knowing that the agent does not search, the first consulted expert adopts a cutoff $s^* = \frac{L+l}{2+L+l}$, which is efficient in the single-expert benchmark. The analysis becomes much more challenging if the experts can publicly post and commit to, possibly a menu of fee-and-compensation

[35] The monopoly fee ϕ can be found by choosing ϕ and s^* to maximize $\pi (1 - F(s^*|1))(1 + \phi) + (1 - \pi)(1 - F(s^*|0))(\phi - l)$ subject to constraints

$$-\phi - L + (1 + L) \frac{1}{1 + \frac{1-\pi}{\pi} \frac{1-F(s^*|0)}{1-F(s^*|1)}} \geq 0; \text{ and } \phi - l + (1 + l) \frac{1}{1 + \frac{1-\pi}{\pi} \frac{1}{s^*}} = 0.$$

The first constraint ensures the agent follows the expert's recommendation Y; whereas the second constraint governs the adoption of cutoff s^* by the expert under operation fee ϕ. The second constraint implies that an increase in ϕ lowers the cutoff s^*, and increases the expert's expected payoff. Therefore, the fee should be set as high as possible given the first constraint, thus completely eliminating the agent's surplus.

contracts, before the agent begins his search. Now experts are competing with each other not only in prices (fee and compensation) but also in the implied information service (i.e., recommendation rule) they provide. An equilibrium may involve each expert posting a menu of contracts, and signaling his observed s through the offered contract. A full analysis is left for future research.

Communication of experts' signal In the main model, we impose a restrictive binary message space for the experts. If we allow for a general message space, more information can be transmitted in some equilibrium, though the equilibrium we consider and characterize still exists. For instance, along with the recommendation decision, the expert can send a cheap-talk message. Now if an expert does not recommend the operation, she may just as well fully reveal the signal learned through her message, as her payoff is constant at zero. However, as Lemma 4 shows, the experts almost always recommend the operation as the search cost vanishes. This use of the cheap-talk message therefore does not affect our limit results.

Heterogeneity of experts In the main model, we assume that the agent samples experts randomly, an assumption that seems reasonable as all experts are assumed to ex-ante identical. Below, we consider a version of expert heterogeneity and show that the distortion in information transmission caused by the loser's curse remains. Suppose there are $k \in \mathbb{N}$ prominent experts whose signal structure $F'(s|\omega)$ are more informative than other experts, and that these prominent experts are consulted first before others (but the search among the k prominent experts remains random). Suppose also that k is large and/or F' is informative so each prominent expert is pivotal with positive probability. It is clear that the prominent and non-prominent experts would adopt different recommendation cutoffs. Moreover, the agent's cutoff pair $(p_{0,t}, p_{1,t})$ would now depend on the number of remaining prominent experts $t \leq k$ to be consulted (with $(p_{0,0}, p_{1,0})$ meaning that all prominent experts have been consulted). Suppose that in equilibrium the prominent experts' advice is indeed more informative (so that it is reasonable to seek their advice first). Then $p_{0,t}$ is increasing in t, and $p_{1,t}$ is decreasing in t. Now the prominent experts would suffer from a more severe loser's curse than nonprominent experts, as $p_{1,t} \geq p_{1,0}$ for all t and strictly so for $t > 1$. Therefore, for small enough search cost, they are likely to adopt a cutoff lower than that of non-prominent experts, partially cancelling out the improvement in the quality of information the agent gathers.[36] Finally, the limit result for vanishing search cost would remain unchanged, as the agent almost surely searches beyond the prominent experts.

Alternative expert preference In the model, we assume that the expert derives a non-zero

[36]Note that the agent may still find it wothwhile to seek the advice of the prominent experts first because their signal structure is more informative.

payoff from the interaction with the agent if and only if the agent takes the operation with her. Alternatively, each expert may care about the agent's eventual payoff, provided that she has recommended the operation to him. This may arise, for instance, if the experts are altruistic, or if they have reputational concern. Specifically, suppose the expert's payoff is given by the table below:

	$\omega = 1$	$\omega = 0$
Recommend Y and agent takes operation eventually (not necessarily with her)	1	$-l$
Recommend N, or agent does not take operation in the end	0	0

With such payoffs, the experts face essentially the same incentive structure as in the case of search with recall discussed above. Therefore, all of our results remain valid, except for those concerning the social welfare, as we have changed the experts' payoff.

Appendix

Proof of Lemma 1: The upper bound on $U_1(F)$ can be computed by considering an auxiliary game that makes the following two modifications to the benchmark game. First, the agent chooses F; and second, the expert is forced to report truthfully the learned signal, along with her recommendation. It is clear that the agent's payoff in this auxiliary game is no less than what he can obtain in the benchmark model considered here. The agent's payoff in this auxiliary game can be computed using the technique developed by Kamenica and Gentzkow (2011): the optimal signal structure can be found by looking for the concave closure of the payoff as a function of realized posterior.

As the lower bound of the signal space $\underline{s}$ is fixed, the lowest possible posterior belief after learning the signal is $\underline{p} \equiv \left(1 + \frac{1-\pi}{\pi}\frac{1}{\underline{s}}\right)^{-1}$. Denote by $\tilde{U}_{\underline{s}} : [\underline{p}, 1] \to \mathbb{R}$ the agent's payoff as a function of the expert's posterior after learning the signal:

$$\tilde{U}_{\underline{s}}(p) = \begin{cases} 0 & \text{if } p < \frac{l}{1+l} \\ \max\{0, -L + p(1+L)\} & \text{if } p \geq \frac{l}{1+l} \end{cases}.$$

To understand the payoff function, note that if the expert has a posterior belief $p < \frac{l}{1+l}$, then she recommends no operation, and the agent gets a zero payoff. If the expert has a posterior belief $p > \frac{l}{1+l}$, then she recommends the operation, and the agent's expected payoff is $\max\{0, -L + p(1+L)\}$. The lemma below follows immediately from Corollary 2 of Kamenica and Gentzkow (2011).

Lemma 7 *Fix the lower bound of the signal space* $\underline{s} \in [0, 1)$. *The agent's payoff in the auxiliary game is given by the concave closure of* $\tilde{U}_{\underline{s}}(\cdot)$, *denoted by* $\text{con}\left[\tilde{U}_{\underline{s}}\right](\cdot)$, *evaluated at the prior* π. *The concave* *closure is defined by*

$$con\left[\tilde{U}_{\underline{s}}\right](\cdot) \equiv sup\left\{z \in \mathbb{R} : (p, z) \in co(\tilde{U}_{\underline{s}}(p))\right\},$$

where $co\left(\tilde{U}_{\underline{s}}(p)\right)$ *is the convex hull of the graph of* $\tilde{U}_{\underline{s}}(p)$.

The lemma allows us to compute the agent's payoff in the auxiliary game by constructing the concave closure of $\tilde{U}_{\underline{s}}$. Given $l < L$ and $\pi < \frac{l}{l+\underline{s}}$, a plot of the function $\tilde{U}_{\underline{s}}(p)$ is given below.

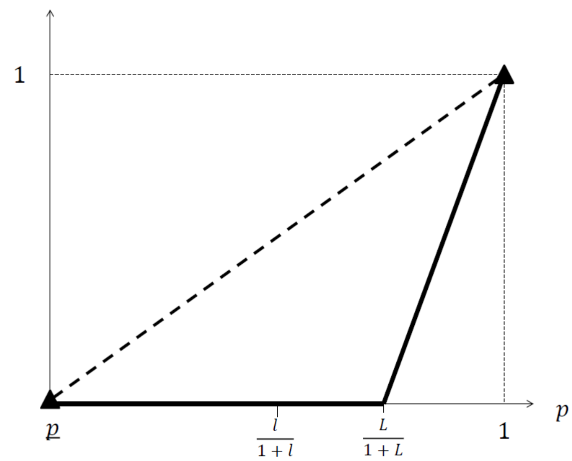

Figure 1: Plot of $\tilde{U}_{\underline{s}}(p)$.

In the figure, the solid curve is the function $\tilde{U}_{\underline{s}}(p)$, and its concave closure is represented by the dotted line. It is clear that the dotted line is obtained by connecting the two points represented by the triangles. As a result, the optimal signal structure is exactly $F_{\underline{s}}$. Consequently, in this case, we have $con\left[\tilde{U}_{\underline{s}}\right](\pi) = \pi(1 - \underline{s})$.

We explain below why the upper bound on the agent's payoff $con\left[\tilde{U}_{\underline{s}}\right](\pi)$ identified above is achievable in the benchmark model in which the signal realization is privately observed by the expert. Note that the optimal signal structure has a binary support: the lower signal leads to recommendation N, thus a zero payoff to the agent; the higher signal leads to recommendation Y, thus a positive expected payoff to the agent. Consequently, the agent's expected payoff remains unchanged even if he does not know the expert's signal realization.

Next, the signal structure that maximizes $T_1(F)$ can be computed similarly. Consider an auxiliary game that modifies the benchmark game as follows. First, the expert chooses F. Second, the expert decides whether or not the operation is carried out without needing the agent's approval. It is clear that the expert's payoff in this auxiliary game is no less than that in the benchmark game, thus serving as an upper bound of the latter. Denote by $\tilde{T}_{\underline{s}} : [\underline{p}, 1] \to \mathbb{R}$ the expert's payoff as a function of the realized posterior:

$$\tilde{T}_{\underline{s}}(p) = \begin{cases} 0 & \text{if } p < \frac{l}{1+l} \\ -l + p(1+l) & \text{if } p \geq \frac{l}{1+l} \end{cases}.$$

The optimal signal structure in the auxiliary game can be obtained by finding the concave closure of $\tilde{T}_{\underline{s}}(\cdot)$. Given $l < L$ and $\pi < \frac{l}{l+\underline{s}}$, a plot of $\tilde{T}_{\underline{s}}$ in this case is given by the figure below:

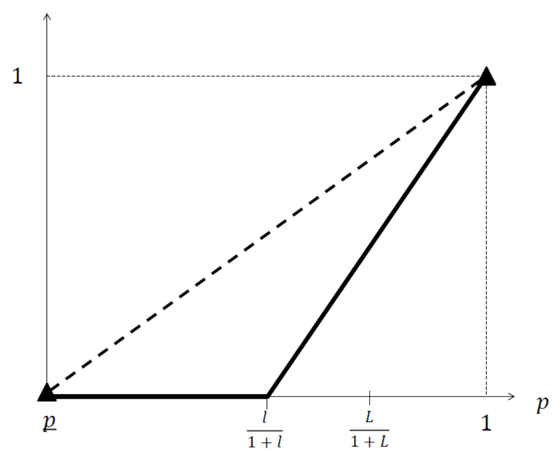

Figure 2: Plot of $\tilde{T}_{\underline{s}}(p)$.

It is therefore clear that the concave closure is obtained by connecting the two triangles in Figure 2, which corresponds to signal structure $F_{\underline{s}}$, and $con\left[\tilde{T}_{\underline{s}}\right](\pi) = \pi(1-\underline{s})$. It is easy to see that with $F_{\underline{s}}$, the agent is willing to follow the expert's recommendation, so the payoff $\pi(1-\underline{s})$ can indeed be achieved in the benchmark game. Q.E.D.

Proof of Proposition 1: We explicitly construct a sequence of conditional distribution functions that converge pointwise to $F_{\underline{s}}$. As pointed out in the proof of Lemma 1 above, the lowest possible posterior belief after learning a signal is $\underline{p} \equiv \left(1 + \frac{1-\pi}{\pi}\frac{1}{\underline{s}}\right)^{-1}$.

For each $n \in \mathbb{N}$, define a density function over posterior belief $f_n : [\underline{p}, 1] \to \mathbb{R}_+$ as follows:

$$f_n(p) \equiv \begin{cases} n\left(1 - \pi(1-\underline{s}) - \frac{m_n^1}{n}\right) & \text{if } p \in \left[\underline{p}, \underline{p} + \frac{1}{n}\right] \\ \frac{1}{1-\frac{2}{n}-\underline{p}}\left(\frac{m_n^1+m_n^2}{n}\right) & \text{if } p \in \left[\underline{p} + \frac{1}{n}, 1 - \frac{1}{n}\right] \\ n\left(\pi(1-\underline{s}) - \frac{m_n^2}{n}\right) & \text{if } p \in \left[1 - \frac{1}{n}, 1\right] \end{cases},$$

where m_n^1, m_n^2 satisfies, for all n,

$$\left(1 - \underline{p} + \frac{1}{n}\right)m_n^1 - \left(1 - \underline{p} - \frac{1}{n}\right)m_n^2 = 1,\ m_n^1, m_n^2 > 0, \text{ and } \frac{m_n^1}{n}, \frac{m_n^2}{n} \to 0.$$

It is straightforward to verify that for all n sufficiently large, there exists a pair of sequences $\{m_n^1\}$ and $\{m_n^2\}$ that satisfies the conditions above. With such a choice of sequences, f_n is a well-defined density function such that the expected value of the posterior distribution is equal to π. Moreover, it is clear that, for all $p \in (\underline{p}, 1)$, $f_n(p)$ converges to 0 as $n \to \infty$.

