Competition in Information Disclosure

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May 9, 2017

Abstract

We analyze a model of competition in Bayesian persuasion in which two or more senders vie for the patronage of a receiver by disclosing information about their respective proposal qualities. The model's primitives that define the competitive environment are: the ex-ante correlation of senders' proposal qualities; the respective ex-ante expected proposal qualities; and the number of competing senders. Using the finding that each sender must face a payoff function with a linear structure in equilibria, we fully characterize the unique equilibrium. We then identify the effects of the primitives on senders' disclosure strategies. We find that an increase in the ex-ante correlation has two opposing effects on the incentives for information disclosure. Full disclosure arises in the limit as the correlation approaches its maximum value if and only if there exists an ex-ante difference in expected qualities. Furthermore, we show that as the number of senders increases, each sender discloses information more aggressively. Full disclosure by each sender arises in the limit of infinitely many senders.

Keywords Information Transmission, Bayesian Persuasion, Multiple Senders

JEL Codes D83

1 Introduction

Situations where self-interested persuaders attempt to influence the action of a decision-maker are ubiquitous. They often persuade by selectively disclosing information. An example occurs in the job market of

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[‡]We are grateful to Richard Holden, Anton Kolotilin, Hongyi Li, Peter Norman, Carlos Pimienta, Marek Pycia, Satoru Takahashi, Yosuke Yasuda, two anonymous referees, an associate editor, and the editor for useful discussions and valuable comments; and Gary Liang for excellent research assistance. We would also like to thank seminar participants at University of Technology Sydney, UNSW Sydney, Singapore Management University, Sun Yat-sen University, National University of Singapore, Econometric Society Asian Meeting, Chinese University of Hong Kong Shenzhen, Nanjing University, Korea University, Nanyang Technological University, Japan-Taiwan-Hong Kong Contract Theory Workshop, the Australian National University, Otaru University of Commerce. The second author greatly acknowledges the financial support from UNSW Sydney and Australian Research Council.

university graduates. To boost their placement records, professional schools and universities design their grading and disclosure policies to convince prospective employers that the qualities of their graduates are higher than those of competing institutions.[(1)]

In designing their grading policies, universities must take into account the average quality of students in each university, the similarity or correlation in students' abilities across universities,[(2)] and the number of universities competing in placing their students. Universities, especially highly-ranked universities, may inflate their grades, arguing that the higher proportion of good grades reflects the superior qualities of their students. By suppressing information using grade inflation (i.e. occasionally passing or giving good grades to bad students), a university can boost the career prospects of its bad students. However, the cost involved is a decline in the career prospects of its good students. Facing this behavior from higher-ranked universities, competing universities have an opportunity to improve their overall job placement record by improving the perception of their top students. One strategy to achieve this is to adopt a grading system with a finer scale (for example, a letter grade system from A^+ to F) which distinguishes the top students from the rest. Alternatively, they may mandate the use of a forced curve, or report an average grade in each class alongside the grade of an individual student. In anticipation of such a response, higher-ranked universities may also seek to differentiate their top students by adopting grading systems with finer scale.

Pharmaceutical companies face similar competition when submitting clinical trial plans for the authority's approval of their drugs that treat a common medical problem. Prior to the trials, no party has superior information about the efficacy and side-effects of the new drugs. Since each pharmaceutical company can advertise its drugs' efficacy based only on the results of clinical trials, positive results would help convince physicians to adopt its drug. In designing clinical trial plans, pharmaceutical companies would take into account the ex-ante expectation of the efficacy of the new drugs, the similarity of the mechanisms to tackle a medical problem among competing drugs, and the number of competing companies.

In the examples above, self-interested senders (universities/pharmaceutical companies) seek to influence a receiver's beliefs (students' ability/drugs' efficacy) and consequently her actions (hiring decision/prescription decisions). Also, at the time of choosing their disclosure policies, the senders are not superiorly informed about their actual qualities (their students' abilities/their drugs' efficacy). Moreover, unlike typical principal-agent models, standard tools, such as contracts or monetary incentive schemes, are not the primary tools used by the senders. Instead, they must convince the receiver by designing a dis-

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[(1)]Kolotilin (2016) analyzes the situation where there is only one university, but a potential employer can acquire information not only from the university but also from other sources. He uses a linear-programming approach to characterize the optimal disclosure mechanism, and derives the necessary and sufficient conditions for full and no information revelation.

[(2)]The abilities of students from different universities can be positively correlated because they were educated in a common primary and secondary education system.

closure policy (grading policies/clinical trials) that selectively reveals the relevant information (students' abilities/drugs' efficacy).

In this paper, we study the equilibria of this type of disclosure game by analyzing a model of competition in Bayesian persuasion. In our model, two or more senders vie for the patronage of a receiver by disclosing information about their respective proposals. Our aim is to conduct comparative statics with respect to primitives that define the competitive environment including the ex-ante correlation of senders' proposal qualities, the respective ex-ante expected proposal qualities, and the number of competing senders.

In our model, there is no ex-ante information asymmetry, and all players share a common prior belief concerning the joint distribution over the proposals' qualities. We assume that each sender can only directly control the disclosure of information regarding his own proposal's quality, but he has full flexibility in choosing his disclosure policy concerning his own proposals' qualities. The flexibility in their choice of disclosure policies in turn implies that we can reformulate the problem of each sender as choosing a distribution over marginal posterior beliefs that respects Bayes' rule (as pointed out by Kamenica and Gentzkow (2011)). Moreover, optimizing the distribution over posterior distributions is equivalent to finding the concave closure of the sender's payoff as a function of realized posteriors.

This reformulation leads us to the observation that a strategy profile is an equilibrium if and only if the induced payoff functions (of posterior distributions) exhibit a particular linear structure. An implication of this observation is that, in an equilibrium, a sender does not find it profitable to induce, with positive probability, a posterior which no other senders' messages imply. A related implication is that each sender must use a rich message space in equilibrium, for otherwise, competing senders would take advantage by inducing marginally better posteriors. Therefore, loosely speaking, each sender only induces messages that matter, and the set of messages is rich in an equilibrium.(3) The linear structure of payoff functions encapsulates these necessary equilibrium conditions. More importantly, it allows us to construct the unique equilibrium of the game.

In Section 2, we illustrate the intuition behind the results discussed above, using the simplest version of our model, in which two symmetric senders are endowed with proposals of independent and binary qualities. By utilizing the linear structure of equilibrium payoff functions, we then analyze the effect of (weakly positive) correlation in proposal qualities in the competition between two asymmetric senders (in Section 3); and the number of (symmetric) senders with independent proposal qualities (in Section 4).

At first glance, it appears that a stronger correlation in senders' proposal qualities results in more transparent disclosure. Our analysis in Section 3 shows that the effect of an increase in correlation de-

(3)This is consistent with a casual observation that whilst potential recruiters are known to use GPAs only as a threshold to manage recruitment, many universities adopt a similar and rich set of letter grades (e.g., 13 possible letter grades of A^+ to F).

pends on whether the senders are symmetric or not. This is because the information externality – the news disclosed by a sender contains information about the other sender's proposal – entails two opposing effects on the incentives for information disclosure. First, when the news delivered by a sender is good, it is more likely that the other sender's news is also good. Therefore, fixing the other sender's strategy, a stronger information externality implies a smaller marginal benefit of delivering good news. This effect, which we call *the good-signal curse*, softens competition. Second, we find that the information externality always favors the strong sender if the two senders differ in the ex-ante expected qualities of their proposals. Specifically, the receiver chooses the weak sender's proposal only if the weak sender's news is sufficiently better than that delivered by the strong sender. This unequal treatment induces the weak sender to disclose aggressively, and in turn, so does the strong sender. As a result, an increase in the positive correlation in the proposals' qualities would intensify competition, and lead to more equilibrium disclosure by both senders. We call this *the receiver-treatment effect.*

If the proposal qualities are ex-ante symmetric, then only the effect of the good-signal curse exists. Therefore, as the correlation in qualities increases, the senders disclose less information. On the other hand, if the senders' ex-ante expected qualities are different, then the effect of an increase in correlation on equilibrium disclosure is ambiguous in general. However, we can show that as correlation approaches its maximal value, the receiver-treatment effect dominates. In fact, competition becomes extremely intense in the limit, and both senders engage in full disclosure in the unique limit equilibrium.

In Section 4, we analyze the effect of the number of competing senders on equilibrium disclosure policies. For simplicity and transparency, we abstract away from any correlation among senders' qualities, as well as any asymmetry in expected qualities. We establish that there exists a unique symmetric equilibrium. Intuitively, a larger number of senders would intensify competition, and hence each sender finds that a more transparent disclosure policy is necessary to stand a chance in persuading the receiver. More specifically, fixing the other senders' strategy, an increase in the number of senders implies that a sender now faces a "more convex" payoff function in posteriors, which gives him an incentive to disclose more information. Consequently, in the symmetric equilibrium with more senders, each sender adopts a more informative disclosure policy. Moreover, we find that as the number of senders approaches infinity, each sender's strategy converges to full disclosure.

We then generalize the model to allow for an arbitrary number of possible quality realizations. Unlike the binary case, each posterior belief is multi-dimensional, so a sender's payoff function over posteriors is no longer homeomorphic to his payoff function over expected qualities induced by respective posteriors. Consequently, the linearity of equilibrium payoff function of posteriors does not translate into the linearity of equilibrium payoff function of expected qualities in a straightforward manner. Nevertheless, if we recast the problem to one of choosing a distribution over expected qualities (rather than choosing a distribution over posteriors), the linearity of equilibrium payoff function remains valid locally, with a possibility of upward kinks at interim qualities. We establish that this property of payoff functions over expected qualities, together with other related properties, are necessary and sufficient conditions for an equilibrium. We then use these properties to develop a simple algorithm that constructs the unique symmetric equilibrium. Finally, the algorithm allows us to show that the equilibrium strategy approaches full disclosure as the number of senders goes to infinity.

1.1 Related Literature

As discussed above, the technique developed by Kamenica and Gentzkow (2011) plays a key role in our analysis. Their article has stimulated an active literature on information disclosure game in which the sender(s) can commit to the disclosure mechanism. Below, we discuss a number of articles from the literature that study competition among senders.(4) Ostrovsky and Schwarz (2010) consider a model setup similar to ours. In their model, schools disclose information about the ability of their students, with the objective of maximizing their students' overall placement. The main result of Ostrovsky and Schwarz (2010) is that the equilibrium outcome of their game is independent of the distribution of students' abilities across the schools. An implication is that fixing the prize structure and the distribution of students' abilities, increasing the number of schools have no impact on the equilibrium disclosure. Whereas some of our results concern the characterization of the equilibrium disclosure policies, we are primarily interested in the effect of changes in the competitive environment on the equilibrium disclosure policies. In particular, we find that an increase in the number of senders leads to more aggressive disclosure by every sender.(5) Moreover, we investigate how equilibrium disclosure policies respond to changes in the correlation in students' abilities across schools.

Boleslavsky and Cotton (2016) analyze a game that is a special case of ours; specifically, their game has two senders with independent proposals and the underlying state space is binary.(6) Their equilibrium construction is based on the observation that a sender's incentive is similar to that of a complete-information all-pay auction. In contrast, our approach builds upon the linear structure of payoff functions. The versatility of our approach allows us to tackle more general settings with state-correlation and mul-

(4)Information transmission with multiple senders has been studied using frameworks different from Bayesian persuasion. For example, Milgrom and Roberts (1986) study a multi-senders persuasion game in which the receiver is unsophisticated. There is also a large literature that examines the conflict of interests among senders in the cheap-talk settings. Morgan and Krishna (2001) extend Crawford and Sobel (1982) to a setting with two senders and show that a full-revelation equilibrium exists if the senders have opposing bias. In addition, Battaglini (2002) shows that with two senders and a multidimensional state space, a full-revelation equilibrium generically exists. Kawai (2015) generalizes the finding of Morgan and Krishna (2001) to multi-dimensional state space.

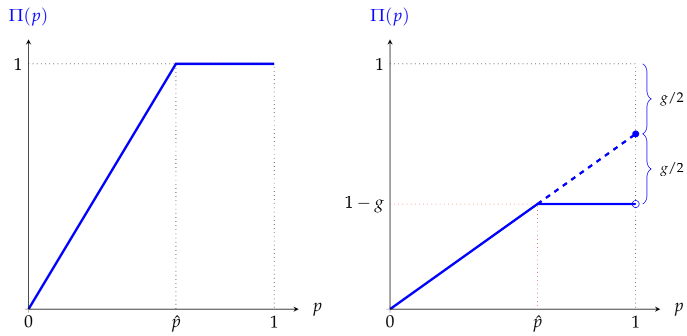
(5)Also, in Ostrovsky and Schwarz (2010), there exist some prize structures and distributions of students' abilities such that full disclosure is an equilibrium outcome. In contrast, full disclosure is never an equilibrium in our game

(6)We thank Raphael Boleslavsky for bringing this paper to our attention.

tiple senders. This in turn allows us to study the effect of changes in state-correlation and the number of senders on equilibrium disclosure policies. Furthermore, our analysis highlights an important difference between competitive Bayesian persuasion games and all-pay auctions. Whereas a bid is one-dimensional, a posterior distribution is a multi-dimensional object if there are more than two states. With many states, the equilibrium strategies in our competitive Bayesian persuasion game display a linear structure very different from those of all-pay auctions. In particular, the equilibrium distributions of expected qualities in our game typically feature upward kinks at intermediate utilities, whereas no analogous features exist in all-pay auctions.

Hoffmann et al. (2014) also study competitive disclosure using the framework of a persuasion game with information acquisition by persuaders. The key features that distinguish their game from Bayesian persuasion settings is that senders are privately informed about the state (with positive probability) and the set of feasible disclosure policies is constrained (to be effectively binary). They focus their analysis on independent proposals and find that senders adopt the most informative disclosure policy when the number of competing senders is sufficiently large. Our model, in contrast, puts no restrictions on feasible disclosure mechanisms, and this flexibility in the choice of disclosure mechanisms allows us to show that equilibrium disclosure gets strictly more informative as the number of senders increases. Furthermore, we consider correlated qualities in our analysis of two-senders competition.

Except the aforementioned papers, most studies on competitive Bayesian persuasion assume that the senders share a common state of the world, and each one can independently disclose information on the common state to a single receiver. Allowing each sender to adopt a mechanism that is arbitrarily correlated with each other, Gentzkow and Kamenica (2016) provide a simple equilibrium characterization. Furthermore, Gentzkow and Kamenica (2017) identify a necessary and sufficient condition on the set of feasible disclosure mechanisms under which the equilibrium outcome is more informative with an additional sender (regardless of preferences). The game we analyze does not satisfy their condition, so their result is not applicable in this scenario. Li and Norman (2017a) provide an example that if only (conditionally) independent mechanisms are feasible for each sender, the equilibrium outcome can be less informative with an additional sender. Board and Lu (2016) consider a search environment in which a buyer (receiver) sequentially learns from senders of a homogeneous product about its attributes. Restricting to (conditionally) independent mechanisms, they show that if the buyer's search history is private, full disclosure is the unique equilibrium outcome as the search cost vanishes. We obtain a somewhat similar result, but in the context of simultaneous disclosure about differentiated products. Au (2015) analyzes a dynamic disclosure setting with a single sender. In the absence of commitment power, the sender faces competition with his future selves. Our paper differs from these articles in that we assume that each sender can only control the disclosure of information about his proposal and that it is infeasible for him



(a) Form 1 ($\pi \le 1/2$)

(b) Form 2 ($\pi > 1/2$)

Figure 1: Shape of Equilibrium Payoff

to directly reveal information about other senders' proposal.

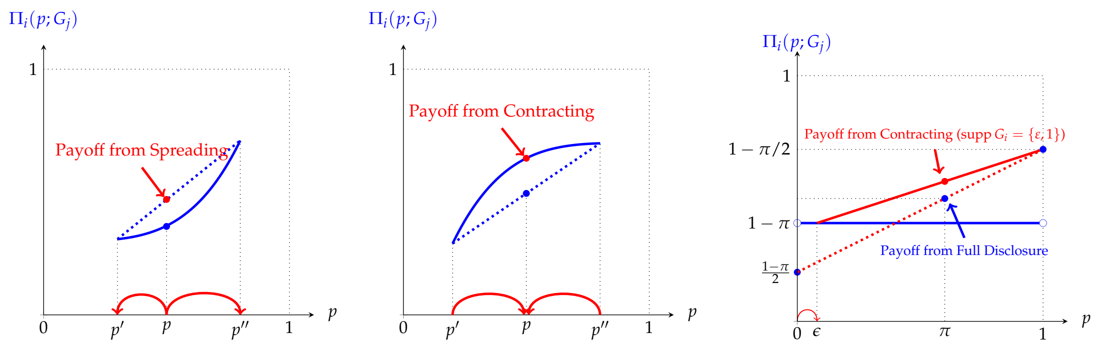
2 Linear Structure of Equilibrium Payoff

As the linear structure of equilibrium payoff function plays a crucial role in subsequent analysis, we illustrate its intuition in the simplest version of our model in this section. There are two (male) risk-neutral senders, each of whom is endowed with a proposal. They engage in competition for the endorsement of a single (female) receiver. The quality of proposal by sender $i = 1, 2$ is denoted by U_i, which takes a value of either u_0 or $u_1 > u_0$. For each i, U_i is independently and identically distributed with a commonly known prior distribution, characterized by $\pi \equiv \Pr(U_i = u_1)$.

Denote a generic element of $\Delta(\{u_0, u_1\})$ by $p_i \in [0,1]$, standing for the probability that $U_i = u_1$. Sender i's strategy space is the set of Bayes-plausible distributions over posteriors, i.e., the set of distribution functions $G_i : [0,1] \to [0,1]$ such that $E_{G_i}[p] = \pi$. The receiver is an expected-utility-maximizer. Therefore, given a pair of realized posteriors (p_1, p_2), she chooses sender i with probability one if $p_i > p_j$; and with probability $1/2$ if $p_i = p_j$. A sender's objective is to maximize the probability that the receiver chooses his proposal. Thus, when sender i's strategy is G_i, sender j's expected payoff of inducing posterior p is

$$\Pi(p; G_i) = \Pr[p_i < p] + \frac{1}{2}\Pr[p_i = p] = \frac{1}{2}\left(G_i(p) + \lim_{p' \to p^-} G_i(p')\right). \tag{1}$$

Below, we show that a Bayes-plausible distribution G is a symmetric equilibrium strategy if and only if it induces a payoff function $\Pi(p; G)$ that has the linear structure, as depicted in Figure 1. Formally, we say G induces the payoff function $\Pi(p; G)$ with a linear structure if there exist a $\hat{p} \in (0,1]$ and a linear function $\overline{\Pi}(p)$ such that (i) $\overline{\Pi}(p) \ge \Pi(p; G)$ for all $p \in [0,1]$; and (ii) $\operatorname{supp} G = P \in \{[0,\hat{p}], [0,\hat{p}] \cup \{1\}\}$,



(a) convexity/ upward-kink (b) concavity/ non-upward-kink (c) Π_i from Full Disclosure

Figure 2: Incentives for Spreading and Contracting

where $P \equiv \text{cl}\left(\left\{p : \Pi\left(p;G\right) = \overline{\Pi}\left(p\right)\right\}\right)$.[(7)]

We first explain below why a strategy G constitutes a symmetric equilibrium if $\Pi\left(p;G\right)$ has the linear structure. Given a strategy G_i, sender i can disclose more information than G_i by a mean-preserving spread of G_i. Conversely, sender i can disclose less information than G_i by a mean-preserving contraction of G_i. In an equilibrium, no sender strictly benefits from engaging in such spreading (more information disclosure) or contraction (less information disclosure).

More information disclosure through spreading increases a sender's payoff if his payoff function is "locally convex" or has an "upward-kink". In Figure 2-(a), given sender j's strategy G_j, $\Pi\left(p;G_j\right) < \alpha\Pi\left(p';G_j\right) + \left(1-\alpha\right)\Pi\left(p'';G_j\right)$ for all $p = \alpha p' + \left(1-\alpha\right)p'' \in \left(p', p''\right)$. In this case, sender i has incentives to spread the weight in the neighborhood of p to those of p' and p'', so p is not in the support of sender i's best response. Therefore, such a G_j cannot be a symmetric equilibrium strategy. In contrast, less information disclosure through contraction increases a sender's payoff if his payoff function is "locally concave" or has an "non-upward-kink". In Figure 2-(b), given some G_j, $\Pi\left(p;G_j\right)$ is increasing at both p' and p'', and that $\Pi\left(p;G_j\right) > \alpha\Pi\left(p',G_j\right) + \left(1-\alpha\right)\Pi\left(p'',G_j\right)$ for all $p = \alpha p' + \left(1-\alpha\right)p'' \in \left(p', p''\right)$. In this case, sender i would not find it optimal to use a strategy that assigns positive measures on the neighborhoods of both p' and p''. Using another strategy that contracts these measures to p in a mean-preserving manner would strictly increase sender i's payoff. Therefore, it cannot be the case that sender i's best response puts positive weights on both the neighborhoods of p' and p''. Consequently, such a G_j cannot be a symmetric equilibrium strategy. When the payoff function facing a sender is linear, he cannot strictly benefit from any form of spreading or contraction. As a result, using the same strategy as the other sender is one of the best responses.