Next, define conditional density $f_n(s|\omega) : [\underline{s}, \infty] \times \{0,1\} \to \mathbb{R}_+$ by

$$f_n(s|0) \equiv \frac{f_n\left(\left(1 + \frac{1-\pi}{\pi}\frac{1}{s}\right)^{-1}\right)}{\pi s + (1-\pi)}, \text{ and } f_n(s|1) \equiv s f_n(s|0).$$

In an informative equilibrium, as $\pi < \frac{l}{\underline{s}+l}$, the expert recommends the operation if and only if $s \in \left[\frac{1-\pi}{\pi} l, \infty\right]$. This event happens with probability $\int_{\frac{l}{1+l}}^{1} f_n(p)\,dp$, which converges to $\pi(1-\underline{s})$ as $n \to \infty$. It is straightforward that conditional on a recommendation for the operation, the agent's belief that $\omega = 1$ converges to 1, because $\frac{1-F_n(\tilde{s}(l)|0)}{1-F_n(\tilde{s}(l)|1)} = \frac{\int_{\frac{1-\pi}{\pi}l}^{\infty} f_n(s|0)ds}{\int_{\frac{1-\pi}{\pi}l}^{\infty} f_n(s|1)ds} \to 0$ as $n \to \infty$. This implies that (2) holds for n sufficiently large. Therefore, as $n \to \infty$, the payoff of both the agent and the expert in the informative equilibrium converge to $\pi(1-\underline{s})$. Q.E.D.

Computation of $p_0(\cdot)$ and $p_1(\cdot)$: In the discussion below, we fix a $\hat{s} \in [\underline{s}, \bar{s})$ and assume that the experts adopt cutoff strategy $\hat{s}$: recommends Y if and only if $s \geq \hat{s}$. As explained in the text, they must adopt a cutoff strategy in any equilibrium. If the agent approaches the expert with a prior belief p (that $\omega = 1$), then conditional on a recommendation Y, his posterior belief becomes $\left(1 + \frac{1-p}{p}\frac{1-F(\hat{s}|0)}{1-F(\hat{s}|1)}\right)^{-1}$; whereas conditional on a recommendation N, his posterior belief becomes $\left(1 + \frac{1-p}{p}\frac{F(\hat{s}|0)}{F(\hat{s}|1)}\right)^{-1}$. Denote by $V : [0,1] \to \mathbb{R}$ the agent's beginning-of-period continuation value, as a function of his current belief p, assuming that he decides to search this period. Using observations above, $V(p)$ can be recursively defined by

$$\begin{aligned} V(p;\hat{s}) &= -c + \left[p\left(1 - F(\hat{s}|1)\right) + (1-p)\left(1 - F(\hat{s}|0)\right)\right] \\ &\quad \times \max\left\{0, V\left(\left(1 + \frac{1-p}{p}\frac{1-F(\hat{s}|0)}{1-F(\hat{s}|1)}\right)^{-1}; \hat{s}\right), -L + \left(1 + \frac{1-p}{p}\frac{1-F(\hat{s}|0)}{1-F(\hat{s}|1)}\right)^{-1}(1+L)\right\} \\ &\quad + \left[pF(\hat{s}|1) + (1-p)F(\hat{s}|0)\right]\max\left\{0, V\left(\left(1 + \frac{1-p}{p}\frac{F(\hat{s}|0)}{F(\hat{s}|1)}\right)^{-1}; \hat{s}\right)\right\}. \end{aligned} \quad (13)$$

To understand equation (13), note that after paying a search cost c, the agent may receive a recommendation Y, at which point, he can either (i) leave the current expert without undergoing the operation, which gives him a payoff $\max\left\{0, V\left(\left(1 + \frac{1-p}{p}\frac{1-F(\hat{s}|0)}{1-F(\hat{s}|1)}\right)^{-1}; \hat{s}\right)\right\}$; or (ii) agree to have the operation with the current expert, which gives him a payoff $-L + \left(1 + \frac{1-p}{p}\frac{1-F(\hat{s}|0)}{1-F(\hat{s}|1)}\right)^{-1}(1+L)$. If he receives a recommendation N, then he must leave the current expert without undergoing the operation.

It remains to identify the function V, which is done in Lemma 8 below.

Lemma 8 *There exists a unique function $V : [0,1] \to \mathbb{R}$ that satisfies (13). Moreover, V is nondecreasing and weakly convex.*

Proof. See Lemma 3.1 and Theorem 3.2 of Ross (1983). ■

Given the value function $V(p;\hat{s})$, the agent's optimal search strategy can be computed by solving for $p_0(\hat{s})$ and $p_1(\hat{s})$. Specifically, if $V(1;\hat{s}) > 0$, then $p_0(\hat{s})$ is the unique solution to $V(p;\hat{s}) = 0$. If $V(1;\hat{s}) \leq 0$, then $p_0(\hat{s}) = 1$. Also,

$$p_1(\hat{s}) = \min\left\{p \in \left[\frac{L}{1+L}, 1\right] : V(p;\hat{s}) \leq -L + p(1+L)\right\}. \tag{14}$$

The value $p_0(\hat{s})$ is well-defined. To see this, note that $V(\cdot;\hat{s})$ is nondecreasing and convex (thus continuous). As $V(0) = -c$, the equation $V(p;\hat{s}) = 0$ has a unique solution if $V(1;\hat{s}) > 0$. Moreover, as $\left\{p \in \left[\frac{L}{1+L}, 1\right] : V(p;\hat{s}) \leq -L + p(1+L)\right\}$ is a compact interval, $p_1(\hat{s})$ is also well-defined. We conclude this discussion with the following observations on $p_0(\hat{s})$ and $p_1(\hat{s})$.

Corollary 4 *(i) If $p_0(\hat{s}) < \frac{L}{1+L}$, then $V(p_1(\hat{s});\hat{s}) = -L + p_1(\hat{s})(1+L)$, and $p_1(\hat{s}) > \frac{L}{1+L}$.*
(ii) If $p_0(\hat{s}) \geq \frac{L}{1+L}$, then $p_1(\hat{s}) = \frac{L}{1+L}$.

Proof. As $\left\{p \in \left[\frac{L}{1+L}, 1\right] : V(p;\hat{s}) \leq -L + p(\hat{s})(1+L)\right\}$ is a closed interval, either $p_1(\hat{s}) = \frac{L}{1+L}$ or $V(p_1(\hat{s});\hat{s}) = -L + p_1(\hat{s})(1+L)$. As $V(p_0(\hat{s});\hat{s}) = 0$ and $V(\cdot;\hat{s})$ is strictly increasing at $p_0(\hat{s})$, $p_0(\hat{s}) < \frac{L}{1+L}$ if and only if $V\left(\frac{L}{1+L};\hat{s}\right) > 0$. ■

Finally, if $\hat{s} = \bar{s}$, i.e., the experts always recommend N, then it is clear that the optimal strategy of the agent, for all $\pi \in (0,1)$, is never searching at all. In this case, $V(p;\bar{s}) = -c$, so $p_0(\bar{s}) = 1$ and $p_1(\bar{s}) = \frac{L}{1+L}$.

Proof of Proposition 2: It suffices to consider the case $\pi \in \left(\max\left\{\frac{l}{l+\bar{s}}, \frac{L}{1+L}\right\}, \frac{l}{l+\underline{s}}\right)$, as other cases are covered by Lemma 3. In particular, we show below the existence of an informative equilibrium for this range of π. Take a candidate cutoff strategy $\hat{s}$ of the experts. Denote by $\Psi(\hat{s})$ the set of agent's best response to experts' cutoff $\hat{s}$. Define $J : \mathbb{R}_+ \times [\underline{s}, \bar{s}) \times \Lambda \to \mathbb{R}_+$ by

$$J(s,\hat{s},b) \equiv \left(1 + \frac{1}{s}\frac{1-\pi}{\pi}\frac{\sum_{h\in H} b_1(h,Y)\, q_0(h;\hat{s},b)}{\sum_{h\in H} b_1(h,Y)\, q_1(h;\hat{s},b)}\right)^{-1}. \tag{15}$$

It is immediate that $J(s,\hat{s},b)$ is strictly increasing in s. Define $x : [\underline{s},\bar{s}) \times \Lambda \to \mathbb{R}_+$ as the unique solution to the equation $J(\cdot,\hat{s},b) = \frac{l}{1+l}$. Define a correspondence $Z : [\underline{s},\bar{s}] \rightrightarrows [\underline{s},\bar{s}]$ as follows:

$$Z(\hat{s}) \equiv \begin{cases} \{\max\{\underline{s},\min\{\bar{s},x(\hat{s},b)\}\} : b \in \Psi(\hat{s})\} & \text{if } \hat{s} \in [\underline{s},\bar{s}) \\ \frac{1-\pi}{\pi} l & \text{if } \hat{s} = \bar{s} \end{cases}.$$

The correspondence Z can be interpreted as the set of best responses of an individual expert, given all other experts' cutoff strategy $\hat{s}$, and that the agent playing a best response to $\hat{s}$. Specifically, if $x(\hat{s},b) \in [\underline{s},\bar{s}]$, then an individual expert finds it optimal to adopt cutoff $x(\hat{s},b)$, given all other experts' adopting cutoff $\hat{s}$ and the agent playing $b \in \Psi(\hat{s})$. If $x(\hat{s},b) > \bar{s}$, then given others players' strategy profile, an individual expert finds it optimal to always recommend N, i.e., adopting cutoff $\bar{s}$. Likewise, if $x(\hat{s},b) < \underline{s}$, then an individual expert's best response is to always recommend Y, i.e., adopting cutoff $\underline{s}$. Finally, defining $Z(\bar{s}) = \frac{1-\pi}{\pi} l \in (\underline{s},\bar{s})$ is enough to ensure the graph of $Z(\cdot)$ is closed at $\hat{s} = \bar{s}$. To see this, note that if $\hat{s}$ is sufficiently close to $\bar{s}$, then $p_0(\hat{s}) > \pi$ and $p_1(\hat{s}) = \frac{L}{1+L}$. As $\pi > \frac{L}{1+L}$, this implies that $b_1(Y) = 1$ and $b_0(N) = 1$. With such a best response $b \in \Psi(\hat{s})$, $J(s,\hat{s},b) = \left(1 + \frac{1}{s}\frac{1-\pi}{\pi}\right)^{-1}$, giving $x(\hat{s},b) = \frac{1-\pi}{\pi} l$.

We show below that the correspondence Z has a fixed point. To this end, we invoke the Kakutani's fixed point theorem. In the remainder of the proof, we show that (i) Z is a non-empty-valued self-map, (ii) Z is convex-valued, and (iii) Z is upper semi-continuous.

(i) We have established the existence of the agent's best response in Lemma 2. Moreover, as $\pi > \frac{L}{1+L}$, it is clear that whenever $\hat{s} < \bar{s}$, any strategy with $b_1(h,Y) = 0$ for all h on the equilibrium path is not optimal. Therefore, $J(s,\hat{s},b)$ is well-defined. The fact that Z is a non-empty valued self-map follows from the definition above.