[(7)]Here, $\text{cl}\left(A\right)$ is the closure of set A. The support of distribution G, supp G, is defined as $\text{cl}\left(\left\{p \in \triangle\Omega : G\left(p\right) > 0\right\}\right)$.

Next, we explain why it is necessary that $\Pi(p; G)$ has the linear structure in an equilibrium. This is less straightforward than the sufficiency of the linear structure for equilibrium. For example, one may wonder why it is not an equilibrium for both senders ro engage in full information disclosure, i.e., equilibrium in which senders use strategy $G(p) = 1 - \pi$ for $p \in [0, 1)$ and $G(1) = 1$. In this case, sender 1's payoff function has a jump at $p = 0$. Therefore, sender 1 can profitably deviate by replacing the atom at 0 with another atom at a small but positive posterior, which ensures that he always wins whenever sender 2's realized posterior is 0. This is illustrated graphically in Figure 2-(c). Suppose sender 2 engages in full disclosure. If sender 1 also engages in the full disclosure, then his payoff would be given by the blue point on the red dotted line in Figure 2-(c). Now fix a small $\varepsilon > 0$, and consider a strategy of sender 1 that has a support of $\{\varepsilon, 1\}$, with the respective probabilities of realization $\frac{1-\pi}{1-\varepsilon}$ and $\frac{\pi-\varepsilon}{1-\varepsilon}$. The expected payoff of this strategy is represented by the red point on the solid red line in Figure 2-(c), which lies above the blue point. The same logic implies that if sender i assigns an atom at some $p \in (0, 1)$, sender j would have a strictly profitable deviation by replacing the atom at p with one at $p + \varepsilon$. As a result, in every symmetric equilibrium, no sender assigns an atom at any $p \in [0, 1)$, i.e., G has to be continuous on $[0, 1)$, and $\Pi(p, G) = G(p)$ for all $p \in [0, 1)$. However, an atom at $p = 1$ is possible in equilibrium, as it is the maximum feasible posterior, and the profitable deviation suggested above is not available.[8]

Moreover, an equilibrium payoff function can be flat only at the top. To see this, suppose that G_j is constant on interval (a, b) and is increasing in interval (b, c), then $\Pi\left(p; G_j\right)$ would have an upward-kink at b. As argued above, sender i would find it strictly suboptimal to put a positive weight in the neighborhood of b, and G_j cannot be a symmetric equilibrium strategy. Thus, a necessary condition for G being a symmetric equilibrium strategy is that G is increasing and continuous in interval $[0, \hat{p}]$ for some $\hat{p} < 1$.

Furthermore, if the equilibrium strategy has an atom at $p = 1$, then it necessarily has a gap below $p = 1$, i.e., there is some $\hat{p} < 1$ such that $G(p) = G(\hat{p})$ for all $p \in [\hat{p}, 1)$ (see Figure 1-(b)). The reason is that the jump of $\Pi(p; G)$ at $p = 1$ makes it strictly suboptimal for a sender to induce posteriors slightly below 1. If G assigned a positive weight to these posteriors, a sender could gain by spreading this weight to $p = 1$ and some lower posteriors. Intuitively, an atom at the top (and a gap below it) arises in equilibrium if the prior π is high enough that a uniform distribution over the set of feasible posteriors does not satisfy the Bayes-plausibility condition.

Finally, if G is a symmetric equilibrium strategy, then it is necessary that it induces a payoff function $\Pi(p; G)$ such that for all posterior p in the support of G, $(p, \Pi(p; G))$ lies on the linear line that connects $(0, \Pi(0; G))$ and $(\hat{p}, \Pi(\hat{p}; G))$ on the graph of $\Pi(p; G)$. Were this not the case, there would exist a p in the support of G such that $\Pi(p; G) < \overline{\Pi}(p)$, or $\Pi(p; G) > \overline{\Pi}(p)$. Recall the discussion about Figure 2-(a) and

[8] As we show in Section 3, an atom at the bottom, i.e., $p = 0$, is possible if the two senders are ex-ante asymmetric.

(b). In the former case, a sender facing $\Pi(p; G)$ can increase his payoff by disclosing more information by spreading posterior p (to posterior 0 and $\bar{p}$). In the latter case, a sender facing $\Pi(p; G)$ can increase his payoff by disclosing less information by contracting posteriors in the neighborhoods of 0 and $\bar{p}$ into p. The observations above therefore imply that $\Pi(p; G)$ must have the linear structure, i.e., it takes the form of either Figure 1-(a) or Figure 1-(b).

It is straightforward to verify that the linear structure of the equilibrium payoff function, along with the Bayes-plausibility condition, give a system of equations in $\hat{p}$ and $G(\hat{p})$ with a unique solution. Therefore, the unique symmetric equilibrium can be identified. Moreover, as the game is zero-sum, the interchangeability property of zero-sum games[(9)] then immediately implies that there is no asymmetric equilibrium. Consequently, the symmetric equilibrium is the unique equilibrium.

In the following sections, we show that the observation that the equilibrium payoff function must have the linear structure can be generalized to environments in which the two senders are asymmetric and their proposals' qualities are correlated; as well as environments in which there are more than two senders and more than three possible proposal qualities. In these settings, the linear structure of payoff functions allows us to fully characterize the equilibrium, and establish a certain form of equilibrium uniqueness.

3 Correlated States and Asymmetric Senders

3.1 Model

In this section, we study the disclosure game played between two (possibly) asymmetric senders, whose proposal's qualities U_1 and U_2 are (possibly) positively correlated. Specifically, given a pair of prior expected qualities $E[U_1] = \pi_1$ and $E[U_2] = \pi_2$, and the covariance between the qualities of two proposals $\rho \equiv cov(U_1, U_2)$, the joint distribution of U_1 and U_2 is tabulated below.

	$U_2 = u_0$	$U_2 = u_1$
$U_1 = u_0$	$(1-\pi_1)(1-\pi_2)+\rho$	$\pi_2(1-\pi_1)-\rho$
$U_1 = u_1$	$\pi_1(1-\pi_2)-\rho$	$\pi_1\pi_2+\rho$

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It is without loss of generality to assume $\pi_1 \geq \pi_2 > 0$. Under this assumption, sender 1 is "weakly stronger" than sender 2. Moreover, we assume that $\rho \in [0, \bar{\rho})$, where $\bar{\rho} \equiv \pi_2(1-\pi_1)$ which ensures that the joint distribution of U_1 and U_2 has a full support.

Each sender i simultaneously chooses an information disclosure mechanism on U_i, which consists of a signal space M_i and a conditional distribution $\Phi_i : \{u_0, u_1\} \times M_i \to [0,1]$. The choices of disclosure

[(9)] The interchangibility property of zero-sum games is as follows. If strategy profiles (σ_1, σ_2) and (τ_1, τ_2) are two distinct Nash equilibria, then so are (σ_1, τ_2) and (τ_1, σ_2).

mechanisms are known to the receiver before she makes her decision. After observing the disclosure mechanisms and realized signals of both senders, the receiver decides which sender's proposal to adopt.

The information disclosure mechanism on U_i induces a distribution of marginal distributions over U_i. A realized marginal distribution over U_i is one-dimensional, and a generic distribution is denoted by $p_i = \Pr(U_i = u_1 | p_i)$. We will refer this as sender i's realized signal, or sender i's signal for short. As the receiver is an expected-utility maximizer, for a realized pair of signals (p_1, p_2), she chooses sender i with probability one if $\Pr(U_i = u_1 | p_i, p_j) > \Pr(U_j = u_1 | p_i, p_j)$. In the case of a tie, i.e., $\Pr(U_i = u_1 | p_i, p_j) = \Pr(U_j = u_1 | p_i, p_j)$, we assume that she randomizes equally between the two senders. A sender's objective is to maximize the probability that the receiver chooses his proposal.

As pointed out by Kamenica and Gentzkow (2011), there is a one-to-one correspondence between the set of feasible disclosure mechanisms and Bayes-plausible (marginal) distributions of posterior beliefs over $\{u_0, u_1\}$. It is therefore without loss of generality to focus on the game of information disclosure played among the senders, in which the set of pure strategies of sender i consists of all Bayes-plausible (marginal) distributions over signals.[(10)] Furthermore, for each mixed strategy, there exists a pure strategy that preserves the expected payoffs of all players. Therefore, without loss of generality, we restrict our attention to pure-strategy Nash equilibria of the game described above.

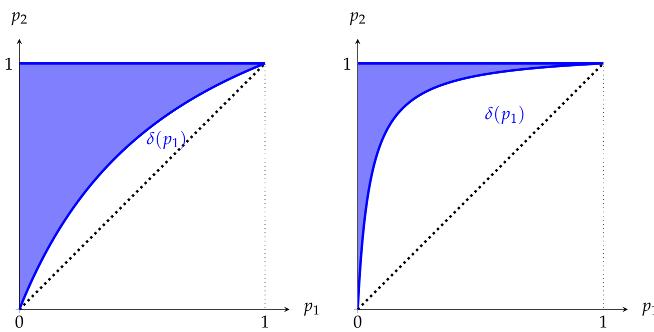
3.2 Equilibrium

In this subsection, we show that an equilibrium in the disclosure game with quality correlation exhibits a linear structure similar to that described in Section 2, with a few notable modifications. We first illustrate a key implication of the correlation between senders' qualities. Due to the correlation in qualities, sender i's signal p_i is informative not only of its own quality U_i, but also of the other sender's quality U_j, i.e., $\Pr(U_i = u_1 | p_i) \neq \Pr(U_i = u_1 | p_i, p_j)$. Whenever $\pi_1 > \pi_2$, this information externality effect always favors the strong sender, i.e., sender 1. To gain some intuition on why this is true, suppose the disclosure mechanism of the weak sender, sender 2, generates a good signal $p_2 > \pi_2$, whereas that of sender 1 generates a neutral signal $p_1 = \pi_1$. Then sender 1 benefits from sender 2's good signal because

$$\Pr(U_1 = u_1 | p_1 = \pi_1, p_2) = \pi_1 + \frac{\rho}{\pi_2 (1 - \pi_2)} (p_2 - \pi_2) > \pi_1. \tag{2}$$

As $\Pr(U_2 = u_1 | p_1 = \pi_1, p_2) = p_2$, the receiver chooses sender 2's proposal only if his signal p_2 exceeds π_1 by a sufficiently large margin, i.e., $p_2 \geq \pi_1 + \frac{\rho}{\pi_2(1-\pi_2)} (p_2 - \pi_2)$. In contrast, sender 1's good news does not benefit sender 2. To see this, suppose sender 1 has a good signal $p_1 > \pi_1$, whereas sender 2 has

(10) See Proposition 1 of Kamenica and Gentzkow (2011).



(a) small ρ (b) large ρ

Figure 3: Lemma 1

a neutral signal $p_2 = \pi_2$. Then

$$\begin{aligned}\Pr(U_2 = u_1 | p_1, p_2 = \pi_2) &= \pi_2 + \frac{\rho}{\pi_1 (1 - \pi_1)} (p_1 - \pi_1) \\ &< p_1 = \Pr(U_1 = u_1 | p_1, p_2 = \pi_2),\end{aligned}$$

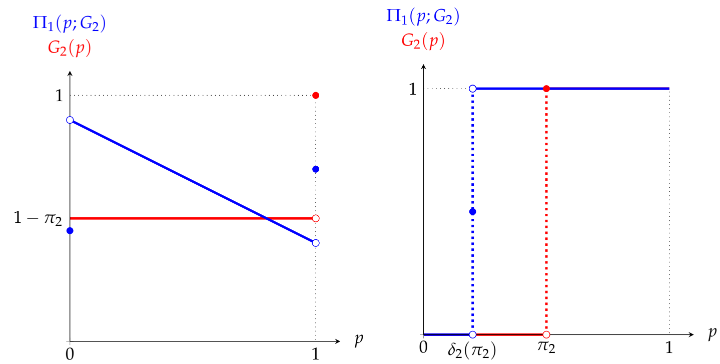
so sender 1 is chosen with probability one.[11] Although a good signal of sender 1 raises the posterior of sender 2's proposal, the increase is never enough to make the receiver adopt sender 2's proposal.

The examples above illustrate that the information externality effect works in the favor of the strong sender. It is straightforward to show, by a direct application of Bayes' rule, that for all $p \in (0, 1)$, $\Pr(U_1 = u_1 | p_1 = p_2 = p) > \Pr(U_2 = u_1 | p_1 = p_2 = p)$. Therefore, the receiver chooses the strong sender's proposal with certainty even if his signal is slightly worse than that of the weak sender. This unequal treatment of senders is illustrated in Figure 3. The shaded area in Figure 3 represents the set of signal pairs following which the receiver chooses sender 2. It can be seen that the area lies strictly above the 45-degree line.

If $\rho = 0$, i.e., the proposals' qualities are independent, then the aforementioned unequal treatment disappears and sender 1 is chosen with certainty if and only of $p_1 > p_2$. As ρ increases, the favorable treatment for sender 1 becomes more significant, as illustrated in Figure 3. The region of signal pairs following which sender 2 is chosen shrinks as ρ gets larger. The observations above are formally stated in the lemma below.

Lemma 1 *Suppose (p_1, p_2) is a pair of realized signals. There exists an increasing concave function $\delta : [0, 1] \to [0, 1]$ with $\delta(0) = 0$ and $\delta(1) = 1$ such that $\Pr(U_1 = u_1 | p_1, p_2) \geq \Pr(U_2 = u_1 | p_1, p_2)$ if and only if $p_2 \leq \delta(p_1)$. Furthermore, the function $\delta(p)$ has the following properties: (i) $\delta(p)$ is increasing in ρ, (ii) if $\rho = 0$ or*

[11]The inequality above holds because $\pi_1 > \pi_2$ and $\rho < \bar{\rho} = \pi_2 (1 - \pi_1)$.



(a) G_2 : Full Disclosure

(b) G_2: No Disclosure

Figure 4: Sender 1's Payoff

$\pi_1 = \pi_2$, *then* $\delta(p) = p$, *and (iii)* $\lim_{\rho\to\bar{\rho}} \delta(p) = 1$ *for all* $p \in (0,1]$.

Using Lemma 1, we can express the senders' payoff functions $\Pi_i(p_i; G_j) = \Pr(p_j < \delta_i(p_i) | G_j) + \frac{1}{2}\Pr(p_j = \delta_i(p_i) | G_j)$ as follows.

$$\Pi_i(p_i; G_j) = \left(1 + \frac{\rho(p_i - \pi_i)(\delta_i(p_i) - \pi_j)}{\pi_i(1-\pi_i)\pi_j(1-\pi_j)}\right) \frac{G_j(\delta_i(p_i)) + \lim_{p\to p_i^-} G_j(\delta_i(p))}{2} - \frac{\rho(p_i - \pi_i)}{\pi_i(1-\pi_i)\pi_j(1-\pi_j)} \int_0^{\delta_i(p_i)} G_j(s)\, ds,$$

where $\delta_1(p) \equiv \delta(p)$ and $\delta_2(p) \equiv \delta^{-1}(p)$.(12)

Figure 4-(a) and (b) illustrate sender 1's payoff function if sender 2 engages in full disclosure and no disclosure respectively, given $\pi_1 > \pi_2$ and $\rho > 0$.(13) Note that in Figure 4-(a), sender 1's payoff function is downward-sloping in the interval $(0,1)$, over which sender 2 puts zero probability. The reason is that a

(12)To see how payoff function is derived, suppose G_j does not assign an atom at posterior $\delta_i(p)$. Then

$$\Pi_i(p, G_j) = G_j(\delta_i(p) | U_j = u_1) \Pr(U_j = u_1 | p_i = p) + G_j(\delta_i(p) | U_j = u_0) \Pr(U_j = u_0 | p_i = p). \tag{3}$$

Next observe that by definition, $G_j(\cdot) = \pi_j G_j(\cdot | U_j = u_1) + (1-\pi_j) G_j(\cdot | U_j = u_0)$ and $p_j = \Pr(U_j = u_1 | p_j) = \left(1 + \frac{1-\pi_j}{\pi_j} \frac{g_j(p_j | U_j = u_0)}{g_j(p_j | U_j = u_1)}\right)^{-1}$, where $g_j(\cdot | U_j)$ is the density function of of $G_j(\cdot | U_j)$. Using these definitions, we get

$$G_j(p | U_j = u_1) = \frac{pG_j(p) - \int_0^p G_j(s)\, ds}{\pi_j}, \text{ and } G_j(p | U_j = u_0) = \frac{(1-p)G_j(p) + \int_0^p G_j(s)\, ds}{1-\pi_j}. \tag{4}$$

Moreover, $\Pr(U_j | p_i = p)$ can be computed using a direct application of Bayes' rule. Substituting (4) into (3) gives the payoff function with $G_j(\delta_i(p_i)) = \lim_{p\to p_i^-} G_j(\delta_i(p))$ (i.e., G_j assigns no atom at $\delta_i(p_i)$). For the case that G_j assigns an atom at $\delta_i(p_i)$, the probability $\Pr(p_j = \delta_i(p_i) | G_j)$ can be computed in a similar way as above.

(13)That is, $G_2(p) = 1 - \pi_2$ for all $p \in [0,1)$ in Figure 4-(a); and $G_2(p) = 0$ for $p \in [0, \pi_2)$, and $G_2(p) = 1$ for all $p \in [\pi_2, 1]$ in Figure 4-(b).

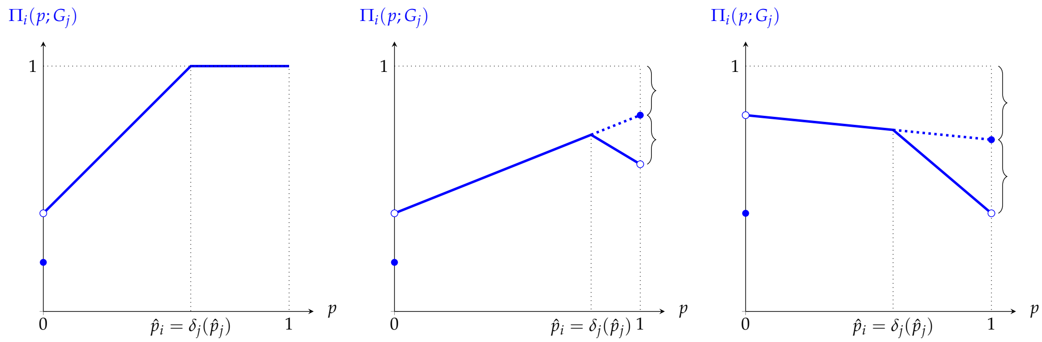


Figure 5: Linear Structure of Payoff Functions

higher value of p_1 implies a higher likelihood that $U_2 = u_1$, and consequently $p_2 = 1$. For $p_1 \in (0,1)$, this means sender 1 has a lower chance of winning.

According to Lemma 1, $\delta_1(p) = \delta_2(p) = p$ if U_1 and U_2 are independently distributed or if their marginal distributions are identical. We have shown in Section 2 that if $\rho = 0$ and $\pi_1 = \pi_2$, the necessary and sufficient condition for the unique symmetric equilibrium is the linearity of senders' payoff function. Also, the heuristic argument in the previous section suggests that the senders cannot both assign an atom at posterior 0, as each sender would have a profitable deviation to replace the atom at posterior 0 with another atom at a small and positive posterior (in a mean-preserving manner).

The finding that the linearity of payoff functions is necessary and sufficient for an equilibrium can be extended to the current environment, with a few modifications. First, due to the senders' asymmetry, sender i's payoff function is different from that of sender j. Second, with a positive correlation, a payoff function is not necessarily weakly increasing in the signal a sender induces, as shown in Figure 4-(a). Also, following an argument similar to that in Section 2, the support of sender i's equilibrium strategy takes the form of either $[0, \hat{p}_i]$, or $[0, \hat{p}_i] \cup \{1\}$. Correspondingly, sender j's equilibrium strategy must have a support $[0, \delta_i(\hat{p}_i)] = [0, \hat{p}_j]$ in the former case, and $[0, \delta_i(\hat{p}_i)] \cup \{1\} = [0, \hat{p}_j] \cup \{1\}$ in the latter case. Figure 5 illustrates the possible forms that $\Pi_i(p; G_j)$ can take in an equilibrium.

Formally, we say a pair of payoff functions $(\Pi_1(p_1; G_2), \Pi_2(p_2; G_1))$ induced by a pair of strategies (G_1, G_2) has the linear structure if there exist a pair $(\hat{p}_1, \hat{p}_2) \in (0,1]^2$ and a pair of linear functions $(\overline{\Pi}_1, \overline{\Pi}_2)$ such that for $i = 1,2$ and $j \neq i$, (i) $\overline{\Pi}_i(p_i) \geq \Pi_i(p_i; G_j)$ for all $p_i \in [0,1]$; (ii) $\operatorname{supp} G_i = P_i \in \{[0, \hat{p}_i], [0, \hat{p}_i] \cup \{1\}\}$, where $P_i \equiv \operatorname{cl}(\{p : \Pi_i(p; G_j) = \overline{\Pi}_i(p)\})$ and $P_j = \{\delta_i(p_i) : p_i \in P_i\}$; and (iii) $G_1(0) \times G_2(0) = 0$. The following theorem states that this linear structure of payoff functions fully characterizes the unique equilibrium of the game.