(ii) It is immediate that $Z(\bar{s})$ is convex-valued. Consider a $\hat{s} < \bar{s}$. Suppose $z', z^{\#} \in Z(\hat{s})$, and $z^{\&} \in (z', z^{\#})$. Denote by b' and $b^{\#}$ the corresponding optimal strategy respectively. As this is a game of perfect recall, by the Kuhn's Theorem (Kuhn (1953)), every behavioral strategy is equivalent to some mixed strategy. Denote by α' and $\alpha^{\#}$ the mixed-strategy equivalents of b' and $b^{\#}$ respectively. With a slight abuse of notation, $J(s,\hat{s},\alpha)$ is defined analogous to (15) for each mixed strategy α of the agent as follows:

$$J(s,\hat{s},\alpha) \equiv \int \left(1 + \frac{1}{s}\frac{1-\pi}{\pi}\frac{\sum_{h\in H}\beta_1(h,Y)\,q_0(h;\hat{s},\beta)}{\sum_{h\in H}\beta_1(h,Y)\,q_1(h;\hat{s},\beta)}\right)^{-1} d\alpha(\beta),$$

where β are the agent's pure strategies on the support of α. Define $T : [0,1] \to \mathbb{R}$ by

$$T(\gamma) \equiv \gamma J\left(z^{\&},\hat{s},\alpha\right) + (1-\gamma)\, J\left(z^{\&},\hat{s},\alpha^{\#}\right) - \frac{l}{1+l}.$$

We have $T(1) > 0$ and $T(0) < 0$. As T is continuous and increasing, by the intermediate value theorem, there exists a unique $\gamma^* \in (0,1)$ such that $T(\gamma^*) = 0$. As every pure strategy on the support of α and $\alpha^{\#}$ is a best response to $\hat{s}$, it is clear that $\gamma^*\alpha + (1-\gamma^*)\alpha^{\#}$ is also a best response to $\hat{s}$. Using Kuhn's Theorem again, $z^{\&} \in Z(\hat{s})$.

(iii) Recall the set of agent's best response $\Psi(\hat{s})$ is characterized by two cutoffs. The cutoff $p_0(\hat{s})$ calls for stopping the search altogether when belief falls below it, and $p_1(\hat{s})$ calls for taking the operation immediately upon being recommended to do so. The cutoffs $p_0(\hat{s})$ and $p_1(\hat{s})$ in the best response of the agent are given respectively by $V(p_0(\hat{s});\hat{s}) = 0$ and $p_1(\hat{s}) = \min\left\{p \in \left[\frac{L}{1+L}, 1\right] : V(p;\hat{s}) \leq -L + p(1+L)\right\}$,[37] where $V(p;\hat{s})$ is defined in (13).

Lemma 9 *$p_0(\hat{s})$ and $p_1(\hat{s})$ are continuous in $\hat{s}$.*

Proof. We first show the continuity of $p_0(\hat{s})$ and $p_1(\hat{s})$ at $\hat{s} = \bar{s}$. Fix a $p \in (0,1)$. For $\hat{s}$ sufficiently close to $\bar{s}$, $p(1 - F(\hat{s}|1)) + (1-p)(1 - F(\hat{s}|0)) < \frac{c}{2}$. The definition of $V(\cdot;\hat{s})$ in equation (13) then implies

$$V(p;\hat{s}) \leq -c + \frac{c}{2} + \left(1 - \frac{c}{2}\right)\max\{0, V(p;\hat{s})\}.$$

It is immediate that $V(p;\hat{s}) < 0$. Consequently, $\lim_{n\to\infty} p_0(\hat{s}_n) = 1$. That $\lim_{n\to\infty} p_1(\hat{s}_n) = \frac{L}{1+L}$ then follows immediately from part (ii) of Corollary 4.

Below we focus on the continuity in the interval $[\underline{s}, \bar{s})$. Fix a $\hat{s} \in [\underline{s}, \bar{s})$ and an $\varepsilon > 0$ such that $\hat{s} + \varepsilon < \bar{s}$. Take an arbitrary sequence $\{\hat{s}_n\}$ such that $\lim_{n\to\infty} \hat{s}_n = \hat{s}$ and that $\hat{s}_n < \hat{s} + \varepsilon$ for all $n \in \mathbb{N}$. This gives two sequences of cutoffs $\{p_0(\hat{s}_n)\}_n$ and $\{p_1(\hat{s}_n)\}_n$ satisfying

$$V(p_0(\hat{s}_n);\hat{s}_n) = 0, \text{ and} \tag{16}$$

$$p_1(\hat{s}_n) = \min\left\{p \in \left[\frac{L}{1+L}, 1\right] : V(p;\hat{s}_n) \leq -L + p(1+L)\right\} \tag{17}$$

Suppose $\{p_0(\hat{s}_n)\}_n$ and $\{p_1(\hat{s}_n)\}_n$ are convergent (otherwise take a subsequence). We want to show that $\lim_{n\to\infty} p_0(\hat{s}_n) = p_0(\hat{s})$ and $\lim_{n\to\infty} p_1(\hat{s}_n) = p_1(\hat{s})$.

First we show that the family of functions $\{V(p;\hat{s}_n)\}_{n\in\mathbb{N}}$ is Lipschitz continuous with a common modulus $1 + L + c(\hat{s} + \varepsilon - 1)$. Recall from Lemma 8 that both $V(\cdot;\hat{s}_n)$ is weakly convex on $[0,1]$. Thus, $V(\cdot;\hat{s}_n)$ is differentiable for almost all $p \in [0,1]$, and their derivatives are nondecreasing in p. If p is sufficiently large, according to (13), $V(p;\hat{s}_n)$ is given by

$$\begin{aligned} V(p;\hat{s}_n) &= -c + \{-L(1-p)[1 - F(\hat{s}_n|0)] + p[1 - F(\hat{s}_n|1)]\} \\ &\quad + [pF(\hat{s}_n|1) + (1-p)F(\hat{s}_n|0)]\, V\left(\frac{1}{1 + \frac{1-p}{p}\frac{F(\hat{s}_n|0)}{F(\hat{s}_n|1)}};\hat{s}_n\right). \end{aligned}$$

[37] The existence and uniqueness of $p_0(\hat{s})$ and $p_1(\hat{s})$ follow from Lemma 2.

To obtain an upper bound on the derivative $\frac{\partial V(p;\hat{s}_n)}{\partial p}$, suppose $V(p;\hat{s}_n)$ is differentiable at $p=1$. Differentiate both sides of the equation above with respect to p,

$$\begin{aligned}\frac{\partial V(p;\hat{s}_n)}{\partial p} &= L[1-F(\hat{s}_n|0)]+[1-F(\hat{s}_n|1)]-[F(\hat{s}_n|0)-F(\hat{s}_n|1)]V\left(\frac{1}{1+\frac{1-p}{p}\frac{F(\hat{s}_n|0)}{F(\hat{s}_n|1)}};\hat{s}_n\right)\\ &\quad+\frac{F(\hat{s}_n|1)F(\hat{s}_n|0)}{pF(\hat{s}_n|1)+(1-p)F(\hat{s}_n|0)}\frac{\partial}{\partial p}V\left(\frac{1}{1+\frac{1-p}{p}\frac{F(\hat{s}_n|0)}{F(\hat{s}_n|1)}};\hat{s}_n\right).\end{aligned}$$

Evaluate the equation above at $p=1$ gives

$$\begin{aligned}\left.\frac{\partial V(p;\hat{s}_n)}{\partial p}\right|_{p=1} &= \frac{L[1-F(\hat{s}_n|0)]+[1-F(\hat{s}_n|1)]-[F(\hat{s}_n|0)-F(\hat{s}_n|1)](1-c)}{1-F(\hat{s}_n|0)}\\ &\leq 1+L+(\hat{s}+\varepsilon-1)c.\end{aligned}$$

Thus, the families of functions $\{V(p;\hat{s}_n)\}_{n\in\mathbb{N}}$ is Lipschitz continuous.

As a result, $\{V(p;\hat{s}_n)\}_{n\in\mathbb{N}}$ are equicontinuous and uniformly bounded (by $[-1,1]$), by the Arzela-Ascoli Theorem, there exists a subsequence $\{\hat{s}_{n_k}\}$ such that $V(p;\hat{s}_{n_k})$ converge uniformly. It is clear that the limiting functions is $V(p;\hat{s})$, as by Lemma 8, there exists a unique function that satisfies (13). Now pass (16) to the limit, we get $V(\lim_{k\to\infty}p_0(\hat{s}_{n_k});\hat{s})=0$. By Lemma 2, the only subsequential limit of $\{p_0(\hat{s}_n)\}$ is thus $p_0(\hat{s})$. Therefore, $\lim_{n\to\infty}p_0(\hat{s}_n)=p_0(\hat{s})$.

If $p_0(\hat{s})<\frac{L}{1+L}$, then $p_0(\hat{s}_{n_k})<\frac{L}{1+L}$ for k sufficiently large. According to Corollary 4, $V(p_1(\hat{s}_{n_k});\hat{s}_{n_k})=-L+(1+L)p_1(\hat{s}_{n_k})$. Passing to the limit gives $V(\lim_{k\to\infty}p_1(\hat{s}_{n_k});\hat{s})=-L+(1+L)\lim_{k\to\infty}p_1(\hat{s}_{n_k})$. By Lemma 2, the only subsequential limit of $\{p_1(\hat{s}_n)\}$ is thus $p_1(\hat{s})$. Therefore, $\lim_{n\to\infty}p_1(\hat{s}_n)=p_1(\hat{s})$.

If $p_0(\hat{s})\geq\frac{L}{1+L}$, then $p_1(\hat{s})=\frac{L}{1+L}$. By Corollary 4, either one of the following subsequences $\{\hat{s}_{n_k}\}$ must exist: (i) a subsequence such that $p_0(\hat{s}_{m_k})\leq\frac{L}{1+L}$ and $V(p_1(\hat{s}_{m_k});\hat{s}_{m_k})=-L+(1+L)p_1(\hat{s}_{m_k})$, or (ii) a subsequence such that $p_1(\hat{s}_{m_k})=\frac{L}{1+L}$. Case (i) is similar to the argument in the paragraph above. For case (ii), it is immediate that $\lim_{k\to\infty}p_1(\hat{s}_{m_k})=\frac{L}{1+L}$. Therefore, in either case above, $\lim_{n\to\infty}p_1(\hat{s}_n)=p_1(\hat{s})$. ■

Now take a pair of sequences $\{\hat{s}_m\},\{z_m\}$ such that $\hat{s}_m\to\hat{s}$, $z_m\in Z(\hat{s}_m)$, and $z_m\to z$. To prove the upper semi-continuity of Z, we need to show that $z\in Z(\hat{s})$. As we have explained in the definition of correspondence Z, it is continuos at $\hat{s}=\bar{s}$. Below we consider the case $\hat{s}<\bar{s}$.

Suppose first that $z\in(\underline{s},\bar{s})$. Then it is without loss to assume $z_m\in(\underline{s},\bar{s})$ for all $m\in\mathbb{N}$ (otherwise, take a subsequence). Consequently, for all $m\in\mathbb{N}$, $J(z_m,\hat{s}_m,b^m)=\frac{l}{1+l}$ for some behavioral strategy $b^m\in\Psi(\hat{s}_m)$. As the set of histories H is countable, following a standard diagonalization argument, one can construct a subsequence $\{b^{m_k}\}$ that converges pointwise to some $b^{\#}:H\to[0,1]^2$. The following claim shows that $b^{\#}$ is a best response to $\hat{s}$ because of the continuity of $p_0(\cdot)$ and $p_1(\cdot)$ established in Lemma 9.

Claim 2 *$b^{\#}$ is a best response to $\hat{s}$.*

Proof. Suppose not. Then there exists a $h \in H$ such that either (i) $p(h;\hat{s}) \in (p_0(\hat{s}), p_1(\hat{s}))$ but either $b_0^{\#}(h) > 0$, or $b_1^{\#}(h) > 0$, or (ii) $p(h;\hat{s}) < p_0(\hat{s})$ but $b_0^{\#}(h) < 1$; or (iii) $p(h;\hat{s}) > p_1(\hat{s})$ but $b_1^{\#}(h) < 1$. Suppose case (i) arises. For either $i = 0, 1$, and for all m sufficiently large, we have $b_i^m(h) > 0$ and $p(h;\hat{s}_m) \notin (p_0(\hat{s}_m), p_1(\hat{s}_m))$. As $p_0(\cdot)$ and $p_1(\cdot)$ are continuous, taking limit with respect to m gives $p(h;\hat{s}) \notin (p_0(\hat{s}), p_1(\hat{s}))$, a contradiction. Suppose case (ii) arises. As $p(h;s_m) \to p(h;\hat{s})$ and $p_0(s_m) \to p_0(\hat{s})$, we have that for all m sufficiently large, $p(h;\hat{s}_m) < p_0(\hat{s}_m)$, so $b_0^m(h) = 1$. Thus, $b_0^{\#}(h) = 1$, a contradiction. Case (iii) is symmetric to case (ii). ■

It remains to show that for $\omega \in \{0,1\}$,

$$\lim_{k\to\infty} \sum_{h\in H} b_1^{m_k}(h,Y)\, q_\omega(h;\hat{s}_{m_k}, b^{m_k}) = \sum_{h\in H} b_1^{\#}(h,Y)\, q_\omega\left(h;\hat{s}, b^{\#}\right). \tag{18}$$

Observe first that the probability $q_\omega(h;\hat{s}_{m_k}, b^{m_k})$ can be decomposed as follows. Suppose $h = \left(r_1, r_2, ..., r_{|h|}\right)$, where $|h|$ stands for the length of the history h.

$$\begin{aligned} q_\omega(h;\hat{s}_{m_k}, b^{m_k}) &= \Pr(r_1|\hat{s}_{m_k},\omega)\left(1-b_1^{m_k}(r_1)\right)\left(1-b_0^{m_k}(r_1)\right) \\ &\quad \times \Pr(r_2|\hat{s}_{m_k},\omega)\left(1-b_1^{m_k}(r_1,r_2)\right)\left(1-b_0^{m_k}(r_1,r_2)\right) \\ &\quad \times .... \times \Pr\left(r_{|h|}|\hat{s}_{m_k},\omega\right)\left(1-b_1^{m_k}(h)\right)\left(1-b_0^{m_k}(h)\right). \end{aligned}$$

Note that as $\Pr(r|\hat{s}_{m_k},\omega)$ is either $F(\hat{s}_{m_k}|\omega)$ or $1-F(\hat{s}_{m_k}|\omega)$, and $F(\cdot|\omega)$ is continuous, it is clear that $\Pr(r|\hat{s}_{m_k},\omega) \to \Pr(r|\hat{s},\omega)$. Together with the fact that $b_0^{m_k}(\cdot)$ and $b_1^{m_k}(\cdot)$ converge pointwise to $b_0^{\#}(\cdot)$ and $b_1^{\#}(\cdot)$ respectively, $q_\omega(h;\hat{s}_{m_k}, b^{m_k})$ converges to $q_\omega\left(h;\hat{s}, b^{\#}\right)$ for each $h \in H$. We now establish (18) using Lebesgue's dominated convergence theorem. For each $k \in \mathbb{N}$, define $Q_k : \mathbb{N} \to \mathbb{R}$ and $Q : \mathbb{N} \to \mathbb{R}$ as follows.

$$\begin{aligned} Q_k(n,\omega) &\equiv \sum_{h\in H:|h|=n} b_1^{m_k}(h,Y)\, q_\omega(h;\hat{s}_{m_k}, b^{m_k})\text{ ;and} \\ Q(n,\omega) &\equiv \sum_{h\in H:|h|=n} b_1^{\#}(h,Y)\, q_\omega\left(h;\hat{s}, b^{\#}\right). \end{aligned}$$

With these definitions, we can write

$$\sum_{h\in H} b_1^{m_k}(h,Y)\, q_\omega(h;\hat{s}_{m_k}, b^{m_k}) = \sum_{n=0}^{\infty} Q_k(n,\omega).$$

Convergence (18) can be restated as $\sum_{n=0}^{\infty} Q_k(n,\omega) \to \sum_{n=0}^{\infty} Q(n,\omega)$. Below we establish this convergence using Lebesgue's dominated convergence theorem. It suffices to show that there exists $N, K \in \mathbb{N}$ such that for all $n > N$ and $k > K$, we have $Q_k(n,\omega) \leq \Phi(n)$ for some function $\Phi(n)$ such that $\sum_{n=0}^{\infty} \Phi(n) < \infty$.

Concerning how the sequences $\{p_0(\hat{s}_{m_k})\}$ and $\{p_1(\hat{s}_{m_k})\}$ approach their respective limits, there are only two possibilities. First, there is a subsequence $\{\hat{s}_{l_k}\}$ of $\{\hat{s}_{m_k}\}$ such that $p_0(\hat{s}_{l_k}) = p_1(\hat{s}_{l_k})$ for all $k \in \mathbb{N}$. Second, there is a subsequence $\{\hat{s}_{l_k}\}$ of $\{\hat{s}_{m_k}\}$ such that $p_0(\hat{s}_{l_k}) < p_1(\hat{s}_{l_k})$ for all $l_k \in \mathbb{N}$.

Consider the first possibility. For all $k \in \mathbb{N}$, period $n > 1$ is reached only if the agent's prior belief exceeds $p_1(\hat{s}_{l_k})$, but has always received recommendation N in all periods up to $n-1$. Thus, $Q_k(n,\omega) \leq F(\hat{s}_{l_k}|\omega)^{n-1}$. Let $\varepsilon \in (0, 1 - F(\hat{s}|0))$. As $\hat{s} < \bar{s}$, there exists a K' such that $F(\hat{s}_{l_k}|\omega) < F(\hat{s}|\omega) + \varepsilon$ for all $k > K'$. Thus, $Q_k(n,\omega) \leq (F(\hat{s}|\omega) + \varepsilon)^{n-1}$ for all $k > K$. It is clear that $\sum_{n=0}^{\infty} (F(\hat{s}|\omega) + \varepsilon)^{n-1} = \frac{1}{1-F(\hat{s}|\omega)-\varepsilon} < \infty$.

Consider the second possibility. For each $k \in \mathbb{N}$, period $n > 1$ is reached only if one of the following events occur: (a) the agent's posterior belief at the end of period $n-1$ is in the interval $[p_0(\hat{s}_{l_k}), p_1(\hat{s}_{l_k})]$, or (b) the agent's prior belief exceeds $p_1(\hat{s}_{l_k})$ but has always received recommendation N in all periods up to period $n-1$. Thus, $Q_k(n,\omega)$ is bounded by

$$
\begin{aligned}
Q_k(n,\omega) &\leq F(\hat{s}_{l_k}|\omega)^{n-1} + \sum_{h \in H:|h|=n-1} \Pr\left(p(h,\hat{s}_{l_k}) \in [p_0(\hat{s}_{l_k}), p_1(\hat{s}_{l_k})] \,|\omega\right) \\
&\leq \begin{cases} F(\hat{s}_{l_k}|1)^{n-1} + \sum_{h \in H:|h|=n-1} \Pr\left(p(h,\hat{s}_{l_k}) \leq p_1(\hat{s}_{l_k}) \,|\omega = 1\right) & \text{if } \omega = 1 \\ F(\hat{s}_{l_k}|0)^{n-1} + \sum_{h \in H:|h|=n-1} \Pr\left(p(h,\hat{s}_{l_k}) \geq p_0(\hat{s}_{l_k}) \,|\omega = 0\right) & \text{if } \omega = 0 \end{cases}. \qquad (19)
\end{aligned}
$$

As $\hat{s} < \bar{s}$, there exists a K' such that $F(\hat{s}_{l_k}|\omega) < F(\hat{s}) + \varepsilon$ for all $k > K'$. An upper bound on $\sum_{h \in H:|h|=n-1} \Pr(p(h,\hat{s}_{l_k}) \leq P_1|\omega = 1)$ can be obtained by noting that for each $\hat{s}_{l_k}$, the expert's recommendation $r \in \{Y, N\}$ is a Bernoulli random variable, with $\Pr(r = Y) = 1 - F(\hat{s}_{l_k}|\omega)$. The agent's posterior after receiving $n-1$ recommendations is weakly less than $p_1(\hat{s}_{l_k})$ if and only if the number y of recommendation Y is sufficiently small:

$$
y \leq \frac{(n-1)\ln\left(\frac{F\left(\hat{s}_{l_k}|1\right)}{F\left(\hat{s}_{l_k}|0\right)}\right) + \ln\left(\frac{1}{p_1\left(\hat{s}_{l_k}\right)} - 1\right) + \ln\frac{\pi}{1-\pi}}{\ln\left(\frac{1-F\left(\hat{s}_{l_k}|0\right)}{F\left(\hat{s}_{l_k}|0\right)}\right) - \ln\left(\frac{1-F\left(\hat{s}_{l_k}|1\right)}{F\left(\hat{s}_{l_k}|1\right)}\right)}.
$$

Applying Hoeffding's inequality,

$$\Pr\left(y \leq \frac{\ln\left(\frac{F\left(\hat{s}_{l_k}|1\right)}{F\left(\hat{s}_{l_k}|0\right)}\right) + \frac{1}{n-1}\left(\ln\left(\frac{1}{p_1\left(\hat{s}_{l_k}\right)} - 1\right) + \ln\frac{\pi}{1-\pi}\right)}{\ln\left(\frac{1-F\left(\hat{s}_{l_k}|0\right)}{F\left(\hat{s}_{l_k}|0\right)}\right) - \ln\left(\frac{1-F\left(\hat{s}_{l_k}|1\right)}{F\left(\hat{s}_{l_k}|1\right)}\right)}(n-1)\,|\omega = 1\right)$$

$$\leq \exp\left(-2\left[(1 - F(\hat{s}_{l_k}|1)) - \frac{\ln\left(\frac{F\left(\hat{s}_{l_k}|1\right)}{F\left(\hat{s}_{l_k}|0\right)}\right) + \frac{1}{n-1}\left(\ln\left(\frac{1}{p_1\left(\hat{s}_{l_k}\right)} - 1\right) + \ln\frac{\pi}{1-\pi}\right)}{\ln\left(\frac{1-F\left(\hat{s}_{l_k}|0\right)}{F\left(\hat{s}_{l_k}|0\right)}\right) - \ln\left(\frac{1-F\left(\hat{s}_{l_k}|1\right)}{F\left(\hat{s}_{l_k}|1\right)}\right)}\right]^2 (n-1)\,|\omega = 1\right).$$

As $n, k \to \infty$, the term in the bracket in the last line approaches

$$(1 - F(\hat{s}|1)) - \frac{\ln F(\hat{s}|0) - \ln F(\hat{s}|1)}{\ln\left(\frac{1-F(\hat{s}|1)}{F(\hat{s}|1)}\right) - \ln\left(\frac{1-F(\hat{s}|0)}{F(\hat{s}|0)}\right)} \equiv L_{\hat{s}} > 0.$$

Therefore, there exists a pair of sufficiently large integers $N_1, K_1 > K'$ such that for all $n > N_1$ and $k > K_1$, we have

$$Q_k(n,1) \leq (F(\hat{s}|1) + \varepsilon)^{n-1} + \sum_{h \in H : |h| = n-1} \Pr\left(p(h, \hat{s}_{l_k}) \leq p_1(\hat{s}_{l_k})\,|\omega = 1\right) \leq (F(\hat{s}|1) + \varepsilon)^{n-1} + \exp\left(-\frac{L_{\hat{s}}^2}{2}(n-1)\right).$$

Define a dominating function $\Phi_1 : \mathbb{N} \to [0,1]$ by

$$\Phi_1(n) \equiv \begin{cases} 1 & \text{if } n < N_1 \\ (F(\hat{s}|1) + \varepsilon)^{n-1} + \exp\left(-\frac{L_{\hat{s}}^2}{2}(n-1)\right) & \text{if } n \geq N_1 \end{cases}.$$

It is clear that

$$\sum_{n=1}^{\infty} \Phi_1(n) = N_1 + \frac{1}{1 - (F(\hat{s}|1) + \varepsilon)} + \frac{\exp\left(-\frac{L_{\hat{s}}^2}{2}(N_1 - 1)\right)}{1 - \exp\left(-\frac{L_{\hat{s}}^2}{2}\right)} < \infty.$$

Therefore by Lebesgue's dominated convergence theorem, $\sum_{n=0}^{\infty} Q_k(n,1) \to \sum_{n=0}^{\infty} Q(n,1)$ as $k \to \infty$.