Theorem 1 *A pair of Bayes-plausible strategies (G_1, G_2) is an equilibrium if and only if it induces a pair of payoff* *functions with the linear structure.*

Using Theorem 1, the search of an equilibrium boils down to solving a system of equations with unknowns $\hat{p}_i$, $G_i(0)$, $G_i(\hat{p}_i)$, for $i = 1, 2$. To see this, observe that the linear structure of payoff functions implies $d^2\Pi_i\left(p_i; G_j\right)/dp_i^2 = 0$ for $p_i \in (0, \hat{p}_i)$, which gives us a differential equation with respect to G_j. Upon solving, we can represent G_j as a function of $G_j(0)$ and $G_j\left(\hat{p}_j\right)$. Consequently, the Bayes-plausibility condition for sender j can also be written as a function of $G_j(0)$ and $G_j\left(\hat{p}_j\right)$. Furthermore, if $G_i(\hat{p}_i) < 1$, i.e., $1 \in \operatorname{supp} G_i$, then $G_j\left(\hat{p}_j\right) < 1$ and $\Pi_i\left(1; G_j\right) = \overline{\Pi}_i(1)$. The linear structure of payoff functions thus implies $\frac{\Pi_i\left(1;G_j\right)-\Pi_i\left(\hat{p}_i;G_j\right)}{1-\hat{p}_i} = \frac{\Pi_i\left(\hat{p}_i;G_j\right)-\lim_{p\to0^+}\Pi_i\left(p;G_j\right)}{\hat{p}_i}$ (see the second and the third panels of Figure 5). The lemma below formally states the equations we need to solve.

Lemma 2 *If (G_1, G_2) is an equilibrium, then G_j takes the following form.*

$$G_j(p) = \begin{cases} G_j(0) + \left(G_j\left(\hat{p}_j\right) - G_j(0)\right) \dfrac{\int_0^{\delta_j(p)} \exp\left(\int_0^{s'} \Lambda_j(s)ds\right)ds'}{\int_0^{\delta_j\left(\hat{p}_j\right)} \exp\left(\int_0^{s'} \Lambda_j(s)ds\right)ds'} & \text{if } p \in (0, \hat{p}_j] \\ G_j\left(\hat{p}_j\right) & \text{if } p \in \left(\hat{p}_j, 1\right) \end{cases}, \tag{5}$$

where $\Lambda_j(s) \equiv -\frac{\rho\left((s-\pi_i)\delta_i'(s)+2\left(\delta_i(s)-\pi_j\right)\right)}{\pi_1\pi_2(1-\pi_1)(1-\pi_2)+\rho(s-\pi_i)\left(\delta_i(s)-\pi_j\right)}$*, for some* $\hat{p}_i \in (0, 1)$, $G_i(0) \in [0, 1-\pi_i)$*, and* $G_i(\hat{p}_i) \in (1-\pi_i, 1]$ *that satisfy the following conditions.*

1. *Simplified Bayes-plausibility:* $1 - \pi_j = G_j\left(\hat{p}_j\right) - \left(G_j\left(\hat{p}_j\right) - G_j(0)\right) T_j(\hat{p}_i)$*, where* $T_j(\hat{p}_i) \equiv \frac{\int_0^{\hat{p}_i} \delta_i(x)\exp\left(\int_0^x \Lambda_j(s)ds\right)dx}{\int_0^{\hat{p}_i}\exp\left(\int_0^x \Lambda_j(s)ds\right)dx}$.

2. *Atom condition at the top: if* $G_i(\hat{p}_i) < 1$*, then* $G_j\left(\hat{p}_j\right) < 1$ *and* $\frac{\Pi_i\left(1;G_j\right)-\Pi_i\left(\hat{p}_i;G_j\right)}{1-\hat{p}_i} = \frac{\Pi_i\left(\hat{p}_i;G_j\right)-\lim_{p\to0^+}\Pi_i\left(p;G_j\right)}{\hat{p}_i}$*, for* $i, j = 1, 2$.[14]

3. *Common support:* $\hat{p}_j = \delta_i(\hat{p}_i)$.

4. *Atom condition at the bottom:* $G_1(0) \times G_2(0) = 0$.

Lemma 2 gives a system of 6 equations in 6 unknowns ($\hat{p}_i$, $G_i(0)$, $G_i(\hat{p}_i)$, for $i = 1, 2$). The following theorem establishes equilibrium existence and uniqueness, thus showing that there exists a unique solution to the system of equations.

Theorem 2 *An equilibrium exists and is unique.*

[14] The values of sender i's profit function at $p = 1, \hat{p}_i$ and 0 are given respectively by $\Pi_i\left(1; G_j\right) = 1 - \frac{1}{2}\left(1 - G_j\left(\hat{p}_j\right)\right)\frac{\pi_1\pi_2+\rho}{\pi_1\pi_2}$; $\Pi_i\left(\hat{p}_i; G_j\right) = 1 - \left(1 - G_j\left(\hat{p}_j\right)\right)\left(1 + \frac{\rho(\hat{p}_i-\pi_i)}{\pi_1\pi_2(1-\pi_i)}\right)$; and $\lim_{p\to0^+}\Pi_i\left(p; G_j\right) = G_j(0)\frac{(1-\pi_1)(1-\pi_2)+\rho}{(1-\pi_1)(1-\pi_2)}$.

3.3 Effects of ρ on Equilibrium Information Disclosure

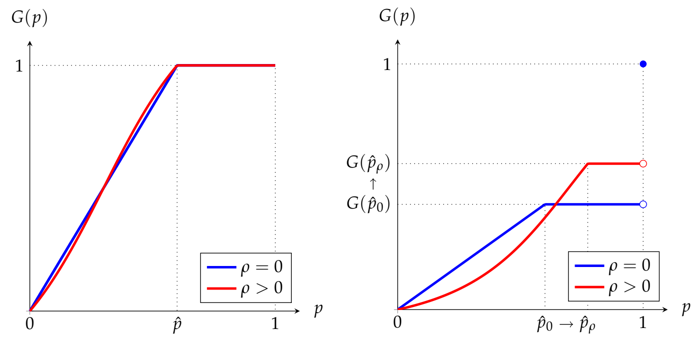
In this subsection, we analyze the effect of the covariance parameter ρ on equilibrium disclosure. We are particularly interested in the limiting disclosure behavior as ρ is taken to its maximum value $\bar{\rho} = \pi_2 (1 - \pi_1)$. An increase in ρ has two opposing effects for the equilibrium information disclosure, both of which arise from the aforementioned information externality effect. The first effect is due to a decrease in the marginal benefit of inducing high signals, which disincentiveizes senders to engage in aggressive disclosure. We call this effect the good-signal curse. The second effect arises from the receiver's asymmetric treatment of senders that exists only when $\pi_1 > \pi_2$, which creates the incentives to engage in aggressive disclosure. We call this the receiver-treatment effect.

We begin with the good-signal curse. Fixing the other senders's strategy, if ρ increases, then a favorable signal by a sender implies a higher likelihood that the other sender would also be able to induce a favorable signal. Consequently, the marginal benefit of inducing a higher signal goes down for each sender. This effect explains why a sender's payoff can become decreasing in his own signal when ρ is large enough. For example, suppose the sender's rival engages in full disclosure. Then the sender's payoff function is flat in the interval $(0, 1)$ if $\rho = 0$. However, if $\rho > 0$, then the sender's payoff is decreasing in the interval $(0, 1)$, as depicted in Figure 4-(a).

To illustrate the good-signal curse most clearly, we consider the case of symmetric senders, i.e., $\pi_1 = \pi_2$, in which the receiver-treatment effect is absent. Formally, we say sender i's strategy G discloses more information than strategy G' if G and G' satisfy the following relation. There exist a $p_L \in (0, 1)$ and a $p_H \in (p_L, 1]$ such that $G(p) > G'(p)$ for all $p \in (0, p_L)$; $G(p) = G'(p)$ for all $p \in \{p_L\} \cup [p_H, 1]$; and $G(p) < G'(p)$ for all $p \in (p_L, p_H)$. We denote this relation by $G \succ G'$. Loosely speaking, G is a "clockwise rotation" of G' in the sense strategy G induces bad signals ($p < p_L$) and good signals ($p > p_L$) with a higher likelihood than strategy G'. Therefore, $G \succ G'$ implies that G is second-order stochastically dominated by G' (but not vice versa). Also, we use G_ρ to denote the unique equilibrium strategy for a given ρ; and G_N to denote the strategy that corresponds to no disclosure, i.e., $G_N(p) = 0$ for all $p \in [0, \pi_1)$ and $G_N(p) = 1$ for all $p \in [\pi_1, 1]$.

Theorem 3 *Suppose the senders are symmetric, i.e., $\pi_1 = \pi_2 = \pi$. If the covariance ρ increases, then each sender discloses less information. Formally, $\rho' > \rho$ implies $G_{\rho'} \prec G_\rho$. However, G_ρ does not converges to G_N in distribution as $\rho \to \bar{\rho}$.*

The theorem above states that for symmetric senders, increasing the correlation in their proposals' qualities would result in each sender engaging in less information disclosure in the sense of an anti-clockwise rotation of distribution. Figure 6 illustrates an anti-clockwise rotation in distribution as ρ increases from zero to a positive number. In the limit as the proposals' qualities become perfectly correlated,



(a) $\pi < 1/2$ (b) $\pi > 1/2$

Figure 6: Equilibrium Strategy

each sender engages in strictly partial disclosure. Observe that in the game with perfectly correlated proposals (i.e., $\rho = \bar{\rho} = \pi_1 (1 - \pi_1)$ and $\pi_1 = \pi_2$), each sender wins with probability $1/2$, regardless of the strategy profile. Thus, in this game, every pair of strategies is an equilibrium, including the one in which both senders engage in full information disclosure. The theorem above shows that a particular partial-disclosure equilibrium is selected, if we take a sequence of games with symmetric senders and increasing covariance ρ such that $\rho \to \bar{\rho}$.

Next, we consider the case with asymmetric senders, i.e., $\pi_1 > \pi_2$, in which both the good-signal curse and the receiver-treatment effect are present. We show that when ρ is sufficiently close to $\bar{\rho}$, the receiver-treatment effect leads to more more aggressive disclosure, and it dominates the good-signal curse (no matter how small the difference in π_1 and π_2 is).

To understand the intuition behind this result, consider a ρ close to $\bar{\rho}$. Irrespective of sender 1's strategy, there exists a $\tilde{p}_2 < 1$ such that sender 2's payoff of inducing a signal $p_2 < \tilde{p}_2$ is close to 0. However, sender 2 can guarantee a payoff of at least $1/2$ by generating signal $p_2 = 1$. Facing such a payoff function, it is sender 2's interest to maximize the probability of inducing $p_2 = 1$, and not to induce any signal in $(\tilde{p}_2, 1)$, so $(\tilde{p}_2, 1) \notin \operatorname{supp} G_2$, and $1 \in \operatorname{supp} G_2$. Thus, irrespective of sender 1's strategy, sender 2 engages in aggressive disclosure.

Since $(\tilde{p}_2, 1) \notin \operatorname{supp} G_2$, sender 1 has no incentive to induce any signal $p_1 \in (\delta_2 (\tilde{p}_2), 1)$, so $(\delta_2 (\tilde{p}_2), 1) \notin \operatorname{supp} G_1$. However, as $\delta_2 (\tilde{p}_2)$ is close to zero (recall Lemma 1), the probability that sender 1 induces interior signals is very small. That is, sender 1 also engages in very aggressive information disclosure. In sum, when ρ is sufficiently large, the receiver-treatment effect forces aggressive disclosure by sender 2, which in turn induces aggressive response by sender 1. In fact, Theorem 4 below states that the equilibrium approaches full disclosure in the limit.

To formally state the result, denote by $G_{i,F}$ the strategy of full disclosure, i.e., $G_{i,F}(p) = 1 - \pi_i$ for all

$p \in [0,1)$, and $G_{i,F}(1) = 1$. Also, we use $\left(G_{1,\rho}, G_{2,\rho}\right)$ to denote the pair of equilibrium strategies for the game in which the covariance of proposal qualities is ρ.

Theorem 4 *Fix the prior expected qualities π_1 and π_2 and suppose $\pi_1 > \pi_2$. In the limit as $\rho \to \bar{\rho}$, both senders engage in full disclosure, i.e., $G_{i,\rho}$ converges to $G_{i,F}$ in distribution as $\rho \to \bar{\rho}$. Furthermore, sender 1's equilibrium payoff converges to $1 - \pi_2/2 > 1/2$ as $\rho \to \bar{\rho}$.*

The limit game with $\rho = \bar{\rho}$ and $\pi_1 > \pi_2$ admits a plethora of equilibria. In fact, every strategy profile in which sender 2 engages in full disclosure, and sender 1 adopts an arbitrary strategy with no atom at $p_1 = 0$ constitutes an equilibrium of the limit game.(15) This includes an almost full-disclosure equilibrium: sender 1 engages in almost full disclosure (i.e., assigns positive weights only to ε and 1, for some arbitrarily small $\varepsilon > 0$), and sender 2 engages in exact full disclosure. The theorem above shows that an almost full-disclosure equilibrium is selected if we take a sequence of games with asymmetric senders and increasing covariance ρ such that $\rho \to \bar{\rho}$.

According to the analysis of this subsection, for two pharmaceutical companies competing in marketing new drugs with similar mechanisms for tackling the disease, the informativeness of the tests they design varies with the similarity of the drugs. If, ex-ante, the qualities of the drugs are expected to be very close, then the more similar the drugs, the less informative their tests are. On the other hand, if the drugs have very different ex-ante expected qualities, then an increase in the similarity of the drugs leads to test designs that are more revealing of the drugs' qualities. Likewise, the effect of correlation in students' qualities on universities' disclosure depends on the difference in their expected qualities.

4 Multiple Senders

4.1 Binary Qualities

In the previous section, we saw that with two symmetric senders, full disclosure never arises in equilibrium regardless of the degree of correlation in their proposals' qualities. In this section, we analyze another element of the competitive environment that affects the equilibrium disclosure: the number of senders. We find that full disclosure arises as a limit equilibrium outcome as the number of senders increases to infinity.

(15)The reason is as follows. Observe that in this game, $U_2 \leq U_1$ with probability 1. If sender 1 does put an atom at $p_1 = 0$, then sender 2's payoff is 0 for all $p_2 < 1$ and equals $\frac{1}{2}$ if $p_2 = 1$. If sender 1 puts an atom at zero, then sender 2's payoff is decreasing linearly on the interval $[0,1)$, and jumps up at $p_2 = 1$. In either case, sender 2's best response is full disclosure. Thus, full disclosure is a dominant strategy for sender 2. When sender 2 engages in full disclosure, $\Pi_1(0; G_2) = 1/2$, and $\Pi_1(p; G_2)$ decreases linearly on $p \in (0,1]$.

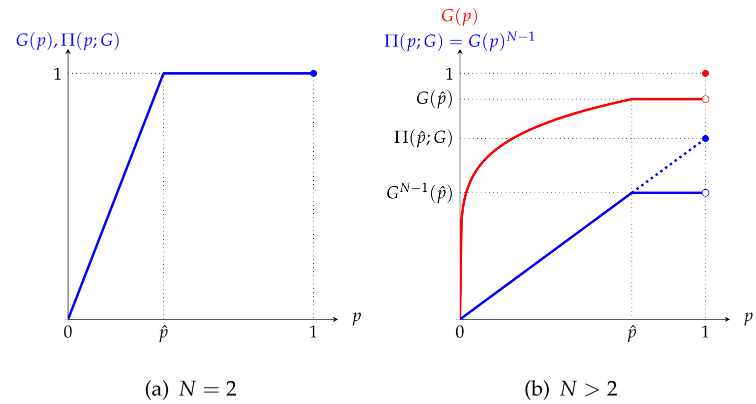


Figure 7: Linear Structure

To highlight the effect of competition that purely arises from the number of senders, we consider the competitive Bayesian persuasion game among N symmetric senders, i.e., $\pi_i = \pi \in (0,1)$ for all $i = 1, \cdots, N$, in which the qualities of theirs proposals are independent. Moreover, we restrict attention to symmetric equilibria.

We begin our analysis with describing sender i's expected payoff when all other senders use strategy $G(p)$. Notice that if he induces a signal p, the receiver chooses him with probability $\frac{1}{k+1}$ if p is the highest signal among all senders and there are k other senders with signal p. Therefore, his expected payoff of inducing signal p is

$$\begin{aligned}\Pi(p,G) &\equiv \lim_{p'\to p^-} \sum_{k=0}^{N-1} \frac{1}{k+1} \frac{(N-1)!}{k!(N-k-1)!} \left(G(p) - G(p')\right)^k \left(G(p')\right)^{N-k-1} \\ &= \lim_{p'\to p^-} \frac{(G(p))^N - (G(p'))^N}{N(G(p)-G(p'))}.\end{aligned}$$

If G is continuous at p, then $\Pi(p,G) = G(p)^{N-1}$.

Recall in Section 2, we consider the special case of 2 symmetric senders, and explain that the linear structure of payoff function is the necessary and sufficient condition for the unique equilibrium (which happens to be symmetric). The same argument applies to the case $N > 2$ considered here, implying that the linear structure of payoff function is the necessary and sufficient condition for the unique symmetric equilibrium. More specifically, we say that G induces a payoff function with the linear structure if there exist a $\hat{p} \in (0,1]$ and a linear function $\overline{\Pi} : [0,1] \to \mathbb{R}$ such that (i) $\overline{\Pi}(p) \geq \Pi(p;G)$ for all $p \in [0,1]$, and (ii) $\operatorname{supp} G = P \in \{[0,\hat{p}], [0,\hat{p}] \cup \{1\}\}$, where $P \equiv \operatorname{cl}\left(\{p : \Pi(p;G) = \overline{\Pi}(p)\}\right)$. Figure 7 illustrates the relationship between a strategy and its induced payoff function with the linear structure.

Theorem 5 *A Bayes-plausible distribution G is a symmetric equilibrium if and only if G induces a payoff function* *that has the linear structure. Moreover, the symmetric equilibrium exists and is unique.*

We now analyze how a change in number of senders affects information disclosure in the unique symmetric equilibrium. Intuitively, as the number of senders increases, the competition for the receiver becomes more intense. To see this, let G_N be a symmetric equilibrium strategy when there are N senders. Then the equilibrium payoff function $\Pi(p, G_N)$ is linear on some interval $[0, \hat{p}_N]$. Now suppose that the number of senders increases to $N' > N$, but $N' - 1$ senders still adopt strategy G_N. Then the payoff function of a sender who faces such $N' - 1$ senders is $(G_N(p))^{N'-1}$, which is convex on $[0, \hat{p}_N]$. Consequently, he benefits from more information disclosure. In response, other senders also engage in more aggressive disclosure. As the number of senders becomes very large, it is extremely likely that an individual sender stands a chance to be chosen by the receiver only if he reveals the most favorable news, i.e., $p = 1$. Consequently, each sender finds it optimal to engage in almost full disclosure, which maximizes the chance that signal $p = 1$ is generated.

Theorem 6 *If the number of senders increases, each sender discloses more information. That is $G_{N'} \succ G_N$ whenever $N' > N$. Moreover, as the number of senders goes to infinity, each sender adopts full disclosure in the limit equilibrium.*

According to the analysis of this section, for pharmaceutical companies competing in marketing similar drugs, an increase in the number of competing companies forces each of them to design and adopt more informative tests. With a sufficiently large number of competitors, the equilibrium designs are almost fully revealing. Likewise, an increase in the number of universities competing in placing their graduates makes each of them reveal more precise information concerning their students. Perloff and Salop (1985) analyze competition in setting prices among symmetric sellers of differentiated products, all of which adopt a full disclosure policy. They show that there exists a unique symmetric equilibrium price, and it converges to the marginal cost of production as the number of sellers approaches infinity.(16) Our results can, therefore, be viewed as counterparts to theirs in the context of competition in information disclosure.

Theorem 6 confirms the intuition that competition among senders leads to more aggressive disclosure. The cheap-talk literature on multiple senders has mainly focused on extreme results, such as establishing conditions that guarantee full revelation as an equilibrium outcome. Gentzkow and Kamenica (2016) consider the effect of competition in a Bayesian persuasion game in which all senders share a common state. They find that adding more senders never makes the set of equilibrium outcomes less informative. However, with equilibrium multiplicity, they also note that the set of outcomes with more competition

(16) The convergence result in their setting requires the tail of the preference distribution to be not too fat. In particular, it holds with a finite support, the case we examine here.

may not be comparable to those with less competition. On the other hand, our setting has a unique symmetric equilibrium, which allows us to obtain a sharp result regarding the effect of competition on information revelation. Board and Lu (2016) consider a search setting in which a receiver, at a positive search cost, sequentially samples senders who provide information concerning a common state. They show that if the receiver's belief is private, and the senders' disclosure mechanisms are independent (conditional on state), then full disclosure is a limit equilibrium as search cost becomes infinitesimally small.[(17)] On the other hand, in our setting, the senders have independent proposals, and they make disclosure simultaneously. There are two notable differences between our results and those of Board and Lu (2016). First, Theorem 6 states that disclosure gets strictly more informative with extra senders, *regardless of the number of existing senders*. On the other hand, Board and Lu (2016)'s result concerns only the limiting case of infinitesimally small search cost. Second, full disclosure is the unique limit equilibrium in our game; whereas in the setting of Board and Lu (2016) with binary states, no disclosure by every sender is always an equilibrium if the prior of the favorable state is relatively high.[(18)] Hoffmann et al. (2014) report a result related to Theorem 6 in their setting of persuasion game with information acquisition by senders. They constrain the set of feasible disclosure policies of each sender to be (effectively) binary, and show a limit result that when the number of competing senders is sufficiently large, the unique equilibrium involves all senders adopting the most informative feasible policy. On the other hand, in our setting, the flexibility in the senders' choice of disclosure mechanism allows us to show that the informativeness of the equilibrium disclosure mechanism is strictly increasing in the number of senders.