Convergence $\sum_{n=0}^{\infty} Q_k(n,0) \to \sum_{n=0}^{\infty} Q(n,0)$ can be established in a similar way. Noting that the agent's posterior after receiving $n-1$ recommendations is weakly larger than $p_0(\hat{s}_{l_k})$ if and only if the number y of recommendation Y is sufficiently large:

$$y \geq \frac{(n-1)\ln\left(\frac{F\left(\hat{s}_{l_k}|1\right)}{F\left(\hat{s}_{l_k}|0\right)}\right) + \ln\left(\frac{1}{p_0\left(\hat{s}_{l_k}\right)} - 1\right) + \ln\frac{\pi}{1-\pi}}{\ln\left(\frac{1-F\left(\hat{s}_{l_k}|0\right)}{F\left(\hat{s}_{l_k}|0\right)}\right) - \ln\left(\frac{1-F\left(\hat{s}_{l_k}|1\right)}{F\left(\hat{s}_{l_k}|1\right)}\right)}.$$

Applying Hoeffding's inequality,

$$\Pr\left(y \geq \frac{\ln\left(\frac{F\left(\hat{s}_{l_k}|1\right)}{F\left(\hat{s}_{l_k}|0\right)}\right)+\frac{1}{n-1}\left(\ln\left(\frac{1}{p_0\left(\hat{s}_{l_k}\right)}-1\right)+\ln\frac{\pi}{1-\pi}\right)}{\ln\left(\frac{1-F\left(\hat{s}_{l_k}|0\right)}{F\left(\hat{s}_{l_k}|0\right)}\right)-\ln\left(\frac{1-F\left(\hat{s}_{l_k}|1\right)}{F\left(\hat{s}_{l_k}|1\right)}\right)}(n-1)\,|\omega=0\right)$$

$$\leq \exp\left(-2\left[\frac{\ln\left(\frac{F\left(\hat{s}_{l_k}|1\right)}{F\left(\hat{s}_{l_k}|0\right)}\right)+\frac{1}{n-1}\left(\ln\left(\frac{1}{p_0\left(\hat{s}_{l_k}\right)}-1\right)+\ln\frac{\pi}{1-\pi}\right)}{\ln\left(\frac{1-F\left(\hat{s}_{l_k}|0\right)}{F\left(\hat{s}_{l_k}|0\right)}\right)-\ln\left(\frac{1-F\left(\hat{s}_{l_k}|1\right)}{F\left(\hat{s}_{l_k}|1\right)}\right)}-\left(1-F\left(\hat{s}_{l_k}|0\right)\right)\right]^2(n-1)\right).$$

As $n,k\to\infty$, the term in the bracket in the last line approaches

$$\frac{\ln F\left(\hat{s}|0\right)-\ln F\left(\hat{s}|1\right)}{\ln\left(\frac{1-F(\hat{s}|1)}{F(\hat{s}|1)}\right)-\ln\left(\frac{1-F(\hat{s}|0)}{F(\hat{s}|0)}\right)}-\left(1-F\left(\hat{s}|0\right)\right)\equiv L'_{\hat{s}}>0.$$

Therefore, there exists a pair of sufficiently large integers $N_0, K_0 > K'$ such that for all $n > N_0$ and $k > K_0$, we have

$$Q_k\left(n,0\right)\leq\left(F\left(\hat{s}|0\right)+\varepsilon\right)^{n-1}+\sum_{h\in H:|h|=n-1}\Pr\left(p\left(h,\hat{s}_{l_k}\right)\geq P_0|\omega=0\right)\leq\left(F\left(\hat{s}|0\right)+\varepsilon\right)^{n-1}+\exp\left(-\frac{L'^2_{\hat{s}}}{2}\left(n-1\right)\right).$$

Define a dominating function $\Phi_0:\mathbb{N}\to[0,1]$ by

$$\Phi_0\left(n\right)\equiv\begin{cases}1 & \text{if } n<N_0\\ \left(F\left(\hat{s}|0\right)+\varepsilon\right)^{n-1}+\exp\left(-\frac{L'^2_{\hat{s}}}{2}\left(n-1\right)\right) & \text{if } n\geq N_0\end{cases}.$$

It is clear that

$$\sum_{n=1}^{\infty}\Phi_0\left(n\right)=N_0+\frac{1}{1-F\left(\hat{s}|0\right)}+\frac{\exp\left(-\frac{L'^2_{\hat{s}}}{2}\left(N_0-1\right)\right)}{1-\exp\left(-\frac{L^2_{\hat{s}}}{2}\right)}<\infty.$$

Therefore by Lebesgue's dominated convergence theorem, we have $\sum_{n=0}^{\infty}Q_k\left(n,0\right)\to\sum_{n=0}^{\infty}Q\left(n,0\right)$.

Next, suppose $z=\bar{s}$. Suppose there exists a subsequence $\{z_{m_k}\}\subset\{z_m\}$ such that for all $k\in\mathbb{N}$, $J\left(z_{m_k},\hat{s}_{m_k},b^{m_k}\right)=\frac{l}{1+l}$ for some $b^{m_k}\in\Psi\left(\hat{s}_{m_k}\right)$. Then the proof above still applies. Suppose next that the subsequence above does not exist. Then for all m sufficiently large, $z_{m_k}=\bar{s}$, and thus $J\left(\bar{s},\hat{s}_{m_k},b^{m_k}\right)\leq\frac{l}{1+l}$ for all $b^{m_k}\in\Psi\left(\hat{s}_{m_k}\right)$. It is without loss that b^{m_k} converges (as otherwise we can take a subsequence) with a limit $b\in\Psi\left(\hat{s}\right)$ (as shown in Claim 2). The analysis above shows that $J\left(\bar{s},\hat{s}_{m_k},b^{m_k}\right)\to J\left(\bar{s},\hat{s},b\right)$. Consequently, $\bar{s}\in Z\left(\hat{s}\right)$.

Now, suppose $z=\underline{s}$. Suppose there exists a subsequence $\{z_{m_k}\}\subset\{z_m\}$ such that for all $k\in\mathbb{N}$, $J\left(z_{m_k},\hat{s}_{m_k},b^{m_k}\right)=\frac{l}{1+l}$ for some $b^{m_k}\in\Psi\left(\hat{s}_{m_k}\right)$. Then the proof above still applies. Suppose next that the subsequence above does not exist. Then for all m sufficiently large, $z_{m_k}=\bar{s}$, and thus $J\left(\underline{s},\hat{s}_{m_k},b^{m_k}\right)\geq$

$\frac{l}{1+l}$ for all $b^{m_k} \in \Psi(\hat{s}_{m_k})$. It is without loss that b^{m_k} converges (as otherwise we can take a subsequence) with a limit $b \in \Psi(\hat{s})$ (as shown in Claim 2). The analysis above shows that $J(\underline{s}, \hat{s}_{m_k}, b^{m_k}) \to J(\underline{s}, \hat{s}, b)$. Consequently, $\underline{s} \in Z(\hat{s})$.

This finishes the proof that Z has a fixed point. Finally, we note that the fixed point occurs at some interior value of s. It is clear that $Z(\bar{s}) = \frac{1-\pi}{\pi} l < \bar{s}$. Moreover, any $b \in \Psi(\underline{s})$ must be such that $b_1(Y) = 1$ (i.e., taking the operation at the first expert), as $\pi > \frac{L}{1+L}$. Consequently, $J(s, \underline{s}, b) = \left(1 + \frac{1}{s}\frac{1-\pi}{\pi}\right)^{-1}$ and $Z(\underline{s}) = \frac{1-\pi}{\pi} l > \underline{s}$. Q.E.D.

Proof of Lemma 4: To ease notation, denote $s_n^* \equiv s^*(c_n)$. We first show that it is impossible to have $\{s_n^*\}$ converging to $\bar{s}$. Suppose there exists a sequence of informative equilibria such that $\lim_{n\to\infty} s_n^* = \bar{s}$. As the agent takes the operation only if his belief is no less than $\frac{L}{1+L}$, we have $p_1(s_n^*) \geq \frac{L}{1+L}$. Using (9), we can derive a lower bound on $\tilde{p}_1(s_n^*)$:

$$\tilde{p}_1(s_n^*) = \frac{1}{\left(\frac{1}{p_1(s_n^*)} - 1\right)\frac{1-F(s_n^*|1)}{1-F(s_n^*|0)} + 1} \geq \frac{1}{\frac{1}{L}\frac{1-F(s_n^*|1)}{1-F(s_n^*|0)} + 1}.$$

Combined with the upper bound on $\tilde{p}_1(s_n^*)$ from (10), we have

$$\frac{L}{l} \leq \frac{1}{s_n^*}\frac{1-F(s_n^*|1)}{1-F(s_n^*|0)}.$$

The right-hand side of the inequality above is less than 1 for all $s_n^* < \bar{s}$ because $\frac{1-F(s|1)}{1-F(s|0)}$ is increasing in s. Therefore, the inequality above implies $L \leq l$, a contradiction to the assumption that $l < L$.

Suppose $\{s_n^*\}$ does not converge to $\underline{s}$. Then there exists an $\varepsilon > 0$ and a convergent subsequence $\{s_{n_k}^*\}$ such that $s_{n_k}^* \to \underline{s} + \varepsilon < \bar{s}$. Consider the agent's search problem. Using (13), his equilibrium value function $V(\cdot; s_{n_k}^*)$ at search cost c_{n_k} is given by

$$\begin{aligned} V(p; s_{n_k}^*) &= -c_{n_k} + \left[p\left(1 - F(s_{n_k}^*|1)\right) + (1-p)\left(1 - F(s_{n_k}^*|0)\right)\right] \\ &\quad \times \max\left\{\begin{array}{l} 0, V\left(\left(1 + \frac{1-p}{p}\frac{1-F(s_{n_k}^*|0)}{1-F(s_{n_k}^*|1)}\right)^{-1}; s_{n_k}^*\right), \\ -L + \left(1 + \frac{1-p}{p}\frac{1-F(s_{n_k}^*|0)}{1-F(s_{n_k}^*|1)}\right)^{-1}(1+L) \end{array}\right\} \\ &\quad + \left[pF(s_{n_k}^*|1) + (1-p)F(s_{n_k}^*|0)\right]\max\left\{0, V\left(\left(1 + \frac{1-p}{p}\frac{F(s_{n_k}^*|0)}{F(s_{n_k}^*|1)}\right)^{-1}; s_{n_k}^*\right)\right\}. \end{aligned}$$

As shown in the proof of Lemma 9, $\{V(\cdot; s_{n_k}^*)\}_{k\in\mathbb{N}}$ is a family of Lipschitz continuous with a common modulus. Thus, there exists a further subsequence $\{m_k\}$ of $\{n_k\}$ such that $V(\cdot; s_{m_k}^*)$ converges uniformly to $V(\cdot; \underline{s} + \varepsilon)$. Replacing n_k with m_k in the equation above and taking limit, we get

$$V(p;\underline{s}+\varepsilon) = [p(1-F(\underline{s}+\varepsilon|1))+(1-p)(1-F(\underline{s}+\varepsilon|0))]\max\left\{\begin{array}{c} 0, V\left(\left(1+\frac{1-p}{p}\frac{1-F(\underline{s}+\varepsilon|0)}{1-F(\underline{s}+\varepsilon|1)}\right)^{-1};\underline{s}+\varepsilon\right), \\ -L+\left(1+\frac{1-p}{p}\frac{1-F(\underline{s}+\varepsilon|0)}{1-F(\underline{s}+\varepsilon|1)}\right)^{-1}(1+L) \end{array}\right\}$$
$$+[pF(\underline{s}+\varepsilon|1)+(1-p)F(\underline{s}+\varepsilon|0)]\max\left\{0, V\left(\left(1+\frac{1-p}{p}\frac{F(\underline{s}+\varepsilon|0)}{F(\underline{s}+\varepsilon|1)}\right)^{-1};\underline{s}+\varepsilon\right)\right\}. \quad (20)$$

It is straightforward to verify that $V(p;\underline{s}+\varepsilon)=p$ is a solution. Moreover, it is unique by Proposition 8. Therefore, $p_1\left(s^*_{n_k}\right)\to 1$. However, using (10), we get

$$\left(1+\frac{1}{s^*_{n_k}}\left(\frac{1-\tilde{p}_1\left(s^*_{n_k}\right)}{\tilde{p}_1\left(s^*_{n_k}\right)}\right)\right)^{-1}\leq\frac{l}{1+l}\Leftrightarrow s^*_{n_k}\leq\frac{1-\tilde{p}_1\left(s^*_{n_k}\right)}{\tilde{p}_1\left(s^*_{n_k}\right)}l,$$

where $\tilde{p}_1\left(s^*_{n_k}\right)$ is defined in (9). Substitute $\tilde{p}_1\left(s^*_{n_k}\right)=\left(1+\left(\frac{1}{p_1\left(s^*_{n_k}\right)}-1\right)\frac{1-F\left(s^*_{n_k}|1\right)}{1-F\left(s^*_{n_k}|0\right)}\right)^{-1}$ into the upper bound above, we get

$$\frac{1-\tilde{p}_1\left(s^*_{n_k}\right)}{\tilde{p}_1\left(s^*_{n_k}\right)}l=\left(\left(\frac{1}{p_1\left(s^*_{n_k}\right)}-1\right)\frac{1-F\left(s^*_{n_k}|1\right)}{1-F\left(s^*_{n_k}|0\right)}\right)l.$$

It converges to 0 as $k\to\infty$, because $p_1\left(s^*_{n_k}\right)\to 1$. Thus, $s^*_{n_k}\to 0$, a contradiction. Q.E.D.