4.2 Extension

In this subsection, we consider a more general set of feasible proposal qualities Ω, which contains M distinct elements. Specifically, denote $\Omega \equiv \{u_0, u_1, ..., u_{M-1}\}$, where $u_m < u_{m+1}$. Following the previous subsection, we assume that each sender's proposal quality U_i is independently and identically distributed according to a commonly known prior distribution $\pi \in \Delta\Omega$, with full support. In the subsequent analysis, we first show that the equilibrium payoff function necessarily exhibits a linear structure, which is counterpart to that of the binary state-space case analyzed above. By exploiting the equilibrium linear structure, we then develop an algorithm that constructs an equilibrium. Finally, the algorithm implies that the equilibrium necessarily converges to full disclosure as the number of competing senders approaches infinity.

[(17)]The intuition is as follows. Suppose every sender provides some information. As the search cost gets very small, the receiver can sample a large number of senders at a small total cost. Then the fact that senders use conditionally independent disclosure mechanisms implies that the receiver has the option of becoming almost fully-informed at a low cost. Therefore, the first sender, upon being sampled, would lose the receiver's patronage if he did not reveal sufficiently informative signals, as the receiver's search is without recall.

[(18)]See Corollary 2 of Board and Lu (2016). The reason is similar to the Diamond paradox in a standard search setting.

We begin with a few preliminary observations. First, a belief about sender i's proposal quality U_i, being an element of $\Delta\Omega$, is represented by an M-dimensional non-negative vector $(p_0, p_1, \cdots, p_{M-1})$ satisfying $\sum_{k=0}^{M-1} p_k = 1$. In particular, the prior belief is denoted by $\pi = (\pi_0, \pi_1, \cdots, \pi_{M-1})$. Next, a sender's strategy space is the set of all Bayes-plausible distributions over posterior distributions. Moreover, a sender's strategy $G \in \Delta(\Delta\Omega)$ induces a distribution over ex-post expected qualities; and the receiver selects a sender that gives her the highest expected quality according to the induced distribution. Therefore, a sender's payoff depends on his realized posterior only through the value of expected quality it induces. More specifically, if the other $N-1$ senders use strategy G that induces the distribution F_G over expected qualities, then the expected payoff of inducing posterior p is

$$\Pi_{post}(p; G) \equiv \lim_{u' \to E_p[U_i]^-} \frac{\left(F_G\left(E_p[U_i]\right)\right)^N - \left(F_G(u')\right)^N}{N\left(F_G\left(E_p[U_i]\right) - F_G(u')\right)}. \tag{6}$$

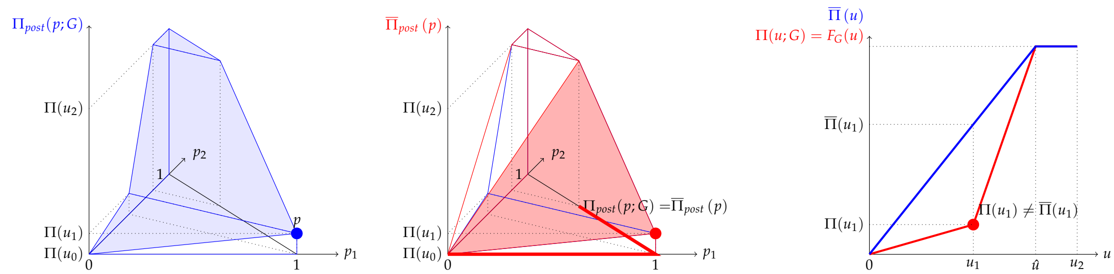
In an equilibrium, a sender neither benefits from more information disclosure, i.e., spreading of posteriors he induces; nor less information disclosure, i.e., contraction of posterior he induces. Therefore, following the same arguments in the previous sections, it is not difficult to see that a Bayes-plausible distribution G over posterior distributions is a symmetric equilibrium strategy if and only if it induces the payoff function $\Pi_{post}(p; G)$ with the linear structure: there exist a $\hat{u} \in (u_0, u_{M-1}]$, and a linear function $\overline{\Pi}_{post}(\cdot) : \Delta\Omega \to \mathbb{R}$ such that (i) $\overline{\Pi}_{post}(p) \geq \Pi_{post}(p; G)$ for all $p \in \Delta\Omega$; and (ii) $\left\{E_p[U] : p \in \text{supp}\, G\right\} = \left\{E_p[U] : p \in P\right\} \in \left\{[0, \hat{u}], [0, \hat{u}] \cup \{u_{M-1}\}\right\}$, where $P \equiv \text{cl}\left(\left\{p : \Pi_{post}(p; G) = \overline{\Pi}_{post}(p)\right\}\right)$.

Figure 8 illustrates an example of the linear structure for the case $N = 2$ and $M = 3$. Figure 8-(a) and (b) are drawn on the simplex $\left\{(p_1, p_2) \in [0,1]^2 : p_1 + p_2 \leq 1\right\}$. Figure 8-(a) illustrates the payoff function $\Pi_{post}(p; G)$ of sender 1 when sender 2 uses strategy G that induces F_G (i.e., the distribution over expected qualities) depicted by the red curve in Figure 8-(c). The support of the strategy, which coincides with P, is $\left\{(p_1, p_2) \in [0,1]^2 : p_2 = 0\right\}$ and $\left\{(p_1, p_2) \in [0,1]^2 : p_1 + p_2 = 1 \text{ and } p_1 u_1 + p_2 u_2 \leq \hat{u}\right\}$ (depicted by the thick red lines on the simplex in 8-(b)). The strategy assigns conditionally uniform weights to the respective intervals. The red plane in Figure 8-(b) illustrates a linear function $\overline{\Pi}_{post}(p)$ on the convex hull of supp G. It satisfies $\overline{\Pi}_{post}(p) \geq \Pi_{post}(p; G)$ for all possible distributions. Furthermore, the set of posterior distributions such that $\overline{\Pi}_{post}(p) = \Pi_{post}(p; G)$ coincides with supp G.

Theorem 7 *G is a symmetric equilibrium strategy if and only if it induces $\Pi_{post}(p; G)$ with the linear structure.*

As a sender's payoff depends on his realized posterior only through the value of expected quality induced by the posterior, we can also define the expected payoff of inducing a certain expected quality $u \in [u_0, u_{M-1}]$. The payoff of inducing expected quality of $u \in [u_0, u_{M-1}]$ when the other $N-1$ senders use strategy G is

$$\Pi(u; G) \equiv \lim_{u' \to u^-} \frac{\left(F_G(u)\right)^N - \left(F_G(u')\right)^N}{N\left(F_G(u) - F_G(u')\right)}. \tag{7}$$



(a) $\Pi_{post}(p;G)$

(b) $\overline{\Pi}_{post}(p)$ and supp G

(c) $\Pi(u;G)$ and $\overline{\Pi}(u)$

Figure 8: $N = 2$ and $M = 3$

When there is no confusion, we omit G from $\Pi(u;G)$ for expositional simplicity.

One may conjecture that the linear structure of $\Pi_{post}(p;G)$ implies the linear structure of $\Pi(u;G)$ defined by (7).[19] However, this conjecture turns out to be false. The reason is that whereas interior posterior can always be spread when the quality is binary, in the current environment, an interior expected quality $u \in (u_0, u_{M-1})$ may be induced by a degenerate posterior,[20] which cannot be spread.

Figure 8 illustrates this point for the case where $N = 2$ and $M = 3$. According to Theorem 7, the strategy G that assigns (conditionally) uniform weights over the intervals $\left\{(p_1, p_2) \in [0,1]^2 : p_2 = 0\right\}$ and $\left\{(p_1, p_2) \in [0,1]^2 : p_1 + p_2 = 1 \text{ and } p_1 u_1 + p_2 u_2 \leq \hat{u}\right\}$ constitutes a symmetric equilibrium. As illustrated in Figure 8-(c), sender 1's payoff function with respect to induced expected quality $\Pi(u;G)$ exhibits an upward-kink at $u = u_1$ and hence $\Pi(u_1;G) \neq \overline{\Pi}(u_1)$ for any linear function $\overline{\Pi}(u)$ such that $\overline{\Pi}(u) \geq \Pi(u;G)$ for all $u \in [u_0, u_{M-1}]$. This is because over the support of G, expected quality u_1 is only induced by a degenerate posterior distribution p such that $p_1 = 1$. Consequently, even though sender 1's payoff function $\Pi(u;G)$ exhibits an upward-kink at $u = u_1$, he cannot spread the posterior distribution at $u = u_1$ to increase his payoff. Thus, the linear structure of $\Pi(u;G)$ is not a necessary condition for an equilibrium.

In the subsequent analysis, we show that the linear structure of $\Pi_{post}(p;G)$ in an equilibrium, as identified in Theorem 7, has a number of implications on the structure of $\Pi(u;G)$. These implications turn out to provide a complete characterization of the unique equilibrium distribution of expected qualities. This characterization gives us a simple algorithm for constructing the unique equilibrium distribution of expected qualities.

[19]That is, there exist a $\hat{u} \in (u_0, u_{M-1}]$ and a linear function $\overline{\Pi}$ defined on $[u_0, u_{M-1}]$ such that $\overline{\Pi}(u) \geq \Pi(u;G)$ for all $u \in [u_0, u_{M-1}]$; and supp $F_G = U \in \{[0, \hat{u}], [0, \hat{u}] \cup \{u_{M-1}\}\}$, where $U \equiv \text{cl}\left(\{u : \Pi(u;G) = \overline{\Pi}(u)\}\right)$.

[20]A posterior $p \in \triangle\Omega$ is degenerate if $p_m = 1$ for some m and $p_{m'} = 0$ for all $m' \neq m$.

4.2.1 Linear Structure of Payoff Functions

The linear structure of $\Pi_{post}(p)$ has the following immediate implications on the corresponding payoff function in expected quality $\Pi(u)$. First, as $\Pi_{post}(p)$ has no atom except possibly at the degenerate posterior with $p_{M-1} = 1$, $\Pi(u)$ is continuous on the interval $[u_0, u_{M-1})$. Second, there is an $\hat{u} \in [u_0, u_{M-1}]$ such that $\Pi(u)$ is increasing on the interval $[u_0, \hat{u}]$ and constant on the interval $[\hat{u}, u_{M-1})$. Moreover, for $u_m < \hat{u}$, $\Pi(u)$ is linear on the interval $[u_m, \min\{u_{m+1}, \hat{u}\}]$. Intuitively, each $u \in (u_m, \min\{u_{m+1}, \hat{u}\})$ is necessarily induced by a non-degenerate posterior, u can be "spread" into u_m and $\min\{u_{m+1}, \hat{u}\}$. The linearity of $\Pi(u)$ over the interval $[u_m, \min\{u_{m+1}, \hat{u}\}]$ then ensures such deviation is not profitable. These necessary equilibrium conditions are summarized by the piecewise-linearity condition below.

Definition 1 (Piecewise-linearity Condition) *A payoff function $\Pi(u)$ is piecewise-linear with $\hat{u} \in (u_0, u_{M-1}]$ if (i) it is continuous on the interval $[u_0, u_{M-1})$ with $\Pi(u_0) = 0$; and (ii) it is linear on the interval $[u_m, \min\{u_{m+1}, \hat{u}\}]$ for each $m = 0, \cdots, M-2$, and constant on the interval $[\hat{u}, u_{M-1})$.*

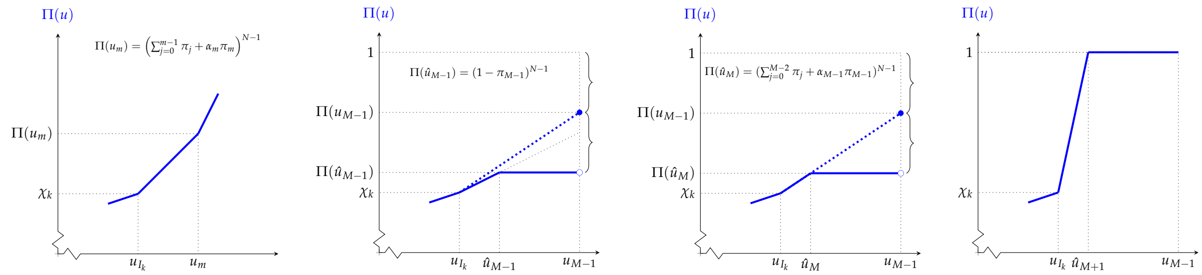
For a piecewise-linear payoff function $\Pi(u)$, define s_m as the slope of $\Pi(u)$ on the interval $[u_m, \min\{u_{m+1}, \hat{u}\}]$. Also, define $s^-(\hat{u})$ as the slope of $\Pi(u)$ on $\left[u_{\tilde{\imath}(\hat{u})}, \hat{u}\right]$, where $\tilde{\imath}(\hat{u}) \equiv \arg\max\{m : u_m < \hat{u}\}$, and define $s^+(\hat{u})$ as the slope of the line that connects $(\hat{u}, \Pi(\hat{u}))$ and $(u_{M-1}, \Pi(u_{M-1}))$ on the graph of Π.

We now look at the property at the "top" of the equilibrium payoff function Π. There are three possibilities. One possibility is that F_G does not have an atom at u_{M-1}; this possibility is covered by case (i) in the definition below. See Figure 9-(d). If F_G has an atom at u_{M-1}, there are two possibilities. One possibility is that $s^-(\hat{u}) < s^+(\hat{u})$ as in Figure 9-(b). This happens only when the atom of F_G at u_{M-1} is exactly π_{M-1}, i.e., $\Pi(\hat{u}) = (1-\pi_{M-1})^{N-1}$.[21] This possibility is covered by case (ii) in the definition below. The last possibility is that $s^-(\hat{u}) = s^+(\hat{u})$, which is covered by case (iii) in the definition below. Also see Figure 9-(c).

Definition 2 (Atom Condition) *A piecewise-linear payoff function $\Pi(u)$ satisfies the atom condition with $\hat{u} \in (u_0, u_{M-1})$ if either (i) $\Pi(\hat{u}) = 1$; (ii) $\Pi(\hat{u}) = (1-\pi_{M-1})^{N-1}$ and $s^-(\hat{u}) < s^+(\hat{u})$; or (iii) $\Pi(\hat{u}) \in \left[(1-\pi_{M-1})^{N-1}, 1\right)$ and $s^-(\hat{u}) = s^+(\hat{u})$.*

As we have seen in the previous subsection, the linear structure of $\Pi_{post}(p)$ does not necessarily imply $s_m = s_{m+1}$ because u_{m+1} may be induced by a degenerate posterior. The linear structure of $\Pi_{post}(p)$, however, rules out the possibility that $s_m > s_{m+1}$. This is because if $s_m > s_{m+1}$, then a sender will benefit from contracting the weights on the expected qualities in the neighborhood of u_{m+1} onto u_{m+1}. Furthermore, if $s_m < s_{m+1}$, i.e., $\Pi(u)$ exhibits an upward-kink at u_{m+1}, then each $u \in supp(F_G)$ is *not*

[21] Otherwise, a positive measure of $u \leq \hat{u}$ is induced by posteriors p with $p_{M-1} > 0$ under G, and facing such a payoff function $\Pi(u)$, the sender can profit from "spreading" the induced utilities to u_{M-1} and some lower utilities.



(a) Property-m (b) Property-$M-1$ (c) Property-M (d) Property-$M+1$

Figure 9: Properties $m, M-1, M, M+1$

induced by a convex combination of $u_k \in \{u_{m+2}, \cdots, u_{M-1}\}$ and $u_{k'} \in \{u_0, \cdots, u_m\}$. Otherwise the upward-kink of $\Pi(u)$ at u_{m+1} implies that a sender can benefit from spreading such a u to some pair of expected qualities $\{u', u''\}$ with $u' < u_{m+1}$ and $u'' > u_{m+1}$.

Definition 3 (Upward-Kink Condition) *A piecewise linear payoff function $\Pi(u)$ satisfies the upward-kink condition if both conditions below hold. First, $s_m \le s_{m+1}$ for all $m \in \{0, 1, ..., \tilde{\imath}(\hat{u}) - 1\}$. Second, if $\Pi(u)$ has an upward kink at u_{m+1}, then for each $p \in supp(G)$, $p_k p_{k'} = 0$ for all $k \in \{0, \cdots, m\}$ and $k' \in \{m+2, \cdots, M-1\}$.*

If a Bayes-plausible distribution G is a symmetric equilibrium strategy, then it necessarily induces a payoff function that satisfies all conditions above.

Definition 4 (Generalized Linear Structure) *A Bayes-plausible strategy G induces a payoff function $\Pi(u; G)$ with the generalized linear structure if there exists a $\hat{u} \in (u_0, u_{M-1})$ such that $\Pi(u; G)$ satisfies the piecewise-linearity condition with $\hat{u}$, the atom condition with $\hat{u}$, and the upward-kink condition.*

In sum, a necessary condition for a strategy to constitute a symmetric equilibrium is that it induces a payoff function with the generalized linear structure. Below, by offering a simple algorithm that constructs a payoff function with the generalized linear structure, we show that the generalized linear structure is also a sufficient condition for an equilibrium.

4.2.2 Algorithm and Sufficiency

In this subsection, we briefly describe an algorithm that constructs a payoff function satisfying the generalized linear structure defined above. As the algorithm yields a unique output, the equilibrium distribution of expected utilities is unique. A formal description of the algorithm is relegated to Appendix A.

Suppose the equilibrium symmetric strategy G induces a distribution of expected utilities F_G such that the κ-th upward kink occurs at u_{I_κ}. As the equilibrium payoff function $\Pi(u)$ satisfies the generalized linear structure, exactly one of the following properties, illustrated by Figure 9, holds.

Property-m : The first upward kink after u_{I_κ} occurs at $u_m \in \{u_{I_\kappa+1}, \cdots, u_{M-2}\}$, i.e., $\Pi(u)$ is linear on the interval $[u_{I_\kappa}, u_m]$ and has an upward kink at u_m.

Property-$M-1$: $\Pi(u)$ does not have any upward kink at $u_i \in \{u_{I_\kappa+1}, \cdots, u_{M-2}\}$. Moreover, there exists a $\hat{u}_{M-1} \in (u_{I_\kappa}, u_{M-1})$ such that $F_G(\hat{u}_{M-1}) = 1 - \pi_{M-1}$ and $\frac{\Pi(\hat{u}_{M-1}) - \Pi(u_{I_\kappa})}{\hat{u}_{M-1} - u_{I_\kappa}} < \frac{\Pi(u_{M-1}) - \Pi(\hat{u}_{M-1})}{u_{M-1} - \hat{u}_{M-1}}$.

Property-M : $\Pi(u)$ does not have any upward-kink at $u_i \in \{u_{I_\kappa+1}, \cdots, u_{M-2}\}$. Moreover, there exists a $\hat{u}_M \in (u_{I_\kappa}, u_{M-1})$ such that $\frac{\Pi(\hat{u}_M) - \Pi(u_{I_\kappa})}{\hat{u}_M - u_{I_\kappa}} = \frac{\Pi(u_{M-1}) - \Pi(\hat{u}_M)}{u_{M-1} - \hat{u}_M}$.

Property-$M+1$: $\Pi(u)$ does not have any upward-kink at $u_i \in \{u_{I_\kappa+1}, \cdots, u_{M-2}\}$. Moreover, $F_G(\hat{u}_{M+1}) = 1$ for some $\hat{u}_{M+1} \in (u_{I_\kappa}, u_{M-1}]$.

The key of the algorithm is to identify which property above holds, given the κ-th upward kink occurs at u_{I_κ}. For simplicity of exposition, we illustrate the case of $N = 2$ here. See Appendix A for the formal description that covers any finite number of senders.

We first define a sequence of *potential slopes*, $s_m^{\kappa+1}$ for each $m \in \{I_\kappa, I_\kappa + 1, ..., M+1\}$. For $m \in \{I_\kappa + 1, \cdots, M-2\}$, $s_m^{\kappa+1}$ is the potential slope of Π, assuming that Π has the $(\kappa+1)$-th upward kink at u_m, and that it satisfies the piecewise-linearity condition and the upward kink condition. Specifically, it is defined by equating the two expressions for $E_G[u|u \in [u_{I_\kappa}, u_m]]$ discussed below. First, as $\Pi(u) = F_G(u)$ is linear on the interval $[u_{I_\kappa}, u_m]$, $E_G[u|u \in [u_{I_\kappa}, u_m]] = \frac{1}{2}(u_{I_\kappa} + u_m)$. Second, by the upward-kink condition at u_m, there exists an $\alpha_m \in [0,1]$ such that $F_G(u_m) = \sum_{j=0}^{m-1} \pi_j + \alpha_m \pi_m$ and

$$E_G[u|u \in [u_{I_\kappa}, u_m]] = \frac{\left(\sum_{j=0}^{I_\kappa} \pi_j - F_G(u_{I_\kappa})\right) \pi_{I_\kappa} u_{I_\kappa} + \sum_{j=I_\kappa+1}^{m-1} \pi_j u_j + \alpha_m \pi_m u_m}{\left(\sum_{j=0}^{I_\kappa} \pi_j - F_G(u_{I_\kappa})\right) \pi_{I_\kappa} + \sum_{j=I_\kappa+1}^{m-1} \pi_j + \alpha_m \pi_m}.$$

If there exists an $\alpha_m \in (0,1)$ that equates the two expressions for $E_G[u|u \in [u_{I_\kappa}, u_m]]$, then the slope $s_m^{\kappa+1}$ is defined to be $\frac{\sum_{j=0}^{m-1} \pi_j + \alpha_m \pi_m - F_G(u_{I_\kappa})}{u_m - u_{I_\kappa}}$. If no such α_m exists, then define $s_m^{\kappa+1} \equiv \infty$. We define $s_\ell^{\kappa+1}, \ell \in \{M-1, M, M+1\}$ in a similar manner.

If $s_\ell^{\kappa+1} = \infty$, then it is clear that Π does not satisfy Property-ℓ. The converse is not necessarily true, i.e., even if $s_\ell^{\kappa+1} < \infty$, Π may not satisfy Property-ℓ. However, as we formally prove in the appendix (Lemma 6), if $s_\ell^{\kappa+1} < s_{\ell'}^{\kappa+1} < \infty$ for some ℓ and ℓ', or $s_\ell = s_{\ell'}$ and $\ell' < \ell$, then we can show that Π does not satisfy Property-ℓ'. The intuition is that keeping the value of $\Pi(u)$ as low as possible slacks the constraints imposed by the upward-kink condition and the atom condition at the top for higher values of u. Using this result, we can identify the property that Π satisfies by finding the index $\ell \in \{I_\kappa, I_\kappa + 1, ..., M+1\}$ that minimizes $s_{\ell}^{\kappa+1}$.[22] Therefore, by initiating the algorithm with $I_0 = 0$, we can construct a payoff function by identifying the locations of all upward kinks.

The theorem below establishes the existence of a symmetric equilibrium. As a result, the algorithm described above necessarily identifies a payoff function satisfying the generalized linear structure. Moreover, as is clear from the description of the algorithm, it yields at most one output. The uniqueness of the algorithm's output in turn implies that the generalized linear structure of induced payoff function is also a sufficient condition for an equilibrium.

Theorem 8 *A Bayes-plausible strategy G is an equilibrium if and only if the induced payoff function $\Pi(u; G)$ has the generalized linear structure. A symmetric equilibrium exists and is unique up to the induced distribution of expected qualities. The algorithm described above constructs the unique equilibrium expected-quality distribution. If $N = 2$, then the distribution of expected qualities is necessarily symmetric in equilibrium.*

Note that whereas the distribution of expected qualities is unique in equilibrium, there may, in general, be multiple posterior distributions that induce it.

Finally, using the algorithm, we can show that the symmetric equilibrium approaches full disclosure in the limit as the number of senders goes to infinity. More formally, let $F_{Full}(u)$ be the expected-quality distribution that corresponds to full disclosure, i.e., $F_{Full}(u) = \sum_{j=0}^{m} \pi_j$ for all $u \in [u_m, u_{m+1})$, and $m = 0, \cdots, M-2$.

Theorem 9 *Let $F_{G,N}$ be the unique symmetric equilibrium expected-quality distribution if there are N symmetric senders. Then $F_{G,N}$ converges to F_{Full} in distribution as the number of senders approaches infinity.*

The intuition of the result is as follows. When a sender is facing a large number of competing senders, he understands that with a very high probability, some other senders would generate a signal with expected utility very close to the maximum equilibrium value. This creates a strong incentives for each sender to maximize the probability of generating the most favorable equilibrium signal, so when the number of senders is sufficiently large, each sender would almost fully reveal the state u_{M-1}. In a similar manner, for any $m \in \{1, \cdots, M-2\}$, conditional on the highest signal among the other senders being no higher than u_m, each sender understands that with a very high probability, some other senders would generate a signal with expected utility very close to the maximum equilibrium value conditional on it being no higher than u_m. Therefore, each sender fully reveals u_m.

[22] In the case of a tie, pick the largest index.

5 Concluding Remarks

In this paper, we analyzed how differences in competitive environments affect equilibrium information disclosure. Our results highlight notable differences between competition in setting prices a la Perloff and Salop (1985) and competition in information disclosure among firms with differentiated products.

In the price setting environment of Perloff and Salop (1985), fixing other firms' strategies, a firm can increase its demand by lowering its price. In our setting of information disclosure, the corresponding tool for increasing demand (i.e., the probability that the receiver adopts his proposal) is the generation of favorable signals. As we have shown in Section 3, however, due to the good-signal curse, a higher signal realization does not necessarily lead to a higher probability of his proposal being adopted. Consequently, an increase in the substitutability of proposals among symmetric senders, represented by an increase in the ex-ante correlation of qualities, results in less information disclosure in the unique equilibrium; and the equilibrium strategy does not converge to full disclosure. This finding is in sharp contrast to the corresponding result in a price-setting environment: prices decrease as the products offered by symmetric firms become more substitutable, and converge to the marginal cost of production as the products become perfectly substitutable.

We believe that the main insights obtained in the analysis of the specific settings we considered carry over to more general settings. Below we discuss the potential extensions of our model, and their possible outcomes.

First, in the presence of ex-ante correlation in proposal qualities, we identify the receiver-treatment effect when there is a difference in the expected qualities. As shown in Section 3, this effect intensifies the competition in information disclosure to a point where both senders disclose almost full information. This finding is indicative that, if possible, the receiver may benefit from committing to treat the senders unequally. Relatedly, we assumed that the receiver always chooses one proposal with a specific tie-breaking rule. As the insights from the auction literature suggests, the receiver may potentially benefit from committing to an inefficient level of "reserve" expected proposal quality, as it induces more aggressive disclosure. Much of our analysis can be adapted in a straightforward manner to games with a binding reserve quality. A couple of notable implications of reserve quality are that some sender may assign an atom at the reserve quality, and that full disclosure may be an equilibrium for a sufficiently high reserve quality. Investigating the receiver's optimal use of these tools can be an interesting avenue for future research.

Second, we have shown that a strategy profile is an equilibrium if and only if the payoff functions it induces possess the linear structure. We note that the sufficiency of this finding holds true in general. However, it requires further investigation to find out whether the linear structure of payoff functions is a necessary condition in more general settings. For example, in a setting in which the qualities of proposals take multiple values, and are independently but non-identically distributed, it is likely that the linear structure of payoff functions (with respect to posteriors) remains a necessary condition for an equilibrium. Furthermore, the existence of an equilibrium in this environment can be established by slightly modifying the proof of Theorem 2. Although we can easily extend our finding in Section 4 to establish local linearity of payoff functions with respect to expected qualities, whether there exists a simple algorithm that identifies an equilibrium in this setting is an open question.(23)

Relatedly, throughout the analysis, we assumed that the supports of possible qualities are common across all senders. This is a crucial assumption for the equilibrium existence under the tie-breaking used in this paper.(24) The reason of the non-existence of an equilibrium is quite similar to that of an asymmetric Bertrand game. Therefore, if we allow the tie-breaking rule to be determined as a part of solution a la Simon and Zame (1990), then it is expected that the upper hemi-continuity of payoff correspondences would guarantee the existence of an equilibrium.(25)

Third, it is a natural question to ask what would happen if senders move sequentially. Consider the simple setting in Section 2, but suppose the second sender observes the first sender's realized signal before choosing his disclosure policy. Moreover, to ensure the existence of a subgame-perfect equilibrium, assume the receiver always chooses the second sender if there is a tie.(26) Provided that the common expected quality π exceeds 0.5, the unique equilibrium outcome would be full disclosure by the first sender; followed by no disclosure by the second sender if the first sender's signal is poor, and full disclosure by the second sender if the first sender's signal is good.(27) An alternative specification of sequential move is that the second sender observes only the first sender's disclosure policy but not the latter's realized signal. This sequential game cannot be solved by techniques used in this article. The reason is that as the second sender's disclosure choice depends on that of the first sender, it is impossible to specify the first sender's payoff function in his realized signals independent of his strategy.(28)

(23)One of the difficulties arises from the possibility that the lower bounds of the support of equilibrium distributions over expected utilities differ.

(24)Suppose there are two senders with U_1 and U_2 being independently distributed. Suppose also that $U_1 \in \{0, 1\}$ with prior $\Pr(U_1 = 1) = 0.7$, whereas $U_2 \in \{0, 2\}$ with prior $\Pr(U_2 = 2) = 0.1$. Then it is not difficult to see that there exists no equilibrium. Intuitively, given the relative high prior of sender 1, an atom at the top is needed for Bayes-plausibility. The atom in turn implies a jump in sender 2's payoff function at $p_2 = 0.5$, resulting in no best-response for sender 2.

(25)We appreciate an anonymous referee for pointing this out.

(26)If we maintain the tie-breaking rule adopted, i.e., randomly selects one sender with equal probabilities, a subgame-perfect equilibrium does not exist.

(27)The reason is as follows. If the first sender's realized signal p_1 exceeds π, the second sender would always maximize the probability of matching p_1 by choosing a disclosure mechanism with support $\{0, p_1\}$. Thus, the first sender can back out the payoff of inducing each posterior p_1: $\Pi(p_1) = 1 - \frac{\pi}{p_1}$ for $p_1 > \pi$ and $\Pi(p_1) = 0$ for $p_1 \leq \pi$. It is straightforward that the first sender's optimal mechanism has a binary support $\{0, \min\{2\pi, 1\}\}$.

(28)For recent progress, see Li and Norman (2017b) that utilize a linear programming approach to analyze a sequential game with

Finally, we analyzed the case of positively correlated proposal qualities in Section 3, and did not explore the case of negative correlation. A reason is that we find positive correlation to be more relevant to the applications considered. Extending our analysis to the case of negative correlation is interesting but not trivial. Although the equilibrium-existence proof in Theorem 2 can cover the negative-correlation case and the linearity of equilibrium payoff function is likely to remain valid, obtaining an explicit equilibrium characterization is complicated by the possibility that the support of equilibrium strategies may contain multiple gaps. We leave this problem for future research.

Appendix A: Formal Description of Algorithm

Output of Algorithm:

Define $I_0 = 0, \sigma_0 \equiv 0$ and $\chi_0 \equiv 0$. By the end of κ-th step, the algorithm generates $\left(\{I_j\}_{j=0}^{\kappa}, \{\sigma_j\}_{j=0}^{\kappa}, \{\chi_j\}_{j=0}^{\kappa}\right)$, where I_j is the j-th upward kink of Π, σ_j is the right-derivative of Π at u_{I_j}, and $\chi_j = \Pi\left(u_{I_j}\right)$.

Description of Algorithm:

The $\kappa + 1$-th step of the algorithm proceeds as follows:

1. Calculate $\left\{s_j^{\kappa+1}\right\}_{j=I_{\kappa+1}}^{M+1}$ and $\{\hat{u}_j\}_{j=M-1}^{M+1}$ defined by $(8), (9), (11)$, and (14).
2. Define $I_{\kappa+1} \equiv \max\left\{\arg\min_{\ell\in\{I_\kappa+1,\cdots,M+1\}}\left\{s_\ell^{\kappa+1}\right\}\right\}$.
3. If $I_{\kappa+1} = M-1, M$, or $M+1$, then the algorithm constructs F_G by

$$F_G(u) = \begin{cases} \left(\sum_{j=0}^{\kappa}\sigma_j \max\left\{0, \left(\min\left\{u_{I_j}, u\right\} - u_{I_{j-1}}\right)\right\}\right)^{\frac{1}{N-1}} & u \in [u_0, u_{I_\kappa}] \\ \left(\sum_{j=0}^{\kappa}\sigma_j\left(u_{I_j} - u_{I_j-1}\right) + s_{I_{\kappa+1}}^{\kappa+1}\left(\min\left\{u, \hat{u}_{I_{\kappa+1}}\right\} - u_{I_\kappa}\right)\right)^{\frac{1}{N-1}} & u \in (u_{I_\kappa}, u_{M-1}) \\ 1 & u = u_{M-1} \end{cases}.$$

4. If $I_{\kappa+1} \in \{I_\kappa + 1, \cdots, M-2\}$, then define $\sigma_{\kappa+1} \equiv \min_{\ell\in\{I_\kappa+1,\cdots,M+1\}}\left\{s_\ell^{\kappa+1}\right\}$ and $\chi_{\kappa+1} \equiv \sum_{j=0}^{\kappa+1}\sigma_j\left(u_{I_j} - u_{I_{j-1}}\right)$. The algorithm proceeds to $\kappa + 2$-th step with $\left(\{I_j\}_{j=0}^{\kappa+1}, \{\sigma_j\}_{j=0}^{\kappa+1}, \{\chi_j\}_{j=0}^{\kappa+1}\right)$.

Definitions of $\left\{s_j^{\kappa+1}\right\}_{j=I_{\kappa+1}}^{M+1}$ and $\{\hat{u}_j\}_{j=M-1}^{M+1}$:

- $s_m^{\kappa+1}$: For each $m \in \{I_\kappa + 1, \cdots, M-1\}$, define

$$s_m^{\kappa+1} \equiv \begin{cases} \frac{\left(\sum_{j=0}^{m-1}\pi_j + \tilde{\alpha}_m\pi_m\right)^{N-1} - \chi_\kappa}{u_m - u_{I_\kappa}} & \text{if } \tilde{\alpha}_m \in (0,1) \\ \infty & \text{otherwise} \end{cases}, \quad (8)$$

a common state.

where $\tilde{\alpha}_m$ solves

$$\frac{N-1}{N}\frac{\left(\sum_{j=0}^{m-1}\pi_j+\tilde{\alpha}_m\pi_m\right)^N-\chi_\kappa^{\frac{N}{N-1}}}{\left(\sum_{j=0}^{i-1}\pi_j+\tilde{\alpha}_m\pi_m\right)^{N-1}-\chi_\kappa}=\sum_{j=0}^{I_\kappa}\pi_j+\sum_{j=I_\kappa+1}^{m-1}\pi_j\frac{u_m-u_j}{u_m-u_{I_\kappa}}.$$

- $\left(s_{M-1}^{\kappa+1},\hat{u}_{M-1}\right)$:

$$\left(s_{M-1}^{\kappa+1},\hat{u}_{M-1}\right)\equiv\begin{cases}\left(\frac{\left(\sum_{j=0}^{M-2}\pi_j\right)^{N-1}-\chi_\kappa}{\hat{u}_{M-1}-u_{I_\kappa}},\hat{u}_{M-1}\right) & \text{if } \hat{u}_{M-1}\in\left(u_{I_\kappa},u_{M-1}\right) \text{ and } I_\kappa\in\{0,\cdots,M-3\}\\ (\infty,\infty) & \text{otherwise}\end{cases}, \tag{9}$$

where $\hat{u}_{M-1}$ solves

$$\frac{N-1}{N}\frac{\left(1-\pi_{M-1}\right)^N-\chi_\kappa^{\frac{N}{N-1}}}{\left(1-\pi_{M-1}\right)^{N-1}-\chi_\kappa}=\sum_{j=0}^{I_\kappa}\pi_j+\sum_{j=I_\kappa+1}^{M-2}\pi_j\frac{\hat{u}_{M-1}-u_j}{\hat{u}_{M-1}-u_{I_\kappa}}. \tag{10}$$

- $\left(s_{M}^{\kappa+1},\hat{u}_{M}\right)$:

$$\left(s_{M}^{\kappa+1},\hat{u}_{M}\right)\equiv\begin{cases}\left(\frac{\left(\sum_{j=0}^{M-2}\pi_j+\tilde{\alpha}_M\pi_{M-1}\right)^{N-1}-\chi_\kappa}{\hat{u}_{M}-u_{I_\kappa}},\hat{u}_{M}\right) & \text{if } \hat{u}_{M}\in\left(u_{I_\kappa},u_{M-1}\right) \text{ and } \tilde{\alpha}_M\in[0,1)\\ (\infty,\infty) & \text{otherwise}\end{cases}. \tag{11}$$

where $(\tilde{\alpha}_M,\hat{u}_M)$ is the solution to the system of equations

$$\begin{aligned}&\frac{N-1}{N}\frac{\left(\sum_{j=0}^{M-2}\pi_j+\tilde{\alpha}_M\pi_{M-1}\right)^N-\chi_\kappa^{\frac{N}{N-1}}}{\left(\sum_{j=0}^{M-2}\pi_j+\tilde{\alpha}_M\pi_{M-1}\right)^{N-1}-\chi_\kappa}\\&=\sum_{j=0}^{I_\kappa}\pi_j+\sum_{j=I_\kappa+1}^{M-2}\pi_j\frac{\hat{u}_M-u_j}{\hat{u}_M-u_{I_\kappa}}+\tilde{\alpha}_M\pi_{M-1}\frac{\hat{u}_M-u_{M-1}}{\hat{u}_M-u_{I_\kappa}}.\end{aligned} \tag{12}$$

and

$$\begin{aligned}&\frac{\left(\sum_{j=0}^{M-2}\pi_j+\alpha_M\pi_{M-1}\right)^{N-1}-\chi_\kappa}{\hat{u}_M-u_{I_\kappa}}\\&=\frac{1}{u_{M-1}-\hat{u}_M}\left(\frac{1-\left(\sum_{j=0}^{M-2}\pi_j+\alpha_M\pi_{M-1}\right)^N}{N\left(1-\left(\sum_{j=0}^{M-2}\pi_j+\alpha_M\pi_{M-1}\right)\right)}-\left(\sum_{j=0}^{M-2}\pi_j+\alpha_M\pi_{M-1}\right)^{N-1}\right).\end{aligned} \tag{13}$$

- $\left(s_{M+1}^{\kappa+1},\hat{u}_{M+1}\right)$:

$$\left(s_{M+1}^{\kappa+1},\hat{u}_{M+1}\right)\equiv\begin{cases}\left(\frac{1-\chi_\kappa}{\hat{u}_{M+1}-u_{I_\kappa}},\tilde{u}_{M+1}\right) & \text{if } \hat{u}_{M+1}\in\left(u_{I_\kappa},u_{M-1}\right]\\ (\infty,\infty) & \text{otherwise}\end{cases}, \tag{14}$$

where $\tilde{u}_{M+1}$ is the solution to

$$\frac{N-1}{N}\frac{1-\chi_\kappa^{\frac{N}{N-1}}}{1-\chi_\kappa}=\sum_{j=0}^{I_\kappa}\pi_j+\sum_{j=I_\kappa+1}^{M-1}\pi_j\frac{\hat{u}_{M+1}-u_j}{\hat{u}_{M+1}-u_{I_\kappa}}. \tag{15}$$

Appendix B: Proofs

Proof of Lemma 1 Suppose $p_1, p_2 \in (0,1)$. By a direct application of Bayes' rule,

$$\Pr\left(U_1=u_1|p_1,p_2\right)=\frac{1}{1+\frac{((1-\pi_1)(1-\pi_2)+\rho)\frac{1-p_2}{p_2}\frac{\pi_2}{1-\pi_2}+(\pi_2(1-\pi_1)-\rho)}{(\pi_1(1-\pi_2)-\rho)\frac{1-p_2}{p_2}\frac{\pi_2}{1-\pi_2}+(\pi_1\pi_2+\rho)}\frac{1-p_1}{p_1}\frac{\pi_1}{1-\pi_1}}, \text{ and}$$

$$\Pr\left(U_2=u_1|p_1,p_2\right)=\frac{1}{1+\frac{((1-\pi_1)(1-\pi_2)+\rho)\frac{1-p_1}{p_1}\frac{\pi_1}{1-\pi_1}+(\pi_1(1-\pi_2)-\rho)}{(\pi_2(1-\pi_1)-\rho)\frac{1-p_1}{p_1}\frac{\pi_1}{1-\pi_1}+(\pi_1\pi_2+\rho)}\frac{1-p_2}{p_2}\frac{\pi_2}{1-\pi_2}}.$$

Therefore, $\Pr\left(U_1=u_1|p_1,p_2\right)\geq\Pr\left(U_2=u_1|p_1,p_2\right)$ if and only if

$$\begin{aligned}&\frac{\left(\pi_2\left(1-\pi_1\right)-\rho\right)\frac{1-p_1}{p_1}\frac{\pi_1}{1-\pi_1}+\left(\pi_1\pi_2+\rho\right)}{\left(\left(1-\pi_1\right)\left(1-\pi_2\right)+\rho\right)\frac{1-p_1}{p_1}\frac{\pi_1}{1-\pi_1}+\left(\pi_1\left(1-\pi_2\right)-\rho\right)}\frac{1-p_1}{p_1}\frac{\pi_1}{1-\pi_1}\\&\qquad\leq\frac{\left(\pi_1\left(1-\pi_2\right)-\rho\right)\frac{1-p_2}{p_2}\frac{\pi_2}{1-\pi_2}+\left(\pi_1\pi_2+\rho\right)}{\left(\left(1-\pi_1\right)\left(1-\pi_2\right)+\rho\right)\frac{1-p_2}{p_2}\frac{\pi_2}{1-\pi_2}+\left(\pi_2\left(1-\pi_1\right)-\rho\right)}\frac{1-p_2}{p_2}\frac{\pi_2}{1-\pi_2}.\end{aligned}$$

The inequality above can be rewritten as:

$$\begin{aligned}&\left(\frac{1-p_2}{p_2}\frac{\pi_2}{1-\pi_2}\left(\pi_1\left(1-\pi_2\right)-\rho\right)-\frac{1-p_1}{p_1}\frac{\pi_1}{1-\pi_1}\left(\pi_2\left(1-\pi_1\right)-\rho\right)\right)\times\\&\left(\begin{array}{c}\left(\pi_1\pi_2+\rho\right)+\left(\frac{1-p_1}{p_1}\frac{\pi_1}{1-\pi_1}\right)\left(\pi_2\left(1-\pi_1\right)-\rho\right)+\left(\frac{1-p_2}{p_2}\frac{\pi_2}{1-\pi_2}\right)\left(\pi_1\left(1-\pi_2\right)-\rho\right)\\+\left(\frac{1-p_1}{p_1}\frac{\pi_1}{1-\pi_1}\right)\left(\frac{1-p_2}{p_2}\frac{\pi_2}{1-\pi_2}\right)\left(\left(1-\pi_1\right)\left(1-\pi_2\right)+\rho\right)\end{array}\right)\geq 0.\end{aligned}$$

Consequently, the inequality holds if and only if

$$p_2\leq\frac{1}{1+\frac{\pi_1}{1-\pi_1}\frac{1-\pi_2}{\pi_2}\frac{\pi_2(1-\pi_1)-\rho}{\pi_1(1-\pi_2)-\rho}\frac{1-p_1}{p_1}}.$$

Define $k\equiv\frac{\pi_1}{1-\pi_1}\times\frac{1-\pi_2}{\pi_2}\times\frac{(1-\pi_1)\pi_2-\rho}{\pi_1(1-\pi_2)-\rho}>0$. It is straightforward that $k<1$ if and only if $\rho>0$. Also, define a function $\delta:[0,1]\rightarrow[0,1]$ by $\delta\left(0\right)=0$, $\delta\left(1\right)=1$, and for $p\in(0,1)$,

$$\delta\left(p\right)\equiv\left(1+k\frac{1-p}{p}\right)^{-1}.$$

With these definitions, if $p_1 \in (0,1)$ or $p_2 \in (0,1)$, $\Pr(U_1 = u_1|p_1, p_2) \geq \Pr(U_2 = u_1|p_1, p_2)$ holds if and only if $p_2 \leq \delta(p_1)$.