Proof of Lemma 5: (i) Suppose an informative equilibrium exists for each c_n. To ease notation, denote $s^*_n\equiv s^*(c_n)$. Suppose $\{p_1(s^*_n)\}_n$ converges (otherwise, take a subsequence). By Lemma 4, $s^*_n\to\underline{s}$. Thus, the difference between $p_1(s^*_n)$ and $\tilde{p}_1(s^*_n)$ vanishes in the limit:

$$p_1\left(s^*_{n_k}\right)-\tilde{p}_1(s^*_n)=p_1(s^*_n)-\frac{1}{\left(\frac{1}{p_1(s^*_n)}-1\right)\frac{1-F(s^*_n|1)}{1-F(s^*_n|0)}+1}=p_1(s^*_n)\frac{\frac{1-F(s^*_n|1)}{1-F(s^*_n|0)}-1}{\frac{1-F(s^*_n|1)}{1-F(s^*_n|0)}+1}\to 0 \text{ as } n\to\infty.$$

Rearranging (10) gives $\tilde{p}_1(s^*_n)\leq\frac{l}{l+s^*_n}$. Therefore, $\lim_{n\to\infty}p_1(s^*_n)\leq\frac{l}{l+\underline{s}}$. On the other hand, conditional on being pivotal, the agent's belief prior to learning the current expert's recommendation is bounded from above by $\max\{\pi,p_1(s^*_n)\}$. Condition (8) then implies that

$$\frac{1}{1+\frac{1}{s^*_n}\frac{1-\max\{\pi,p_1(s^*_n)\}}{\max\{\pi,p_1(s^*_n)\}}}\geq\frac{l}{1+l}\Leftrightarrow\max\{\pi,p_1(s^*_n)\}\geq\frac{l}{l+s^*_n}.$$

Taking limit on both sides of the inequality then gives $\max\{\pi,\lim_{n\to\infty}p_1(s^*_n)\}\geq\frac{l}{l+\underline{s}}$. As $\pi<\frac{l}{l+\underline{s}}$ by assumption, it is necessary that $\lim_{n\to\infty}p_1(s^*_n)\geq\frac{l}{l+\underline{s}}$. Therefore, $\lim_{n\to\infty}p_1(s^*_n)=\frac{l}{l+\underline{s}}$.

(ii) Suppose an informative equilibrium exists for each c_n. Then from the proof of part (i), $\lim_{n\to\infty}p_1(s^*_n)\leq\frac{l}{l+\underline{s}}$ regardless of π. As $\underline{s}>\frac{l}{L}$ implies $\frac{l}{l+\underline{s}}<\frac{L}{1+L}$, we have $-L+p_1(s^*_n)(1+L)<0$ for sufficiently large n. This contradicts that the agent is willing to undergo the operation at $p_1(s^*_n)$.

Proof of Proposition 3: (i) Denote $s_n^* \equiv s^*(c_n)$. The agent's equilibrium payoff $U(c_n)$ at search cost c_n is equal to $V(\pi; s_n^*) + c_n$.

Note that by the definition of $p_1(s_n^*)$ in (14), $V(p_1^*(s_n); s_n^*) \leq \max\{0, -L + (1+L) p_1(s_n^*)\}$. Moreover, as each $V(\cdot; s_n^*)$ is weakly convex and $V_n(0; s_n^*) = -c_n$, an upper bound on $V_n(\pi; s_n^*)$ is given by

$$V_n(\pi; s_n^*) \leq -c_n + \frac{\max\{0, -L + (1+L) p_1(s_n^*)\} + c_n}{p_1(s_n^*)} \pi.$$

As $\underline{s} \leq \frac{l}{L}$, according to Lemma 5, $\lim_{n\to\infty} p_1(s_n^*) = \frac{l}{l+\underline{s}} \geq \frac{L}{1+L}$. Taking limsup on both sides of the inequality above gives

$$\limsup_{n\to\infty} V_n(\pi; s_n^*) \leq \left(1 - \frac{L}{l}\underline{s}\right) \pi.$$

Therefore, $\limsup_{n\to\infty} U(c_n) = \limsup_{n\to\infty} V_n(\pi; s_n^*) \leq \left(1 - \frac{L}{l}\underline{s}\right) \pi$.

Denote by E_n the expected payoff of the expert who carries out the operation, in an equilibrium of the game in which the search cost is c_n and the experts' cutoff is s_n^*. Recall in the proof of Lemma 5, we have shown that $\lim_{n\to\infty} \tilde{p}_1(s_n^*) = \lim_{n\to\infty} p_1(s_n^*) = \frac{l}{l+\underline{s}} < \pi$. Therefore, for n sufficiently large, $\pi < \tilde{p}_1(s_n^*) < p_1(s_n^*)$. Consequently, E_n is bounded from above as follows:

$$\begin{aligned} E_n &\leq -l + (1+l) \Pr(\omega = 1 | p = p_1(s_n^*), s \geq s_n^*) \\ &= -l + (1+l) \frac{1}{1 + \frac{1-p_1(s_n^*)}{p_1(s_n^*)} \frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}}, \end{aligned}$$

where $p = p_1(s_n^*)$ stands for the event that the agent enters into the period with a belief $p_1(s_n^*)$. Moreover, using Bayes' rule, starting with a prior belief π, the probability that the agent's posterior reaches $\tilde{p}_1(s_n^*)$ is at most $\frac{\pi}{\tilde{p}_1(s_n^*)}$. The experts' joint payoff $T(c_n)$ is therefore bounded from above by:

$$T(c_n) \leq \frac{\pi}{\tilde{p}_1(s_n^*)} E_n \leq \frac{\pi}{\tilde{p}_1(s_n^*)} \left(-l + (1+l) \frac{1}{1 + \frac{1-p_1(s_n^*)}{p_1(s_n^*)} \frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}} \right).$$

Taking limsup for the inequality above gives

$$\limsup_{n\to\infty} T(c_n) \leq \limsup_{n\to\infty} \frac{\pi}{\tilde{p}_1(s_n^*)} E_n \leq \pi(1 - \underline{s}).$$

Finally, the agent's payoff in an uninformative equilibrium is given by $\max\{0, -L + \pi(1+L)\}$, which is less than $\left(1 - \frac{L}{l}\underline{s}\right)\pi$ (as $\underline{s} \leq \frac{l}{L}$ and $\pi < \frac{l}{l+\underline{s}}$). The experts' joint payoff in an uninformative equilibrium is $-l + \pi(1+l)$ if $\pi \geq \frac{L}{1+L}$ and 0 otherwise. In either case, it is less than $\pi(1-\underline{s})$ (as $\underline{s} < 1$ and $\pi < \frac{l}{l+\underline{s}}$).

(ii) By Lemma 5, there are only uninformative equilibria for c_n sufficiently small. As $\underline{s} > \frac{l}{L}$ implies $\frac{l}{l+\underline{s}} < \frac{L}{1+L}$, if $\pi \geq \frac{L}{1+L}$, then by Lemma 3, the only uninformative equilibrium outcome has $s^* = \underline{s}$ and the agent following the recommendation Y of the first expert. Therefore, $U(c_n) = -L + \pi(1+L)$ and $T(c_n) = -l + \pi(1+l)$. Conversely, if $\pi < \frac{L}{1+L}$, then by Lemma 3, the only uninformative-equilibrium outcome involves the agent not taking the operation, so $U(c_n) = T(c_n) = 0$. Q.E.D.

Proof of Lemma 6: (i) To ease notation, define $s_n^* \equiv s^*(c_n)$. As $\underline{s} = 0$, by part (i) of Lemma 5, $\lim_{n\to\infty} p_1(s_n^*) = 1 > \frac{L}{1+L}$. Using Corollary 4, $V(p_1(s_n^*); s_n^*) = -L + (1+L)p_1(s_n^*) > 0$ and $p_0(s_n^*) < \frac{L}{1+L}$ for n sufficiently large. Using the definition of $V(\cdot;\cdot)$ in (13),

$$\begin{aligned}
& -L + p_1(s_n^*)(1+L) \\
= \; & -c_n + \left[p_1(s_n^*)(1 - F(s_n^*|1)) + (1 - p_1(s_n^*))(1 - F(s_n^*|0))\right] \\
& \times \left(-L + \left(1 + \frac{1-p_1(s_n^*)}{p_1(s_n^*)}\frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}\right)^{-1}(1+L)\right) \\
& + \left[p_1(s_n^*)F(s_n^*|1) + (1-p_1(s_n^*))F(s_n^*|0)\right] V\left(\left(1 + \frac{1-p_1(s_n^*)}{p_1(s_n^*)}\frac{F(s_n^*|0)}{F(s_n^*|1)}\right)^{-1}; s_n^*\right).
\end{aligned}$$

In the equation above, we have used the fact that the definition of $p_1(s_n^*)$ implies $V(p) < -L + p(1+L)$ for $p > p_1(s_n^*)$. Moreover, for n large enough, $\left(1 + \frac{1-p_1(s_n^*)}{p_1(s_n^*)}\frac{F(s_n^*|0)}{F(s_n^*|1)}\right)^{-1}$ is so close to $p_1(s_n^*)$ that $V\left(\left(1 + \frac{1-p_1(s_n^*)}{p_1(s_n^*)}\frac{F(s_n^*|0)}{F(s_n^*|1)}\right)^{-1}; s_n^*\right) > 0$. Re-arranging the equation above gives

$$\begin{aligned}
c_n = \; & -p_1(s_n^*)F(s_n^*|1)(1+L) \\
& + \left[p_1(s_n^*)F(s_n^*|1) + (1-p_1(s_n^*))F(s_n^*|0)\right]\left[V\left(\left(1 + \frac{1-p_1(s_n^*)}{p_1(s_n^*)}\frac{F(s_n^*|0)}{F(s_n^*|1)}\right)^{-1}; s_n^*\right) + L\right]
\end{aligned} \tag{21}$$

As $\pi < 1$, $p_1(s_n^*) > \pi$ for n sufficiently large. For any agent's best response b, the likelihood ratio associated with the event $piv(b)$ in equation (8) can be bounded from below as follows:

$$\frac{1-\pi}{\pi}\frac{\sum_{h\in H} q_0(h; s_n^*, b)\, b_1(h, Y)}{\sum_{h\in H} q_1(h; s_n^*, b)\, b_1(h, Y)} \geq \frac{1-\pi}{\pi}\inf_{\{h\in H: b_1(h,Y)>0\}}\frac{q_0(h; s_n^*, b)}{q_1(h; s_n^*, b)} \geq \frac{1-p_1(s_n^*)}{p_1(s_n^*)}.$$

Substituting this into (8) then gives

$$p_1(s_n^*) \geq \frac{l}{s_n^* + l}. \tag{22}$$

Together with the fact that $V(\cdot;\cdot) \leq 1$, equation (21) implies that

$$c_n \leq (1+L)\frac{s_n^* F(s_n^*|0)}{s_n^* + l}. \tag{23}$$

Now fix a $n \in \mathbb{N}$ and consider the following (necessarily suboptimal) search strategy of the agent: sample a fixed number M_n of experts and take the operation in the end if and only if all of them recommend Y. Here, M_n is chosen such that the posterior reaches $p_1(s_n^*)$ if all M_n experts recommends

Y. The agent's equilibrium payoff is bounded from below by the expected payoff of this strategy, which is given by

$$\pi \left(1 - F\left(s_n^*|1\right)\right)^{M_n} + \left(1-\pi\right)\left(1 - F\left(s_n^*|0\right)\right)^{M_n}\left(-L\right) - c_n M_n.$$

We consider the limit of the following terms respectively: (a) $c_n M_n$, and (b) $\pi \left(1 - F\left(s_n^*|1\right)\right)^{M_n} + \left(1-\pi\right)\left(1 - F\left(s_n^*|0\right)\right)^{M_n}\left(-L\right)$. First consider term (a). The number M_n must satisfy

$$\frac{1}{1+\frac{1-\pi}{\pi}\left(\frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}\right)^{M_n-1}} \leq p_1\left(s_n^*\right) \leq \frac{1}{1+\frac{1-\pi}{\pi}\left(\frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}\right)^{M_n}}$$
$$\Leftrightarrow \quad M_n \in \left[\frac{\ln\left(\frac{\pi}{1-\pi}\left(\frac{1}{p_1(s_n^*)}-1\right)\right)}{\ln\left(\frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}\right)}, \frac{\ln\left(\frac{\pi}{1-\pi}\left(\frac{1}{p_1(s_n^*)}-1\right)\right)}{\ln\left(\frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}\right)}+1\right]. \tag{24}$$

Substituting (9) into (10) gives:

$$p_1\left(s_n^*\right) \leq \left(1 + \frac{1-F\left(s_n^*|0\right)}{1-F\left(s_n^*|1\right)}\frac{s_n^*}{l}\right)^{-1}. \tag{25}$$