It remains to consider $p_1, p_2 \in \{0,1\} \times \{0,1\}$. If $p_1 = 0$, then $\Pr(U_1 = u_1|p_1, p_2) = 0 \geq \Pr(U_2 = u_1|p_1, p_2)$ holds if and only if $p_2 = 0$. If $p_1 = 1$, $\Pr(U_1 = u_1|p_1, p_2) = 1 \geq \Pr(U_2 = u_1|p_1, p_2)$ holds for all $p_2 \in [0,1]$. Finally, if $p_2 = 1$, $\Pr(U_1 = u_1|p_1, p_2) \geq 1 = \Pr(U_2 = u_1|p_1, p_2)$ holds if and only if $p_1 = 1$. In sum, for all $p_1, p_2 \in [0,1]$, $\Pr(U_1 = u_1|p_1, p_2) \geq \Pr(U_2 = u_1|p_1, p_2)$ holds if and only if $p_2 \leq \delta(p_1)$.

The function δ is increasing and concave because

$$\frac{\partial \delta(p)}{\partial p} = \frac{\partial}{\partial p}\left(\left(1 + k\frac{1-p}{p}\right)^{-1}\right) = \frac{k}{(k(1-p)+p)^2} > 0; \text{ and}$$
$$\frac{\partial^2 \delta(p)}{\partial p^2} = \frac{\partial}{\partial p}\frac{k}{(k+p-kp)^2} = \frac{-2k(1-k)}{(k(1-p)+p)^3} < 0.$$

Furthermore, the function δ is increasing in ρ:

$$\frac{\partial \delta(p)}{\partial \rho} = \frac{\pi_1}{1-\pi_1}\frac{1-\pi_2}{\pi_2}\frac{\pi_1 - \pi_2}{(\pi_1(1-\pi_2)-\rho)^2}\frac{p(1-p)}{(k(1-p)+p)^2} > 0.$$

Finally, if $\rho = 0$, then $k = 1$ and $\delta(p) = p$. As $\rho \to (1-\pi_1)\pi_2$, then $k \to 0$ and $\delta(p)$ converges pointwise to $1_{\{p>0\}}$. Q.E.D.

Proof of Theorem 1

Necessary and Sufficient Condition for Linear Stucture: We extensively use the following two claims, which are direct implications of Corollary 2 of Kamenica and Gentzkow (2011). (a) A strategy G_i is a best response to G_j if and only if $E_{G_i}\left[\Pi_i(p; G_j)\right] = C\left(\Pi_i(\pi_i; G_j)\right)$, where $C\left(\Pi_i(p; G_j)\right)$ is the concave closure of Π_i.[29] (b) If G_i is a best response to G_j, then G_i assigns zero measure to $\Lambda \equiv \left\{p : C\left(\Pi_i(p; G_j)\right) > \Pi_i(p; G_j)\right\}$.

The sufficiency is straightforward. To see the necessity, suppose that (G_1, G_2) is a pair of equilibrium strategies. We first make the following preliminary observavtions.

Lemma 3 *(i) Suppose $G_j(q_1) - G_j(q_0) = 0$ and $G_j(q_1) < 1$. Then, $\Pi_i(p; G_j)$ is linear and weakly decreasing on $(\delta_j(q_0), \delta_j(q_1))$; (ii) Suppose (q_0, q_1) is an interval such that $G_j(\delta_i(q_0)) = \lim_{p \to q_1^-} G_j(\delta_i(p))$ and $G_j(\delta_i(q_1)) < G_j(\delta_i(q_2))$ for all $q_2 > q_1$. Then for all $p' \in (q_0, q_1)$ and $q_2 > q_1$, $\frac{d\Pi_i(p'; G_j)}{dp} < \frac{\Pi_i(q_2; G_j) - \Pi_i(q_0; G_j)}{q_2 - q_0}$.*

Proof. *(i) For each $p_i \in (\delta_j(q_0), \delta_j(q_1))$,*

$$\Pi_i(p_i; G_j) = (1 + \lambda(p_i)(\delta_i(p_i) - \pi_j))\, G_j(\delta_i(p_i)) - \frac{\rho(p_i - \pi_i)}{\pi_i(1-\pi_i)\pi_j(1-\pi_j)}\int_0^{\delta_i(p_i)} G_j(s)\, ds$$

[29] That is, $C(\Pi_i(p; G_j)) \equiv \sup\{z | (p, z) \in \mathrm{co}(\Pi_i)\}$, where $\mathrm{co}(\Pi_i)$ is the convex hull of the graph of Π_i.

and

$$\frac{d\Pi_i\left(p_i;G_j\right)}{dp_i} = \frac{\rho}{\pi_i\left(1-\pi_i\right)\pi_j\left(1-\pi_j\right)}\left(\left(\delta_i\left(p_i\right)-\pi_j\right)G_j\left(q_0\right)-\int_0^{\delta_i(p_i)}G_j\left(s\right)ds\right)$$
$$= \frac{\rho}{\pi_i\left(1-\pi_i\right)\pi_j\left(1-\pi_j\right)}\left(\left(q_0-\pi_j\right)G_j\left(q_0\right)-\int_0^{q_0}G_j\left(s\right)ds\right)\leq 0,$$

where $\lambda\left(p_i\right)\equiv\frac{\rho\left(p_i-\pi_i\right)}{\pi_i\left(1-\pi_i\right)\pi_j\left(1-\pi_j\right)}$*. Since* $\left(q_0-\pi_j\right)G_j\left(q_0\right)-\int_0^{q_0}G_j\left(s\right)ds$ *is constant, we have the required result.*

(ii) Notice that $D\equiv\frac{\Pi_i\left(q_2;G_j\right)-\Pi_i\left(q_0;G_j\right)}{q_2-q_0}-\frac{\Pi_i\left(q_1;G_j\right)-\Pi_i\left(q_0;G_j\right)}{q_1-q_0}$,

$$D=\frac{G_j\left(\delta_i\left(q_2\right)\right)-G_j\left(\delta_i\left(q_0\right)\right)}{q_2-q_0}\left(1+\frac{\rho\left(q_2-\pi_i\right)\left(\delta_i\left(q_2\right)-\pi_j\right)}{\pi_i\left(1-\pi_i\right)\pi_j\left(1-\pi_j\right)}\right)$$
$$-\frac{\rho\left(q_2-\pi_i\right)}{\pi_i\left(1-\pi_i\right)\pi_j\left(1-\pi_j\right)}\frac{\int_{\delta_i(q_0)}^{\delta_i(q_2)}\left(G_j\left(s\right)-G_j\left(\delta_i\left(q_0\right)\right)\right)ds}{q_2-q_0}.$$

where $D\equiv\frac{\Pi_i\left(q_2;G_j\right)-\Pi_i\left(q_0;G_j\right)}{q_2-q_0}-\frac{\Pi_i\left(q_1;G_j\right)-\Pi_i\left(q_0;G_j\right)}{q_1-q_0}$*. If* $q_2\leq\pi_i$*, then*

$$D\geq\frac{G_j\left(\delta_i\left(q_2\right)\right)-G_j\left(\delta_i\left(q_0\right)\right)}{q_2-q_0}\left(1+\frac{\rho\left(q_2-\pi_i\right)\left(\delta_i\left(q_2\right)-\pi_j\right)}{\pi_i\left(1-\pi_i\right)\pi_j\left(1-\pi_j\right)}\right)>0,$$

where the last inequality follows from the observation that $1+\frac{\rho\left(q_2-\pi_i\right)\left(\delta_i\left(q_2\right)-\pi_j\right)}{\pi_i\left(1-\pi_i\right)\pi_j\left(1-\pi_j\right)}>0$.[(30)] *If* $q_2>\pi_i$*, then*

$$D\geq\frac{G_j\left(\delta_i\left(q_2\right)\right)-G_j\left(\delta_i\left(q_0\right)\right)}{q_2-q_0}\left(1+\frac{\rho\left(q_2-\pi_i\right)\left(\delta_i\left(q_0\right)-\pi_j\right)}{\pi_i\left(1-\pi_i\right)\pi_j\left(1-\pi_j\right)}\right)>0$$

where the final inequality follows from the observation that $1+\frac{\rho\left(q_2-\pi_i\right)\left(\delta_i\left(q_0\right)-\pi_j\right)}{\pi_i\left(1-\pi_i\right)\pi_j\left(1-\pi_j\right)}>0$.[(31)] ■

Define $\bar{p}_i\equiv\sup\left(\operatorname{supp}G_i\right)$ and $\hat{p}_i\equiv\sup\left(\operatorname{supp}G_i\cap(0,1)\right)$. We show below the following properties.

Lemma 4 *(i)* $\bar{p}_j=\delta_i\left(\bar{p}_i\right)$*; (ii)* $G_i\left(p\right)$ *does not have an atom at any* $p\in(0,1)$ *and* $G_1\left(0\right)\times G_2\left(0\right)=0$*; (iii)* $G_i\left(p\right)$ *is strictly increasing in* $\left(0,\hat{p}_i\right)$*; and (iv)* $\hat{p}_j=\delta_i\left(\hat{p}_i\right)$.

Proof. ***(i)*** $\bar{p}_j=\delta_i\left(\bar{p}_i\right)$***:*** *Suppose* $\bar{p}_i>\delta_j\left(\bar{p}_j\right)$*. Observe that* $\Pi_i\left(p;G_j\right)=1$ *for all* $p\in\left(\delta_j\left(\bar{p}_j\right),1\right)$*. Suppose* $\pi_i>\delta_j\left(\bar{p}_j\right)$*. If such an equilibrium exists, then sender j's payoff is zero. However, sender j can always attain a positive payoff by full disclosure, a contradiction. Next, suppose* $\pi_i\leq\delta_j\left(\bar{p}_j\right)$*. Then it is straightforward to see that there exists a mean-preserving contraction* G_i' *of* G_i *such that* $G_i\left(p\right)>G_i'\left(p\right)$ *for all* $p\in\left[0,\delta_j\left(\bar{p}_j\right)\right]$*, and* $G_i'\left(p'\right)=1$ *at some* $p'\in\left(\delta_j\left(\bar{p}_j\right),\bar{p}_i\right)$*; and* $\int_0^{p'}\Pi_i\left(p;G_j\right)dG_i'>\int_0^{\bar{p}_i}\Pi_i\left(p;G_j\right)dG_i$*, a contradiction.*

(ii) no atom at any $p\in(0,1)$ ***and*** $G_1\left(0\right)\times G_2\left(0\right)=0$***:*** *Suppose* G_i *assigns an atom at* $\tilde{p}\in(0,1)$*. Then* $\Pi_i\left(\tilde{p};G_j\right)=C\left(\Pi_i\left(\tilde{p};G_j\right)\right)$*, and* Π_j *exhibits an upward jump at* $\delta_i\left(\tilde{p}\right)$*. Therefore, there eixists an* $\varepsilon>0$ *such*

[(30)]To see this, notice that the inequality obviously holds if $\left(q_2-\pi_i\right)\left(\delta_i\left(q_2\right)-\pi_j\right)\geq 0$. Now suppose $\left(q_2-\pi_i\right)\left(\delta_i\left(q_2\right)-\pi_j\right)<0$. As $\left(q_2-\pi_i\right)\left(\delta_i\left(q_2\right)-\pi_j\right)>-\pi_1\left(1-\pi_2\right)$ and $\rho\in\left[0,\pi_2\left(1-\pi_1\right)\right)$, we have $\pi_1\left(1-\pi_1\right)\pi_2\left(1-\pi_2\right)+\rho\left(q_2-\pi_i\right)\left(\delta_i\left(q_2\right)-\pi_j\right)>0$.

[(31)]Replacing $\delta_i\left(q_2\right)$ with $\delta_i\left(q_0\right)$ in the previous footnote gives this inequality.

that $\Pi_j(p;G_i) < C(\Pi_j(p;G_i))$ for all $p \in [\delta_i(\tilde{p}) - \varepsilon, \delta_i(\tilde{p})]$. Consequently, $G_j(\delta_i(\tilde{p})) - G_j(\delta_i(\tilde{p}) - \varepsilon) = 0$, and $\Pi_i(p;G_j)$ is weakly decreasing and linear on $[\delta_j(\delta_i(\tilde{p}) - \varepsilon), \tilde{p}]$ by Lemma 3-(i). Let $\Pi_i^L(p;G_j)$ be the linear function such that $\Pi_i^L(p;G_j) = \Pi_i(p;G_j)$ for all $p \in [\delta_j(\delta_i(\tilde{p}) - \varepsilon), \tilde{p}]$. Then, $\tilde{p} \in \operatorname{supp} G_i$, i.e., $\Pi_i(\tilde{p};G_j) = C(\Pi_i(\tilde{p};G_j))$, only if $\Pi_i(p;G_j) \leq \Pi_i^L(p;G_j)$ for all $p \in [\delta_j(\delta_i(\tilde{p}) - \varepsilon), 1]$. By Lemma 3-(ii), this implies $\bar{p}_j \leq \delta_i(\tilde{p})$. However, by the previous claim, we have $\bar{p}_j > \delta_i(\tilde{p})$, a contradiction. Next, suppose G_i assigns an atom at $\tilde{p} = 0$. Then, $\Pi_j(0;G_i) < C(\Pi_j(0;G_i))$. Therefore, $G_i(0) > 0$ implies $G_j(0) = 0$.

***(iii)** $G_i(p)$ **is strictly increasing on** $(0, \hat{p}_i)$**:** Suppose there is an interval (a, b) such that $G_i(b) = G_i(a)$, and $G_i(b') > G_i(b)$ for all $b' > b$. Observe first that $\hat{p}_i > b$ by the definition of $\hat{p}_i$. By Lemma 3-(i), $\Pi_j(p;G_i)$ is weakly decreasing on $(\delta_i(a), \delta_i(b))$, and $\delta_i(b) < \bar{p}_j$. Since G_i is continuous at $\delta_i(b)$ by claim (ii), Lemma 3-(ii) implies that there exists $\varepsilon > 0$ such that $\Pi_j(p;G_i) < C(\Pi_j(p;G_i))$ for all $p \in (\delta_i(a), \delta_i(b) + \varepsilon)$. Thus, G_j is constant on the interval $(\delta_i(a), \delta_i(b) + \varepsilon)$. Then, applying the same argument for $\Pi_i(p;G_j)$, we obtain that there exists an $\varepsilon' > 0$ such that $\Pi_i(p;G_j) < C(\Pi_i(p;G_j))$ for all $p \in (a, \delta_j(\delta_i(b) + \varepsilon) + \varepsilon')$, which contradicts the definition of b.*

***(iv)** $\hat{p}_j = \delta_i(\hat{p}_i)$**:** Suppose $\hat{p}_j > \delta_i(\hat{p}_i)$. By claim (i), $\bar{p}_j = \delta_i(\bar{p}_i)$. Therefore, $\delta_i(\bar{p}_i) > \delta_i(\hat{p}_i)$, or equivalently, $\bar{p}_i > \hat{p}_i$. Then, by the definition of $\hat{p}_i$, we have $\bar{p}_i = \bar{p}_j = 1$, and G_i assigns an atom at $\bar{p}_i$. Also, this implies that for all $p \in [\hat{p}_i, 1)$, $G_i(p) = G_i(\hat{p}_i)$. Then the argument similar to the one in claim (iii) implies that $\Pi_j(p_j;G_i) < C(\Pi_j(p_j;G_i))$ for all $p_j \in (\delta(\hat{p}_i), 1)$, i.e., $\hat{p}_j \leq \delta(\hat{p}_i)$, a contradiction.* ■

Now we argue that properties (i)-(iv) together imply the linear structure of the payoff functions. Take two arbitrary points $p, p' \in \operatorname{supp} G_i \setminus \{0\}$ such that $p' > p$. Suppose also that there exists $\alpha \in (0,1)$ such that $p'' = \alpha p + (1-\alpha) p' \in \operatorname{supp} G$. First, suppose that $\Pi_i(p'';G_j) < \alpha \Pi_i(p;G_j) + (1-\alpha)\Pi_i(p';G_j) \leq C(\Pi_i(p'';G_j))$. By the continuities of $\Pi_i(\cdot;G_j)$ and $C(\Pi_i(\cdot;G_j))$ on $(0,1)$, there exists an open interval $(p_-, p_+) \ni p''$ such that $\Pi_i(\tilde{p};G_j) < C(\Pi_i(\tilde{p};G_j))$ for all $\tilde{p} \in (p_-, p_+)$. Therefore, G_i is a best-response to G_j only if G_i assigns measure zero to (p_-, p_+). This however contradicts that G_i is strictly increasing on (p, p''). Next, suppose $\Pi_i(p'';G_j) > \alpha \Pi_i(p;G_j) + (1-\alpha)\Pi_i(p';G_j)$. Then if G_i is the best response of sender i, then it cannot assign positive measures to the neighbourhoods of all of p, p', and p'', contradicting that $p, p', p'' \in \operatorname{supp} G$. Q.E.D.

Proof of Lemma 2

Suppose (G_1, G_2) is an equilibrium. By Theorem 1, $\Pi_i(p_i;G_j)$ is linear on the interval $(0, \hat{p}_i]$. Therefore,

$$\frac{d^2 \Pi_i(p_i;G_j)}{dp_i^2} = \left(1 + \frac{\rho(p_i - \pi_i)(\delta_i(p_i) - \pi_j)}{\pi_i(1-\pi_i)\pi_j(1-\pi_j)}\right) \frac{d^2(G_j(\delta_i(p_i)))}{dp_i^2} + \left(\frac{2\rho(\delta_i(p_i) - \pi_j) + \rho(p_i - \pi_i)\delta_i'(p_i)}{\pi_i(1-\pi_i)\pi_j(1-\pi_j)}\right) \frac{d(G_j(\delta_i(p_i)))}{dp_i} = 0.$$

The solution to the differential equation above is

$$G_j(p_i) = G_j(0) + C_j \int_0^{\delta_j(p_i)} \exp\left(\int_0^{s'} \Lambda_j(s)\, ds\right) ds',$$

for some integration constant C_j and $\Lambda_j(s) \equiv -\frac{\rho\left((s-\pi_i)\delta_i'(s)+2\left(\delta_i(s)-\pi_j\right)\right)}{\pi_1\pi_2(1-\pi_1)(1-\pi_2)+\rho(s-\pi_i)\left(\delta_i(s)-\pi_j\right)}$. Substituting $p_i = \hat{p}_i$ in the solution above, we get

$$C_j = \frac{G_j(\hat{p}_j) - G_j(0)}{\int_0^{\delta_j(\hat{p}_j)} \exp\left(\int_0^{s'} \Lambda_j(s)\, ds\right) ds'}.$$

Equation (5) is obtained by substituting the integration constant to the solution above.