Consequently, an upper bound on M_n is given by

$$M_n \leq \frac{\ln\left(\frac{\pi}{1-\pi}\left(\frac{1}{p_1(s_n^*)}-1\right)\right)}{\ln\left(\frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}\right)} + 1 \leq \frac{\ln\left(\frac{\pi}{1-\pi}\frac{s_n^*}{l}\frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}\right)}{\ln\left(\frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}\right)} + 1. \tag{26}$$

Using both (23) and (26), we have

$$c_n M_n \leq \left(\frac{1+L}{s_n^*+l}\right)\left[s_n^* \ln\left(\frac{\pi}{1-\pi}\frac{s_n^*}{l}\frac{1-F\left(s_n^*|0\right)}{1-F\left(s_n^*|1\right)}\right)\right]\frac{F\left(s_n^*|0\right)}{\ln\left(\frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}\right)} + \left(1+L\right)\frac{s_n^*}{s_n^*+l}F\left(s_n^*|0\right).$$

Now we take limit on both sides of the inequality above. As $s_n^* \to 0$, the first term on the right hand side has a limit $\lim_{n\to\infty}\left(\frac{1+L}{s_n^*+l}\right) = \frac{1+L}{l}$. For the second (bracketed) term,

$$\lim_{n\to\infty} s_n^* \ln\left(\frac{\pi}{\left(1-\pi\right)l}s_n^*\frac{1-F\left(s_n^*|0\right)}{1-F\left(s_n^*|1\right)}\right) = \lim_{s^*\to 0}\left[s^*\ln\frac{\pi}{\left(1-\pi\right)l} + s^*\ln\frac{1-F\left(s^*|0\right)}{1-F\left(s^*|1\right)} + s^*\ln s^*\right] = 0,$$

where we have used the fact that $\lim_{s^*\to 0} s^* \ln s^* = 0$. For the third term,

$$\lim_{n\to\infty}\frac{F\left(s_n^*|0\right)}{\ln\left(\frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}\right)} = \lim_{s^*\to 0}\frac{F\left(s^*|0\right)}{\ln\left(\frac{1-F(s^*|0)}{1-F(s^*|1)}\right)} = \lim_{s^*\to 0}\frac{\left(1-F\left(s^*|0\right)\right)\left(1-F\left(s^*|1\right)\right)}{s^*\left(1-F\left(s^*|0\right)\right)-\left(1-F\left(s^*|1\right)\right)} = -1,$$

where the second equality uses L'Hospital rule. For the last term, $\lim_{n\to\infty}\left(1+L\right)\frac{s_n^*}{s_n^*+l}F\left(s_n^*|0\right) = 0$. Therefore, we have

$$\lim_{n\to\infty} c_n M_n = 0. \tag{27}$$

Next we consider the limit of term (b): $\pi\left(1-F\left(s_n^*|1\right)\right)^{M_n}+(1-\pi)\left(1-F\left(s_n^*|0\right)\right)^{M_n}(-L)$. Note that for n sufficiently large,

$$\begin{aligned}
& \pi\left(1-F\left(s_n^*|1\right)\right)^{M_n}+(1-\pi)\left(1-F\left(s_n^*|0\right)\right)^{M_n}(-L) \\
\geq\ & (1-\pi)\left(\frac{p_1\left(s_n^*\right)}{1-p_1\left(s_n^*\right)}-L\right)\left(1-F\left(s_n^*|0\right)\right)^{M_n} \\
\geq\ & (1-\pi)\left(\frac{l}{s_n^*}-L\right)\left(1-F\left(s_n^*|0\right)\right)^{M_n} \\
\geq\ & (1-\pi)\left(\frac{l}{s_n^*}-L\right)\left(1-F\left(s_n^*|0\right)\right)^{\frac{\ln\left(\frac{\pi}{1-\pi}\frac{s_n^*}{l}\frac{1-F\left(s_n^*|0\right)}{1-F\left(s_n^*|1\right)}\right)}{\ln\left(\frac{1-F\left(s_n^*|0\right)}{1-F\left(s_n^*|1\right)}\right)}+1} \equiv \Gamma_n.
\end{aligned}$$

where the first inequality uses (24), the second inequality uses (22), and the last inequality uses the fact that $s_n^*\to 0$ and (26). Below we show that $\lim_{n\to\infty}\Gamma_n=\pi$. To this end, we express Γ_n as follows:

$$\begin{aligned}
\Gamma_n &= (1-\pi)\left(1-F\left(s_n^*|0\right)\right)\left[l\exp\left(\Gamma_n^2\right)-L\exp\left(\Gamma_n^1\right)\right], \text{ where} \\
\Gamma_n^1 &\equiv \frac{\ln\left(\frac{\pi}{1-\pi}\frac{s_n^*}{l}\frac{1-F\left(s_n^*|0\right)}{1-F\left(s_n^*|1\right)}\right)}{\ln\left(\frac{1-F\left(s_n^*|0\right)}{1-F\left(s_n^*|1\right)}\right)}\ln\left(1-F\left(s_n^*|0\right)\right), \text{ and} \\
\Gamma_n^2 &\equiv -\ln s_n^*+\frac{\ln\left(\frac{\pi}{1-\pi}\frac{s_n^*}{l}\frac{1-F\left(s_n^*|0\right)}{1-F\left(s_n^*|1\right)}\right)}{\ln\left(\frac{1-F\left(s_n^*|0\right)}{1-F\left(s_n^*|1\right)}\right)}\ln\left(1-F\left(s_n^*|0\right)\right).
\end{aligned}$$

We first show that $\lim_{n\to\infty}\Gamma_n^1=-\infty$. Applying L'Hospital rule, and using the normalization $f\left(s^*|1\right)=s^*f\left(s^*|0\right)$, we have

$$\begin{aligned}
& \lim_{n\to\infty}\Gamma_n^1 \\
=\ & \lim_{s^*\to 0}\frac{\ln\left(1-F\left(s^*|0\right)\right)}{\frac{\ln\left(\frac{1-F\left(s^*|0\right)}{1-F\left(s^*|1\right)}\right)}{\ln\left(\frac{\pi}{1-\pi}\frac{s^*}{l}\frac{1-F\left(s^*|0\right)}{1-F\left(s^*|1\right)}\right)}} \\
=\ & \lim_{s^*\to 0}\frac{\left(\ln\left(\frac{\pi}{l(1-\pi)}\right)+\ln s^*+\ln\left(\frac{1-F(s^*|0)}{1-F(s^*|1)}\right)\right)^2\frac{f(s^*|0)}{1-F(s^*|0)}}{\frac{1}{s^*}\ln\left(\frac{1-F(s^*|0)}{1-F(s^*|1)}\right)-f\left(s^*|0\right)\left(\frac{-1}{1-F(s^*|0)}+\frac{s^*}{1-F(s^*|1)}\right)\left(\ln\left(\frac{\pi}{l(1-\pi)}\right)+\ln s^*\right)}.
\end{aligned}$$

Observe that the denominator in the expression of the last line is negative. Moreover, $\lim_{s^*\to 0}s^*\ln s^*=0$, and

$$\lim_{s^*\to 0}\frac{\ln\left(1-F\left(s^*|0\right)\right)-\ln\left(1-F\left(s^*|1\right)\right)}{s^*}=\lim_{s^*\to 0}\frac{f\left(s^*|0\right)\left(\frac{-1}{1-F(s^*|0)}+\frac{s^*}{1-F(s^*|1)}\right)}{1}=f\left(0|0\right).$$

Therefore, $\lim_{n\to\infty}\Gamma_n^1=-\infty$. Next, we show $\lim_{n\to\infty}\Gamma_n^2=\ln\left(\frac{\pi}{l(1-\pi)}\right)$. We can re-write Γ_n^2 as

$$\Gamma_n^2=\frac{\ln s_n^*}{\frac{\ln(1-F(s_n^*|0))}{\ln(1-F(s_n^*|1))}-1}+\frac{1}{1-\frac{\ln(1-F(s_n^*|1))}{\ln(1-F(s_n^*|0))}}\ln\left(\frac{\pi}{1-\pi}\frac{1}{l}\right)+\ln\left(1-F\left(s_n^*|0\right)\right).$$

Consider the first term. Applying L'Hospital rule,

$$\lim_{s^*\to 0}\frac{\ln s^*}{\frac{\ln(1-F(s^*|0))}{\ln(1-F(s^*|1))}-1} = \lim_{s^*\to 0}\frac{\ln(1-F(s^*|1))}{-(1-F(s^*|1))\ln(1-F(s^*|1))+s^*(1-F(s^*|0))\ln(1-F(s^*|0))} \times\left[\frac{(\ln(1-F(s^*|1)))(1-F(s^*|0))(1-F(s^*|1))}{s^*f(s^*|0)}\right]$$

The limit of the bracketed term above is 0. The limit of the unbracketed term above can be computed by L'Hospital rule again:

$$\begin{aligned}&\lim_{s^*\to 0}\frac{\ln(1-F(s^*|1))}{-(1-F(s^*|1))\ln(1-F(s^*|1))+s^*(1-F(s^*|0))\ln(1-F(s^*|0))}\\ =\ &\lim_{s^*\to 0}\frac{1}{[\ln(1-F(s^*|1))-\ln(1-F(s^*|0))]+\left(\frac{1-F(s^*|0)}{f(s^*|0)}\right)\frac{\ln(1-F(s^*|0))}{s^*}}\frac{-1}{1-F(s^*|1)}\\ =\ &1.\end{aligned}$$

Now consider the second term in Γ_n^2. As

$$\lim_{n\to\infty}\frac{\ln(1-F(s_n^*|1))}{\ln(1-F(s_n^*|0))}=\lim_{s^*\to 0}\frac{1-F(s^*|0)}{1-F(s^*|1)}\frac{f(s^*|1)}{f(s^*|0)}=0,$$

the limit of the second term in Γ_n^2 is $\ln\left(\frac{\pi}{1-\pi}\frac{1}{l}\right)$. The limit of the third term is clearly 0. Taken together, $\lim_{n\to\infty}\Gamma_n^2=\ln\left(\frac{\pi}{l(1-\pi)}\right)$. Therefore, $\lim_{n\to\infty}\Gamma_n=\pi$.

As a result, together with (27), a lower bound on the limit expected payoff of the search strategy under consideration is π. However, the upper bound of the agent's equilibrium payoff is also π. Therefore, the agent's equilibrium payoff in the limit as $c_n\to 0$ is exactly π.

Note that as $V(p;s_n^*)$ is weakly convex in p, $V(\pi;s_n^*)$ is bounded from above by $\frac{\pi-p_0(s_n^*)}{1-p_0(s_n^*)}$. This bound is less than π, as $p_0(s_n^*)\geq 0$. Therefore,

$$\pi=\lim_{n\to\infty}V(\pi;s_n^*)=\lim_{n\to\infty}\frac{\pi-p_0(s_n^*)}{1-p_0(s_n^*)},$$

which implies that $\lim_{n\to\infty}p_0(s_n^*)=0$.

(ii) Fix a $\underline{s}>0$ and assume $l<L$. We show that if the sequence $\{p_0(s_n^*)\}_n$ converges, its limit strictly exceeds 0. Suppose not. Then $\lim_{n\to\infty}p_0(s_n^*)=0$, so by Corollary 4, $p_1(s_n^*)>\frac{L}{1+L}$ for all n sufficiently large. Fix an arbitrarily $q\in\left(0,\frac{1}{2}\frac{l}{l+\underline{s}}\right)$, and an integer N such that $p_0(s_n^*)<q<p_1(s_n^*)$ for all $n>N$. For $n>N$, the agent's continuation value function evaluated at q, $V(q;s_n^*)$, is strictly positive. We first derive an upper bound for $V(q;s_n^*)$. In the best conceivable scenario, the agent with $\omega=0$ learns the state immediately and gets a payoff of 0; whereas the agent with $\omega=1$ gets a consecutive sequence of Y recommendations, leading to a posterior $p_1(s_n^*)$ and taking the operation.

The number of consecutive Y recommendations needed, denoted by M_n, satisfies

$$\frac{1}{1+\frac{1-q}{q}\left(\frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}\right)^{M_n-1}} \leq p_1\left(s_n^*\right) \leq \frac{1}{1+\frac{1-q}{q}\left(\frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}\right)^{M_n}}$$

$$\Leftrightarrow \quad M_n \in \left[\frac{\ln\left(\frac{q}{1-q}\left(\frac{1}{p_1(s_n^*)}-1\right)\right)}{\ln\left(\frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}\right)}, \frac{\ln\left(\frac{q}{1-q}\left(\frac{1}{p_1(s_n^*)}-1\right)\right)}{\ln\left(\frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}\right)}+1\right].$$

Therefore,

$$V\left(q;s_n^*\right) \leq q\left[\left(1-F\left(s_n^*|1\right)\right)^{M_n}-c_n M_n\right]. \tag{28}$$

Below we derive a contradiction by showing that this upper bound cannot be positive for all $q \in (0,1)$. Using (22),

$$M_n \geq \frac{\ln\left(\frac{q}{1-q}\left(\frac{1}{\frac{l}{s_n^*+l}}-1\right)\right)}{\ln\left(\frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}\right)} = \frac{\ln\left(\frac{1-q}{q}\frac{l}{s_n^*}\right)}{\ln\left(\frac{1-F(s_n^*|1)}{1-F(s_n^*|0)}\right)}.$$

As $p_1\left(s_n^*\right) > \frac{L}{1+L}$ for all n sufficiently large, by Corollary 4, we have $V\left(p_1\left(s_n^*\right);s_n^*\right) = -L+(1+L)\,p_1\left(s_n^*\right)$ for n sufficiently large. Therefore, equation (21) holds. Together with the fact that $V(\cdot;\cdot) \geq 0$, we have $c_n \geq -p_1\left(s_n^*\right)\left[LF\left(s_n^*|0\right)+F\left(s_n^*|1\right)\right]+F\left(s_n^*|0\right)L$. Moreover, $p_1\left(s_n^*\right)$ is bounded from above by (25), giving

$$c_n \geq \frac{-F\left(s_n^*|1\right)+\frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}\frac{s_n^*}{l}F\left(s_n^*|0\right)L}{1+\frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}\frac{s_n^*}{l}}.$$

Consequently,

$$c_n M_n \geq \frac{\ln\left(\frac{1-q}{q}\frac{l}{s_n^*}\right)}{1+\frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}\frac{s_n^*}{l}} \frac{-F\left(s_n^*|1\right)+\frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}\frac{s_n^*}{l}F\left(s_n^*|0\right)L}{\ln\left(\frac{1-F(s_n^*|1)}{1-F(s_n^*|0)}\right)}.$$

Taking $\limsup_{n\to\infty}$ on both sides of the inequality above, and using L'Hospital rule,

$$\begin{aligned}
\lim_{n\to\infty}\sup c_n M_n &\geq \frac{\ln\left(\frac{1-q}{q}\frac{l}{\underline{s}}\right)}{\frac{\underline{s}}{l}+1}\lim_{n\to\infty}\frac{-F\left(s_n^*|1\right)+\frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}\frac{s_n^*}{l}F\left(s_n^*|0\right)L}{\ln\left(\frac{1-F(s_n^*|1)}{1-F(s_n^*|0)}\right)} \\
&= \frac{\ln\left(\frac{1-q}{q}\frac{l}{\underline{s}}\right)}{\frac{\underline{s}}{l}+1}\lim_{n\to\infty}\frac{-f\left(s_n^*|1\right)+\left(\begin{array}{c}\frac{-f(s_n^*|0)(1-F(s_n^*|1))+f(s_n^*|1)(1-F(s_n^*|0))}{(1-F(s_n^*|1))^2}s_n^*F\left(s_n^*|0\right) \\ +\frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}\left(s_n^*f\left(s_n^*|0\right)+F\left(s_n^*|0\right)\right)\end{array}\right)\frac{L}{l}}{\frac{f(s_n^*|0)(1-F(s_n^*|1))-f(s_n^*|1)(1-F(s_n^*|0))}{(1-F(s_n^*|0))(1-F(s_n^*|1))}} \\
&= \frac{1}{\underline{s}+l}\frac{L-l}{1-\underline{s}}\underline{s}\ln\left(\frac{1-q}{q}\frac{l}{\underline{s}}\right).
\end{aligned}$$

Taking $\limsup_{n\to\infty}$ on both sides of (28), and using the fact that $\left(1-F\left(s_n^*|1\right)\right)^{M_n} \leq 1$,

$$\lim_{n\to\infty}\sup V\left(q;s_n^*\right) \leq q\left(1-\frac{1}{\underline{s}+l}\frac{L-l}{1-\underline{s}}\underline{s}\ln\left(\frac{1-q}{q}\frac{l}{\underline{s}}\right)\right).$$

As $V(q; s_n^*) > 0$,

$$0 \leq q \left(1 - \ln\left(\frac{1-q}{q}\frac{l}{\underline{s}}\right)\frac{1}{\underline{s}+l}\frac{L-l}{1-\underline{s}}\underline{s}\right).$$

However, this is a contradiction as q can be an arbitrarily small positive number. Q.E.D.

Proof of Claim 1: (i) The conditional probability $\Pr(\omega = 1|piv, s_n^*)$ can be computed as follows:

$$\begin{aligned} &\Pr(\omega = 1|piv, s_n^*) \\ = &\frac{\pi f(s_n^*|1)\begin{pmatrix} n \\ qn \end{pmatrix}(1-F(s_n^*|1))^{qn} F(s_n^*|1)^{(1-q)n}}{\pi f(s_n^*|1)\begin{pmatrix} n \\ qn \end{pmatrix}(1-F(s_n^*|1))^{qn} F(s_n^*|1)^{(1-q)n} + (1-\pi) f(s_n^*|0)\begin{pmatrix} n \\ qn \end{pmatrix}(1-F(s^*|0))^{qn} F(s^*|0)^{(1-q)n}} \\ = &\frac{1}{1 + \frac{1-\pi}{\pi}\frac{1}{s_n^*}\left(\left(\frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}\right)^q \left(\frac{F(s_n^*|0)}{F(s_n^*|1)}\right)^{1-q}\right)^n}. \end{aligned}$$

Note that $\Pr(\omega = 1|piv, s_n^*)$ is continuous in s_n^*. Moreover, for n sufficiently large, $\Pr(\omega = 1|piv, \bar{s}) > \frac{l}{1+l} > \Pr(\omega = 1|piv, \underline{s})$. The existence of a solution to (12) then follows from the intermediate value theorem.

(ii) Suppose the sequence $\{s_n^*\}$ converges (otherwise, take a subsequence). Then the sequence $\left\{\left(\left(\frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}\right)^q \left(\frac{F(s_n^*|0)}{F(s_n^*|1)}\right)^{1-q}\right)^n\right\}_{n\in\mathbb{N}}$ necessarily converges, which implies that

$$\lim_{n\to\infty}\left(\frac{1-F(s_n^*|0)}{1-F(s_n^*|1)}\right)^q \left(\frac{F(s_n^*|0)}{F(s_n^*|1)}\right)^{1-q} = 1.$$

Consequently, $s^* \equiv \lim_{n\to\infty} s_n^*$ is characterized by the equation

$$(1-F(s^*|0))^q (F(s^*|0))^{1-q} = (1-F(s^*|1))^q F(s^*|1)^{1-q}. \tag{29}$$

For each $\omega \in \{0,1\}$, the function $(1-F(s|\omega))^q (F(s|\omega))^{1-q}$ is strictly concave in s, and equals zero at $s = \bar{s}, \underline{s}$. As a result, there exists a unique solution to equation (29). Therefore, the sequence $\{s_n^*\}$ indeed converges, as the subsequential limit is unique.

Denote by F_ω^{-1} the inverse of $F(s|\omega)$. The function $(1-F(s|0))^q (F(s|0))^{1-q}$ is maximized at $F_0^{-1}(1-q)$, whereas the function $(1-F(s|1))^q F(s|1)^{1-q}$ is maximized at $F_1^{-1}(1-q)$. As $F_1^{-1}(\cdot) > F_0^{-1}(\cdot)$, the solution to equation (29) necessarily occurs in the interval $\left(F_0^{-1}(1-q), F_1^{-1}(1-q)\right)$. Q.E.D.

Discussion of conservative experts: Suppose $l \geq L$. Take a sequence $\{c_n\}$ such that $c_n > 0$ and $c_n \to 0$. Let $s_n^* \equiv s^*(c_n)$ be the experts' cutoff in an informative equilibrium with search cost c_n.

First, we explain that if $\pi \in \left(\max\left\{\frac{l}{l+\bar{s}}, \frac{L}{1+L}\right\}, \frac{l}{l+\underline{s}}\right)$, it is possible to pick a sequence of informative-equilibrium cutoff such that $s_n^* \to \underline{s}$. Let $\varepsilon \in \left(0, \frac{\bar{s}-\underline{s}}{2}\right)$. We modify the definition of Z in the proof of Proposition 2 as follows. Let $Z' : [\underline{s}, \underline{s}+\varepsilon] \rightrightarrows [\underline{s}, \underline{s}+\varepsilon]$ be a correspondence defined by

$$Z'(\hat{s}) \equiv \{\max\{\underline{s}, \min\{\underline{s}+\varepsilon, x(\hat{s}, b)\}\} : b \in \Psi(\hat{s})\}.$$

Replacing $\bar{s}$ with $\underline{s}+\varepsilon$ in the proof of Proposition 2 shows that Z' admits a fixed point. It suffices to show that the fixed point does not occur at $\underline{s}+\varepsilon$. To see this, first substituting $\hat{s} = \underline{s}+\varepsilon$ and $c = c_n$ into the agent's value function (13) and then taking limit, we get equation (20), to which $V(p) = p$ is the unique solution. Thus, as $c_n \to 0$, the agent would adopt a best response b with $p_1(\underline{s}+\varepsilon)$ arbitrarily close to 1. Using (10), an individual expert's best response satisfies $x(s_n^*, b) \leq \left(\frac{1}{p_1(\underline{s}+\varepsilon)} - 1\right)\frac{1-F(\underline{s}+\varepsilon|1)}{1-F(\underline{s}+\varepsilon|0)} l$, which is strictly less than $\underline{s}+\varepsilon$ for n sufficiently large. As a result, there exists an $N \in \mathbb{N}$ such that for all $n > N$, the correspondence Z', and hence Z, has a fixed point $s_n^* < \underline{s}+\varepsilon$. It is clear that this sequence $\{s_n^*\}_{n>N}$ does not converge to $\bar{s}$. By Lemma 4, it must converge to $\underline{s}$.

Next we show that if $s_n^* \to \bar{s}$, then $\limsup_{n\to\infty} p_1(s_n^*) \leq \frac{l}{1+l}$. Recall inequality (25) is obtained simply by substituting (9) into (10). The right-hand side of inequality (25) converges to $\frac{l}{l+1}$ as $s_n^* \to \bar{s}$, giving $\limsup_{n\to\infty} p_1(s_n^*) \leq \frac{l}{1+l}$. As a result, we can apply the argument in the proof of Proposition 3 to obtain upper bounds on payoffs. If $\pi < \frac{l}{1+l}$, then $\limsup_{n\to\infty} U(c_n) \leq \pi \frac{1}{\frac{l}{1+l}}\left(-L + (1+L)\frac{l}{1+l}\right) = \pi\left(1 - \frac{L}{l}\right)$. If $\pi \geq \frac{l}{1+l}$, $\limsup_{n\to\infty} U(c_n) \leq -L + \pi(1+L)$. For the experts' payoff, if $\pi < \frac{l}{1+l}$, then $\limsup_{n\to\infty} T(c_n) = 0$; if $\pi \geq \frac{l}{1+l}$, $\limsup_{n\to\infty} T(c_n) \leq -l + \pi(1+l)$.

Finally, we show Corollary 3. We have already shown that if $s_n^* \to \bar{s}$, then $\limsup_{n\to\infty} p_1(s_n^*) \leq \frac{l}{1+l} < 1$, so information is not perfectly aggregated in the limit. Therefore, information is aggregated only if $s_n^* \to \underline{s}$. Moreover, from the proofs of Lemma 5 and part (i) of Lemma 6, their results do not require $l < L$ as long as the sequence of informative equilibria has $s_n^* \to \underline{s}$. We have established above the existence of such a sequence for the case $\pi \in \left(\max\left\{\frac{l}{l+\bar{s}}, \frac{L}{1+L}\right\}, \frac{l}{l+\underline{s}}\right)$. Thus, if $\underline{s} = 0$ and $\pi > \max\left\{\frac{l}{l+\bar{s}}, \frac{L}{1+L}\right\}$, perfect information aggregation is a limit equilibrium outcome. Conversely, if $\underline{s} > 0$, then either there does not exist a sequence of informative equilibria, or $\limsup_{n\to\infty} p_1(s_n^*) < 1$ if such a sequence exists. In either case, information is not perfectly aggregated in the limit.

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