Next, using equation (5), the Bayes-plausibility condition for sender j can be simplified as follows.

$$\begin{aligned} 1 - \pi_j &= \int_0^{\delta_i(\hat{p}_i)} G_j(p)\, dp + G_j(\delta_i(\hat{p}_i))(1 - \delta_i(\hat{p}_i)) \\ &= \delta_i(\hat{p}_i) G_j(0) + \left(G_j(\hat{p}_j) - G_j(0)\right) \frac{\int_0^{\delta_i(\hat{p}_i)} \int_0^{\delta_j(p)} \exp\left(\int_0^{s'} \Lambda_j(s)\, ds\right) ds' dp}{\int_0^{\delta_j(\hat{p}_j)} \exp\left(\int_0^{s'} \Lambda_j(s)\, ds\right) ds'} + G_j(\delta_i(\hat{p}_i))(1 - \delta_i(\hat{p}_i)) \\ &= G_j(\hat{p}_j) - \left(G_j(\hat{p}_j) - G_j(0)\right) T_j(\hat{p}_i). \end{aligned}$$

This gives the simplified Bayes-plausibility condition. The other conditions follow immediately from Theorem 1. Q.E.D.

Proof of Theorem 2

Equilibrium Existence: The strategy space is compact (with respect to weak*-topology). The payoff $V_i(G_i, G_j) \equiv E_{G_i}\left[\Pi_i(p; G_j)\right]$ of sender i is linear in G_i, and hence is quasiconcave in G_i. The game is zero-sum, i.e., $V_1(G_1, G_2) + V_2(G_2, G_1) = 1$, and hence satisfies reciprocal upper-semicontinuity. Therefore, if we show that the payoff function satisfies payoff security, Corollary 3.3 of Reny (1999) guarantees the existence of a pure-strategy equilibrium. Fix an arbitrary strategy profile (G_i, G_{-i}) and $\varepsilon > 0$.

We show below that there exists a strategy $\tilde{G}_i$ of sender i that is continuous on $[0, 1)$ and $V_i\left(\tilde{G}_i, G_{-i}\right) > V_i(G_i, G_{-i}) - \varepsilon/2$. Note that the set of discontinuous points $D \subset [0, 1]$ of G_i is countable. We thus can denote $D = \{d_1, d_2, \cdots, d_l, \cdots\}$, where $d_l < d_{l+1}$. Let t_l be the size of atom at d_l. First, suppose $\Pi_i(p_i; G_{-i})$ is continuous at d_l. Then there exists an interval $(d_l - \varepsilon_l, d_l + \varepsilon_l)$ such that $\Pi_i(p_i; G_{-i}) > \Pi_i(d_l; G_{-i}) - \left(\frac{\varepsilon}{2+\varepsilon}\right)^l$ for all $p_i \in (d_l - \varepsilon_l, d_l + \varepsilon_l)$. Now replacing the atom t_l at d_l with a uniform distribution over the interval $(d_l - \varepsilon_l, d_l + \varepsilon_l)$ gives a new distribution G_i' such that G_i' does not have an atom at d_l and $V_i\left(G_i', G_{-i}\right) > V_i(G_i, G_{-i}) - \left(\frac{\varepsilon}{2+\varepsilon}\right)^l$. Next suppose $\Pi_i(p_i; G_{-i})$ is discontinuous at d_l. Recall that while $\Pi_i(p_i; G_{-i})$ may be decreasing, it cannot jump downwards. Thus, it is necessary that $\lim_{p_i \to d_l^-} \Pi_i(p_i; G_{-i}) < \Pi_i(d_l; G_{-i}) < \lim_{p_i \to d_l^+} \Pi_i(p_i; G_{-i})$. Choose a pair $\varepsilon_l, \varepsilon_l' > 0$ such that (i) for all $p' \in (d_l, d_l + \varepsilon_l)$, $\Pi_i(d_l; G_{-i}) < \Pi_i(p'; G_{-i})$, and (ii) $\frac{\varepsilon_l'}{\varepsilon_l} > t_l\left(\frac{2+\varepsilon}{\varepsilon}\right)^l - 1$ and (iii) $d_l - \varepsilon_l' > 0$. Replace the atom at d_l with a uniform distribution with density $\frac{\varepsilon_l}{\varepsilon'_l(\varepsilon_l+\varepsilon'_l)}t_l$ over the interval $\left(d_l-\varepsilon'_l, d_l\right)$, as well as a uniform distribution with density $\frac{\varepsilon'_l}{\varepsilon_l(\varepsilon_l+\varepsilon'_l)}t_l$ over the interval $(d_l, d_l+\varepsilon_l)$.(32) The new strategy G'_i thus obtained has no atom at d_l. Moreover, as the additional mass assigned to the interval $\left(d_l-\varepsilon'_l, d_l\right)$ is $\frac{\varepsilon_l}{\varepsilon'_l(\varepsilon_l+\varepsilon'_l)}t_l \times \varepsilon'_l < \left(\frac{\varepsilon}{2+\varepsilon}\right)^l$, we have $V_i\left(G'_i, G_{-i}\right) > V_i\left(G_i, G_{-i}\right) - \left(\frac{\varepsilon}{2+\varepsilon}\right)^l$.

Next, as $\tilde{G}_i$ is continuous on $[0,1)$, $V_i\left(\tilde{G}_i, G'_{-i}\right)$ is lower semicontinuous with respect to G'_{-i}.(33) Consequently, there exists a neighborhood $O\left(G_{-i}\right)$ of G_{-i} such that for all $G'_{-i} \in O\left(G_{-i}\right)$, $V_i\left(\tilde{G}_i, G'_{-i}\right) > V_i\left(\tilde{G}_i, G_{-i}\right) - \varepsilon/2$. We thus have the payoff security at $\left(G_i, G_{-i}\right)$.

Equilibrium Uniqueness: Suppose there are two equilibria $\left(G_1, G_2\right)$ and $\left(G'_1, G'_2\right)$. Define $\hat{p}_i$ and $\hat{p}'_i$, as well as $\bar{p}_1$ and $\bar{p}'_1$ accordingly. Then, by the interchangeability of zero-sum games, $\left(G_1, G'_2\right)$ and $\left(G'_1, G_2\right)$ are also equilibria. As a result, G_1 and G'_1 have a common support, i.e., $\hat{p}_1 = \hat{p}'_1$ and $\bar{p}_1 = \bar{p}'_1$. Similarly, G_2, and G'_2 have a common support, i.e., $\hat{p}_2 = \hat{p}'_2$ and $\bar{p}_2 = \bar{p}'_2$. In other words, the support of equilibrium startegies are unique. Finally, we explain why this, together with Bayes-plaubsibility and the necessity of the linear structure of equilibrium strategy uniquely pins down the equilibrium strategy. If $G_i\left(\hat{p}_i\right) = 1$ (and hence $G_j\left(\hat{p}_j\right) = 1$), then the simplified Bayes-plausibility condition in Lemma 2 implies that $G_i\left(0\right) = G'_i\left(0\right)$ and $G_j\left(0\right) = G'_j\left(0\right)$. Next, suppose $G_1\left(\hat{p}_1\right) < 1$. For each $i \in \{1,2\}$, fixing a $\hat{p}_i$, the simplified Bayes-plausibility condition and the atom condition gives a system of two equations in two unknowns ($G_i\left(0\right)$ and $G_i\left(\hat{p}_i\right)$). It is straightforward to verify that there exists a unique solution to the system, so $G_i\left(0\right) = G'_i\left(0\right)$ and $G_i\left(\hat{p}_i\right) = G'_i\left(\hat{p}_i\right)$. Q.E.D.

Proof of Theorem 3

Since the game is zero-sum and symmetric, Theorem 2 implies that the equilibrium is unique and symmetric. Moreover, $\pi_1 = \pi_2 = \pi$ implies that $\delta_1\left(p\right) = \delta_2\left(p\right) = p$. Therefore, $\Lambda\left(s\right) = -\frac{3\rho(s-\pi)}{\pi^2(1-\pi)^2+\rho(s-\pi)^2}$ and $T\left(\hat{p}\right) = \frac{\int_0^{\hat{p}} x \exp\left(\int_0^x \Lambda(s)ds\right)dx}{\int_0^{\hat{p}} \exp\left(\int_0^x \Lambda(s)ds\right)dx}$. By Lemma 4, $G\left(0\right) = 0$ in a symmetric equilibrium. By by Lemma 2, in an equilibrium such that $G\left(\hat{p}\right) = 1$, we have

$$\hat{p} = 2\pi; \text{ and } G\left(p\right) = \min\left\{\frac{1}{2} + \frac{\sqrt{\left(1-\pi\right)^2+\rho}}{2}\frac{p-\pi}{\sqrt{\pi^2\left(1-\pi\right)^2+\rho\left(p-\pi\right)^2}}, 1\right\}. \tag{16}$$

Also, in an equilibrium such that $G\left(\hat{p}\right) < 1$, we have $\pi\Pi\left(1\right) = \frac{1}{2}$ by the linear structure of the payoff

(32)The choice of densities ensure that Bayes-plausibility is preserved.

(33)To see this, define $W\left(p_{-i}\right)$ as the probability that sender i wins by using strategy $\tilde{G}_i$, conditional on the posterior realization of the rival sender being $p_{-i} \in \triangle\Omega$. By definition, $V_i\left(\tilde{G}_i, G_{-i}\right) = \int W\left(p_{-i}\right) dG_{-i}\left(p_{-i}\right)$. As $\tilde{G}_i$ is continuous on $[0,1)$, $W\left(p_{-i}\right)$ is lower-semicontinuous in p_{-i}. Therefore, by the Portmanteau theorem, for every sequence $\left\{G^k_{-i}\right\}_{k\in\mathbb{N}}$ that converges in weak* topology to G_{-i}, we have $\liminf V_i\left(\tilde{G}_i, G^k_{-i}\right) \geq V_i\left(\tilde{G}_i, G_{-i}\right)$. That is, $V_i\left(\tilde{G}_i, G'_{-i}\right)$ is lower semicontiuous with respect to G'_{-i}.

function. Then, Lemma 4, we obtain

$$\hat{p} = \frac{2\pi^2\left((1-\pi)^2+\rho\right)}{\pi^2(1-\pi)+\rho(3\pi-1)}; \text{ and } G(p) = \begin{cases} \frac{1}{2}+\frac{\sqrt{(1-\pi)^2+\rho}}{2}\frac{\min\{p,\hat{p}\}-\pi}{\sqrt{\pi^2(1-\pi)^2+\rho(\min\{p,\hat{p}\}-\pi)^2}} & \text{if } p\in[0,1) \\ 1 & \text{if } p=1 \end{cases}. \tag{17}$$

Let $G_{\tilde{\rho}}(p)$ be the equilibrium strategy when $\rho=\tilde{\rho}$. Similarly, define $\hat{p}_{\tilde{\rho}} \equiv \sup\left(\operatorname{supp} G_{\tilde{\rho}} \cap (0,1)\right)$. Take a pair $\rho,\rho' \in [0,\pi(1-\pi)]$ with $\rho' > \rho$.

First, suppose $\pi \leq 1/2$. By 16, for $p < 2\pi$,

$$\frac{\partial}{\partial\rho}G(p) = \frac{(1-\pi)^2}{4\sqrt{(1-\pi)^2+\rho}}\frac{(\pi-p)\,p\,(2\pi-p)}{\left(\pi^2(1-\pi)^2+\rho(p-\pi)^2\right)^{\frac{3}{2}}}. \tag{18}$$

Thus, $G_\rho(p) > G_{\rho'}(p)$ for all $p\in(0,\pi)$, and $G_\rho(p) < G_{\rho'}(p)$ for all $p\in(\pi,1)$. We thus have $G_{\rho'}(\pi) \prec G_\rho(\pi)$.

Next, suppose $\pi > 1/2$. Observe that both $\hat{p}$ and $G(\hat{p})$, as given by equations (17), are increasing in ρ. Moreover, for $p<\hat{p}$, $\frac{\partial G(p)}{\partial\rho}$ is given by equation (18) above. Therefore, if $\hat{p}_\rho \geq \pi$, then $G_\rho(p) > G_{\rho'}(p)$ for all $p\in(0,\pi)$ and $G_\rho(p) < G_{\rho'}(p)$ for all $p\in(\pi,1)$. On the other hand, if $\hat{p}_\rho < \pi$, then $G_\rho(p) > G_{\rho'}(p)$ for all $p\in\left(0,\hat{p}_\rho\right)$. We thus have $G_{\rho'}(\pi) \prec G_\rho(\pi)$. That $G(p)$ does not converge to $G_N(p)$ in distribution as $\rho\to\bar{\rho}$ is straightforward. Q.E.D.

Proof of Theorem 4

In a game in which the covariance of proposal qualities is ρ, denote by $\Pi_i^\rho\left(p_i;G_j\right)$ the payoff function of sender i, denote by $G_{i,\rho}$ the equilibrium strategy of sender i, and denote by $\delta_i^\rho(p)$ the transformation function identified in Lemma 1. Define $\hat{p}_i^\rho \equiv \sup\left(supp\left(G_{i,\rho}\right)\setminus\{1\}\right)$.

We first show that $G_{1,\rho}\left(\hat{p}_1^\rho\right) < 1$ for ρ sufficiently close to $\bar{\rho}$. Suppose not. Then there exists a sequence $\{\rho_n\}$ such that $\lim_{n\to\infty}\rho_n = \bar{\rho}$, and for all n, $G_{1,\rho_n}\left(\hat{p}_1^{\rho_n}\right) = \Pi_2^{\rho_n}\left(1;G_{1,\rho_n}\right) = 1$. Now for an arbitrary pair $\varepsilon_1,\varepsilon_2>0$, and for $p_2 > 1-\varepsilon_1$, there exists an $\bar{n}$ such that $n>\bar{n}_1$ implies that

$$\Pi_2^{\rho_n}\left(p_2;G_{1,\rho_n}\right) = \left(1+\frac{\rho_n\left(p_2-\pi_2\right)\left(\delta_2^{\rho_n}\left(p_2\right)-\pi_1\right)}{\pi_1\left(1-\pi_1\right)\pi_2\left(1-\pi_2\right)}\right)G_{1,\rho_n}\left(\delta_2^{\rho_n}\left(p_2\right)\right) - \frac{\rho_n\left(p_2-\pi_2\right)}{\pi_1\left(1-\pi_1\right)\pi_2\left(1-\pi_2\right)}\int_0^{\delta_2^{\rho_n}(p_2)} G_{1,\rho_n}(s)\,ds$$

is bounded from above by $\varepsilon_2 \times G_{1,\rho_n}\left(\delta_2^{\rho_n}\left(p_2\right)\right) \leq \varepsilon_2$. However, since $G_{1,\rho_n}\left(\hat{p}_1^{\rho_n}\right) = 1$, we have $\Pi_2^{\rho_n}\left(1;G_{1,\rho_n}\right) < \Pi_2^{\rho_n}\left(p_2;G_{1,\rho_n}\right)/p_2 < 1$, a contradiction.

Next, let $\{\rho_n\}$ be a sequence such that $\lim_{n\to\infty}\rho_n=\bar{\rho}$, and $\lim_{n\to\infty}\hat{p}_2^{\rho_n}$ exists. We show that $\lim_{n\to\infty}\hat{p}_2^{\rho_n} < 1$ and $\lim_{n\to\infty}\hat{p}_1^{\rho_n} = 0$. If $\lim_{n\to\infty}\hat{p}_2^{\rho_n}=1$, then by the argument of the preceding paragraph, for any $\varepsilon_1>0$

and $\varepsilon_2 > 0$; and $p_2 > 1 - \varepsilon_1$, there exists $\bar{n}_2$ such that $n > \bar{n}_2$ implies $\hat{p}_2^{\rho_n} > p_2$ and $\Pi_2^{\rho_n}\left(p_2; G_{1,\rho_n}\right) < \varepsilon_2$. Notice however that $\Pi_2^{\rho_n}\left(1; G_{1,\rho_n}\right) \geq 1/2$. Therefore, the definition of $\hat{p}_2^{\rho_n}$ and the atom condition at the top together imply that for $\varepsilon_1, \varepsilon_2$ sufficiently small and n sufficiently large,

$$\Pi_2^{\rho_n}\left(0; G_{1,\rho_n}\right) \leq \frac{\Pi_2^{\rho_n}\left(p_2; G_{1,\rho_n}\right) - p_2 \Pi_2^{\rho_n}\left(1; G_{1,\rho_n}\right)}{1 - p_2} < \frac{\varepsilon_2 - p_2/2}{1 - p_2} < 0,$$

a contradiction. Furthermore, since $\hat{p}_1^{\rho_n} = \delta_2^{\rho_n}\left(\hat{p}_2\right)$, we have $\lim_{n\to\infty} \hat{p}_1^{\rho_n} = 0$.

We are ready to show that G_{1,ρ_n} converges to $G_{1,F}$ in distribution. Observe that $T_1\left(\hat{p}_2^{\rho_n}\right) < \hat{p}_1^{\rho_n}$. Therefore, the simplified Bayes-plausibility condition for sender 1 implies $1 - \pi_1 \geq G_{1,\rho_n}\left(\hat{p}_1^{\rho_n}\right) - \left(G_{1,\rho_n}\left(\hat{p}_1^{\rho_n}\right) - G_{1,\rho_n}(0)\right)\hat{p}_1^{\rho_n}$. Therefore, $1 - \pi_1 \geq \lim_{n\to\infty} G_{1,\rho_n}\left(\hat{p}_1^{\rho_n}\right)$. At the same time, the simplified Bayes-plausibility condition implies $1 - \pi_1 \leq G_{1,\rho_n}\left(\hat{p}_1^{\rho_n}\right)\left(1 - T_1\left(\hat{p}_2^{\rho_n}\right)\right) \leq G_{1,\rho_n}\left(\hat{p}_1^{\rho_n}\right)$. Therefore, $\lim_{n\to\infty} G_{1,\rho_n}\left(\hat{p}_1^{\rho_n}\right) = 1 - \pi_1$, and G_{1,ρ_n} converges to $G_{1,F}$ in distribution.

We now show that G_{2,ρ_n} converges to $G_{2,F}$ in distribution. We first show that $G_{1,\rho}(0) = 0$ for all ρ sufficiently close to $\bar{\rho}$. Suppose not. Then there exists a sequence $\{\rho_n\}$ such that $\lim_{n\to\infty} \rho_n = \bar{\rho}$ and $G_{1,\rho_n}(0) > 0$ for all n. For each $\varepsilon_3 \in (0, \pi_1), \varepsilon_4 > 0$, and $p_1 \in (0, \varepsilon_3)$, there exists an $\bar{n}_3$ such that $n > \bar{n}_3$ implies that

$$\Pi_1^{\rho_n}\left(p_1; G_{2,\rho_n}\right) = \left(1 + \frac{\rho_n\left(p_1 - \pi_1\right)\left(\delta_1^{\rho_n}\left(p_1\right) - \pi_2\right)}{\pi_1\left(1 - \pi_1\right)\pi_2\left(1 - \pi_2\right)}\right) G_{2,\rho_n}\left(\delta_1^{\rho_n}\left(p_1\right)\right) - \frac{\rho_n\left(p_1 - \pi_1\right)}{\pi_1\left(1 - \pi_1\right)\pi_2\left(1 - \pi_2\right)} \int_0^{\delta_1^{\rho_n}(p_1)} G_{2,\rho_n}(s)\, ds$$

is bounded from below by $1 - \varepsilon_4$. However, $G_{1,\rho_n}(0) > 0$ implies $G_{2,\rho_n}(0) = 0$ and hence $\Pi_1^{\rho_n}\left(0; G_{2,\rho_n}\right) = 0$. Furthermore, $\Pi_1^{\rho_n}\left(1; G_{2,\rho_n}\right) \geq 1/2$. Therefore, $\Pi_1^{\rho_n}\left(0; G_{2,\rho_n}\right) < \Pi_1^{\rho_n}\left(p_1; G_{2,\rho_n}\right)$ and $\Pi_1^{\rho_n}\left(0; G_{2,\rho_n}\right) < \Pi_1^{\rho_n}\left(1; G_{2,\rho_n}\right)$. Therefore, $0 \notin \operatorname{supp} G_{1,\rho_n}$, a contradiction to $G_{1,\rho_n}(0) > 0$.

It remains to show $\hat{p}_2^{\rho} \to 0$ as $\rho \to \bar{\rho}$. Then an argument similar to that for the convergence of G_1 can establish that $G_{2,\rho}$ converges to to $G_{2,F}$ in distribution. To see that $\lim_{\rho\to\bar{\rho}} \hat{p}_2^{\rho} = 0$, suppose there exists a sequence $\{\rho_n\}$ that converges to $\bar{\rho}$ and $\lim_{n\to\infty} \hat{p}_2^{\rho_n} = \tilde{p}_2 \in (0,1)$. Note that we have already established that $G_{1,\rho_n}(0) = 0$ for all n sufficiently large. Thus, for sufficiently large n, the atom condition at the top, as well as that $\Pi_2^{\rho}\left(1; G_{1,\rho_n}\right) \geq 1/2$, implies $\Pi_2^{\rho_n}\left(\tilde{p}_2; G_{1,\rho_n}\right) \geq \tilde{p}_2/2$. On the other hand, $\lim_{n\to\infty} p_1^{\rho_n} = 0$ and $\lim_{n\to\infty} G_{1,\rho_n}\left(\delta_2^{\rho_n}\left(\tilde{p}_2\right)\right) = 0$. Therefore, for any ε_5, there exists an $\bar{n}_4$ such that $n > \bar{n}_4$ implies $\Pi_2^{\rho_n}\left(\tilde{p}_2; G_{1,\rho_n}\right) \leq \frac{1-\tilde{p}_2}{1-\pi_2}\varepsilon_5$, which is a contradiction to $\Pi_2^{\rho_n}\left(\tilde{p}_2; G_{1,\rho_n}\right) \geq \tilde{p}_2/2$.

Lastly, since $\lim_{\rho\to\bar{\rho}} \Pi_1^{\rho}\left(p_1; G_{2,\rho}\right) = 1 - \frac{\pi_2}{2\pi_1} p_1$, $\lim_{\rho\to\bar{\rho}} \Pi_1^{\rho}\left(\pi_1\right) = 1 - \frac{\pi_2}{2}$. Q.E.D.

Proof of Theorem 5

The necessity and the sufficiency of the linear structure of the payoff function for a symmetric equilibrium, as well as the existence of a symmetric equilibrium, can be obtained through a straightforward modification of the proof of Theorem 1, and thus omitted.

To see the uniqueness of the symmetric equilibrium G, notice that $\Pi(p,G) = G^{N-1}(p)$ for $p \in [0,1)$, $\Pi(1,G) = \frac{1-G^N(\hat{p})}{N(1-G(\hat{p}))}$. Therefore, for $p \in [0,\hat{p}]$, we have $G(p) = \left(\frac{p}{\hat{p}} G^{N-1}(\hat{p})\right)^{\frac{1}{N-1}}$. The Bayes-plausibility condition implies

$$\frac{\hat{p}}{N} G(\hat{p}) + 1 - G(\hat{p}) - \pi = 0. \tag{19}$$

Furthermore, the linear structure of Π requires that $G(\hat{p}) < 1$ implies $\frac{1-G^N(\hat{p})}{N(1-G(\hat{p}))} = \frac{G^{N-1}(\hat{p})}{\hat{p}}$, or equivalently,

$$\hat{p} = \frac{N G^{N-1}(\hat{p})(1-G(\hat{p}))}{1-G^N(\hat{p})}. \tag{20}$$

We consider two cases: $N \le 1/\pi$ and $N > 1/\pi$. First, when $N \le 1/\pi$, there exists an equilibirum such that $G(\hat{p}) = 1$ and $\hat{p} = N\pi$. For notational simplicity, we use G_1 to denote this equilbrium strategy. Suppose there exists another equilibrium G_2. Then there exists a pair of $\hat{p} \in (0,1)$ and $G(\hat{p}) \in (0,1)$ that simultaneously solves (19) and (20). Observe that $G_2(\hat{p}) > G_1(\hat{p})$. Otherwise, G_2 first-order stochastically dominates G_1. Notice that $G_2(\hat{p}) > G_1(\hat{p})$ implies $\hat{p} < N\pi$ and $\frac{\Pi(\hat{p},G_2)}{\hat{p}} > \frac{1}{N\pi}$. However, these together imply $\Pi(N\pi, G_2) = \frac{\Pi(\hat{p},G_2)}{\hat{p}} \times N\pi > 1$, a contradiction.

Next, suppose $N > 1/\pi$. It is straightforward to see that there exists no equilibrium such that $G(\hat{p}) = 1$ and $\hat{p} \in [0,1]$. Suppose there exist $(\hat{p}_1, G_1(\hat{p}_1))$ and $(\hat{p}_2, G_2(\hat{p}_2))$ that solve (19) and (20). Without loss of generality, we assume $\frac{G_1^{N-1}(\hat{p}_1)}{\hat{p}_1} \le \frac{G_2^{N-1}(\hat{p}_2)}{\hat{p}_2}$. First, suppose that $\frac{G_1^{N-1}(\hat{p}_1)}{\hat{p}_1} = \frac{G_2^{N-1}(\hat{p}_2)}{\hat{p}_2}$ and $\hat{p}_1 < \hat{p}_2$. Then, $G_1^{N-1}(\hat{p}_1) < G_2^{N-1}(\hat{p}_2)$. Therefore, $\Pi(1;G_1) = \frac{1-G_1^N(\hat{p})}{N(1-G_1(\hat{p}))} < \frac{1-G_2^N(\hat{p})}{N(1-G_2(\hat{p}))} = \Pi(1;G_1)$. This contradicts that $\Pi(1;G_1) = \frac{G_1^{N-1}(\hat{p}_1)}{\hat{p}_1} = \frac{G_2^{N-1}(\hat{p}_2)}{\hat{p}_2} = \Pi(1;G_2)$. Next, suppose $\frac{G_1^{N-1}(\hat{p}_1)}{\hat{p}_1} < \frac{G_2^{N-1}(\hat{p}_2)}{\hat{p}_2}$ and $G_1^{N-1}(\hat{p}_1) \le G_2^{N-1}(\hat{p}_2)$. Then G_1 first-order stochastically dominates G_2, a contradiction. Lastly, suppose $\frac{G_1^{N-1}(\hat{p}_1)}{\hat{p}_1} < \frac{G_2^{N-1}(\hat{p}_2)}{\hat{p}_2}$ and $G_1^{N-1}(\hat{p}_1) > G_2^{N-1}(\hat{p}_2)$. Then, since $\frac{1-g^N}{N(1-g)}$ is increasing in g, $\frac{1-G_1^N(\hat{p}_1)}{N(1-G_1(\hat{p}))} > \frac{1-G_2^N(\hat{p}_1)}{N(1-G_2(\hat{p}))}$. However, the linear structure of the payoff functions implies $\frac{G_1^{N-1}(\hat{p}_1)}{\hat{p}_1} = \frac{1-G_1^N(\hat{p}_1)}{N(1-G_1(\hat{p}))} > \frac{1-G_2^N(\hat{p}_1)}{N(1-G_2(\hat{p}))} = \frac{G_2^{N-1}(\hat{p}_2)}{\hat{p}_2}$, which is a contradiction. Q.E.D.

Proof of Theorem 6

Consider a pair of integers N_1 and $N_2 > N_1$. In light of the proof of Theorem 5, we start with $N_1 > 1/\pi$. In this case, there exists a $\hat{p}_N \in (0,1)$ such that $G_N^{N-1}(p)$, $N = N_1, N_2$, is linear on $p \in [0, \hat{p}_N]$. For notational simplicity, let $g_N \equiv G_N(\hat{p}_N)$. We show that $g_{N_1} > g_{N_2}$, $\hat{p}_{N_1} > \hat{p}_{N_2}$ and $\Pi(1, G_{N_1}) > \Pi(1, G_{N_2})$.

By (19) and (20), we obtain $\pi = \frac{1-g_N}{1-g_N^N}$. Since $\frac{1-x}{1-x^N}$ is decreasing in both x and N, we have $g_{N_1} > g_{N_2}$. Furthermore, observe that $\frac{Nx^{N-1}(1-x)}{1-x^N}$ is decreasing in N and increasing in x. Therefore, that g_N is decreasing in N and (20) together imply $\hat{p}_{N_1} > \hat{p}_{N_2}$.[34] Lastly, since $\frac{1-g^N}{N(1-g)}$ is decreasing in N, $\Pi(1, G_{N_1}) =$

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[34]Since $\frac{\partial(1-x^n+n\ln x)}{\partial x} = n\frac{1-x^n}{x} > 0$ for $x \in (0,1]$, $\frac{\partial\left(\frac{nx^{n-1}(1-x)}{1-x^n}\right)}{\partial n} = (1-x)\, x^{n-1} \frac{1-x^n+n\ln x}{(1-x^n)^2} < 0$. Furthermore, $\frac{\partial(n(1-x)-(1-x^n))}{\partial x} =$

$\frac{1-g_{N_1}^{N_1}}{N_1(1-g_{N_1})} > \frac{1-g_{N_1}^{N_2}}{N_2(1-g_{N_1})} > \frac{1-g_{N_2}^{N_2}}{N_2(1-g_{N_2})} = \Pi\left(1, G_{N_2}\right)$.

Therefore, on the interval $\left[0, \hat{p}_{N_1}\right]$, while $G_{N_1}^{N_1-1}(p)$ is linear, $G_{N_2}^{N_1-1}(p)$ is concave. Also, $G_{N_1}^{N_1-1}(0) = G_{N_2}^{N_1-1}(0)$ and $G_{N_1}^{N_1-1}\left(\hat{p}_{N_1}\right) = g_{N_1}^{N_1-1} > g_{N_2}^{N_1-1} = G_{N_2}^{N_1-1}\left(\hat{p}_{N_2}\right) = G_{N_2}^{N_1-1}\left(\hat{p}_{N_1}\right)$. If $G_{N_1}^{N_1-1}(p) > G_{N_2}^{N_1-1}(p)$ for all $p \in \left(0, \hat{p}_{N_2}\right)$, then G_{N_2} first-order stochastically dominates G_{N_1}. This implies $G_{N_2} \succ G_{N_1}$.

The cases where $N_2 \leq 1/\pi$ and $N_1 \leq 1/\pi < N_2$ can be shown in a similar manner, and hence omitted.

Lastly, for $N > 1/\pi$, g_N is decreasing in N and $\pi = \frac{1-g_N}{1-g_N^N}$. Therefore, $\lim_{N\to\infty} g_N = 1 - \pi$. Then by the Bayes-plausibility condition, we have $\lim_{N\to\infty} G_N(p) = 1 - \pi$ for all $p \in (0, 1)$. Q.E.D.

Proof of Theorem 7

The sufficiency is straightforward. We thus only show the necessity. We first introduce a few notations. For $u \in [u_0, u_{M-1}]$, define $P(u) \equiv \left\{p \in \Delta\Omega : E_p[U_i] = u\right\}$, and $G(A) \equiv \int_A dG$ for any set $A \subset \Delta\Omega$. We use $\tilde{\pi}_m \in \Delta\Omega$ to denote a degenerate posterior p such that $p_m = 1$. We also define $\hat{u} = \sup\left\{u : u = E_p[U_i], p \in (\operatorname{supp} G) \setminus \tilde{\pi}_{M-1}\right\}$; and $\bar{u} = \sup\left\{u : u = E_p[U_i], p \in \operatorname{supp} G\right\}$.

By Corollary 2 of Kamenica and Gentzkow (2011): (a) a strategy G is a best response to payoff function $\Pi_{post}(p; G)$ if and only if $E_G\left[\Pi_{post}(p; G)\right] = C\left(\Pi_{post}(\pi; G)\right)$, where $C\left(\Pi_{post}(p; G)\right)$ is the concave closure of Π. (b) If G is a best response to $\Pi_{post}(p; G)$, then G assigns a zero measure to the set x$\left\{p : C\left(\Pi_{post}(p; G)\right) > \Pi_{post}(p; G)\right\}$.

First, we show that F_G does not have atom at any $u \in [u_0, u_{M-1})$. Suppose F_G has an atom at some $u \in [u_0, u_{M-1})$. Then $\Pi_{post}(p; G) < C\left(\Pi_{post}(p; G)\right)$ for all $p \in P(u)$. This contradicts that G assigns an atom at some $p \in P(u)$.

Next, there exists an $\hat{u} \leq u_{M-1}$ such that $F_G(u)$ is increasing on $[u_0, \hat{u})$. Let (u_l, u_h), where $u_h < \hat{u}$, be a maximal open interval in $[u_0, u_{M-1}]$ such that $F_G(u_h) - F_G(u_l) = 0$. Let $\varepsilon > 0$ and we construct a strategy G_ε as follows. Define $x(\varepsilon)$ by the solution to the following equation:

$$u_l = \frac{\int_{u_l - x(\varepsilon)}^{u_l} u dF_G(u) + \int_{u_h}^{u_h+\varepsilon} u dF_G(u)}{F_G(u_h+\varepsilon) - F_G(u_h) + F_G(u_l) - F_G(u_l - x(\varepsilon))}. \tag{21}$$

Strategy G_ε modifies the equilibrium strategy G by combining weights on the intervals $[u_h, u_h+\varepsilon]$ and $[u_l - x(\varepsilon), u_l]$ to form an atom at u_l. The profit of adopting strategy G_ε exceeds that of G by at least

$$\begin{aligned}&\left(1 - \frac{1}{N}\right)\left[F_G(u_l) - F_G(u_l - x(\varepsilon))\right]\left[F_G^{N-1}(u_l) - F_G^{N-1}(u_l - x(\varepsilon))\right] \\ &- \frac{1}{N}\left[F_G(u_h+\varepsilon) - F_G(u_h)\right]\left[F_G^{N-1}(u_h+\varepsilon) - F_G^{N-1}(u_h)\right].\end{aligned} \tag{22}$$

The reason is that the two strategies yields different payoffs only if both the sender's induced expected utility, as well as that of the highest among the other $N-1$ senders, lie in the intervals $[u_l - x(\varepsilon), u_l]$

$-n\left(1-x^{n-1}\right) < 0$. Therefore, $\frac{\partial\left(\frac{nx^{n-1}(1-x)}{1-x^n}\right)}{\partial x} = \frac{nx^{n-2}}{(1-x^n)^2}\left(n(1-x) - (1-x^n)\right) > 0$.

and $[u_h, u_h+\varepsilon]$. Conditional on the highest expected utility of other $N-1$ senders lie in the interval $[u_h, u_h+\varepsilon]$, strategy G' lowers the sender's probability of winning by $\frac{1}{N}\left[F_G(u_h+\varepsilon)-F_G(u_h)\right]$. On the other hand, conditional on the highest expected utility of other $N-1$ senders lie in the interval $[u_l - x(\varepsilon), u_l]$, strategy G' raises the sender's probability of winning by $\left(1-\frac{1}{N}\right)\left[F_G(u_l)-F_G(u_l-x(\varepsilon))\right]$. It suffices to show that expression (22) is positive for some $\varepsilon>0$, i.e.,

$$\frac{F_G(u_l)-F_G(u_l-x(\varepsilon))}{F_G(u_h+\varepsilon)-F_G(u_h)}\frac{F_G^{N-1}(u_l)-F_G^{N-1}(u_l-x(\varepsilon))}{F_G^{N-1}(u_h+\varepsilon)-F_G^{N-1}(u_h)}>\frac{1}{N-1} \tag{23}$$

holds for some $\varepsilon>0$.

As $u_h, u_l \in supp(F_G)$, the definition of $x(\cdot)$ in equation (21) guarantees that it is locally differentiable at 0. Differentiating equation (21) with respect to ε and rearranging, we get

$$x'(\varepsilon)=\frac{(u_h-u_l+\varepsilon)\,f_G(u_h+\varepsilon)}{x(\varepsilon)\,f_G(u_l-x(\varepsilon))},$$

where f_G is the density function of F_G. As $x(0)=0$, we have $\lim_{\varepsilon\to 0}x'(\varepsilon)=\infty$. Now the limiting value of the left-hand side of inequality (23) is given by

$$\begin{aligned}
&\lim_{\varepsilon\to 0}\frac{F_G(u_l)-F_G(u_l-x(\varepsilon))}{F_G(u_h+\varepsilon)-F_G(u_h)}\frac{F_G^{N-1}(u_l)-F_G^{N-1}(u_l-x(\varepsilon))}{F_G^{N-1}(u_h+\varepsilon)-F_G^{N-1}(u_h)}\\
&=\lim_{\varepsilon\to 0}\frac{f_G(u_l-x(\varepsilon))\,x'(\varepsilon)}{f_G(u_h+\varepsilon)}\lim_{\varepsilon\to 0}\frac{-F_G^{N-2}(u_l-x(\varepsilon))\,f_G(u_l-x(\varepsilon))\,x'(\varepsilon)}{F_G^{N-2}(u_h+\varepsilon)\,f_G(u_h+\varepsilon)}\\
&=\infty.
\end{aligned}$$

We now establish that $C\left(\Pi_{post}(p;G)\right)$ is linear on the convex hull $\mathrm{co}(\mathrm{supp}\,G)$ of $\mathrm{supp}\,G$. By the Minkowski-Caratheodory theorem, for any $p\in\mathrm{co}(\mathrm{supp}\,G)$ there exists a set of posteriors $Q\subset\mathrm{supp}\,G$ such that $|Q|\le M$, and a set of weights $\left\{\alpha_p(q)\in[0,1]:q\in Q\right\}$ such that $p=\sum_{q\in Q}\alpha_p(q)\,q$ and $\sum_{q\in Q}\alpha_p(q)=1$. Therefore, if $C\left(\Pi_{post}(p;G)\right)$ is not linear on $\mathrm{co}(\mathrm{supp}\,G)$, then there exists a $p\in\mathrm{co}(\mathrm{supp}\,G)$ such that $C\left(\Pi_{post}(p;G)\right)>\sum_{q\in Q}\alpha_p(q)\,\Pi_{post}(q;G)$. For each $p'\in\mathrm{co}(\mathrm{supp}\,Q)$, there exists a set of weights $\left\{\alpha_{p'}(q)\in[0,1]:q\in Q\right\}$ such that $p'=\sum_{q\in Q}\alpha_{p'}(q)\,q$. If $\Pi_{post}(p';G)=\sum_{q\in Q}\alpha_{p'}(q)\,\Pi_{post}(q;G)$ for all $p'\in\mathrm{co}(\mathrm{supp}\,Q)$, then $C\left(\Pi_{post}(p;G)\right)=\sum_{q\in Q}\alpha_p(q)\,C\left(\Pi_{post}(q;G)\right)=\sum_{q\in Q}\alpha_p(q)\,\Pi_{post}(q;G)$. Therefore, $C\left(\Pi_{post}(p;G)\right)>\sum_{q\in Q}\alpha_p(q)\,\Pi_{post}(q;G)$ implies that there exists a $p'\in\mathrm{co}(\mathrm{supp}\,Q)$ such that $\Pi_{post}(p';G)>\sum_{q\in Q}\alpha_{p'}(q)\,\Pi_{post}(q;G)$. Since $\Pi_{post}(\cdot;G)$ is continuous on $\left\{\tilde{p}:E_{\tilde{p}}[U_i]\in[u_0,u_{M-1})\right\}$, this implies that there exists a profitable deviation G' such that $Q\not\subset\mathrm{supp}\,G'$, a contradiction.

This proves that $C\left(\Pi_{post}(p;G)\right)$ is linear on $\mathrm{co}(\mathrm{supp}\,G)$. Therefore, there exists a linear function $\overline{\Pi}_{post}(p)$ with the following properties. For all p, $\overline{\Pi}_{post}(p)\ge C\left(\Pi_{post}(p;G)\right)$, and $\overline{\Pi}_{post}(p)=C\left(\Pi_{post}(p;G)\right)$ if and only if $p\in\mathrm{co}(\mathrm{supp}\,G)$. Moreover, if $F_G(\hat{u})=1$, then $\overline{\Pi}_{post}(p)>C\left(\Pi_{post}(p;G)\right)$ for all p such that $E_p[U_i]\in(\hat{u},u_{M-1}]$. If $F_G(\hat{u})<1$, then $C\left(\Pi_{post}(p;G)\right)>\Pi_{post}(p;G)$ for all p such that $E_p[U_i]\in(\hat{u},u_{M-1})$. Q.E.D.

Proof of Theorem 8

We first prove the following two lemmata.

Lemma 5 *Suppose G induces $\Pi(u)$ with the generalized linear structure, and $\Pi(u)$ has a κ-th upward kink at u_{I_κ}. Fix a pair of expected qualities, u' and u'', such that $u_{I_\kappa} \le u' < u'' \le u_{I_{\kappa+1}}$. Then there exist $\beta_j \in [0,1]$, $j = I_\kappa, \cdots, I_{\kappa+1}$ such that $F_G(u') = F_G(u) + \sum_{j=I_\kappa}^{I_{\kappa+1}} \beta_j \pi_j$; and*

$$\frac{N-1}{N} \frac{(F_G(u''))^N - (F_G(u'))^N}{(F_G(u''))^{N-1} - (F_G(u'))^{N-1}} = F_G(u') + \sum_{j=I_\kappa}^{I_{\kappa+1}} \beta_j \pi_j \frac{u'' - u_j}{u'' - u'}. \tag{24}$$

Proof. Suppose $u_{I_\kappa} \le u' < u'' \le u_{I_{\kappa+1}}$. The upward-kink condition implies that $u \in [u', u'']$ is induced only by $u_{I_\kappa}, \cdots, u_{I_{\kappa+1}}$. This implies that there exist $\beta_j \in [0,1]$, $j = I_\kappa, \cdots, I_{\kappa+1}$ such that $F_G(u'') = F_G(u') + \sum_{j=I_\kappa}^{I_{\kappa+1}} \beta_j \pi_j$. Therefore, by Bayes' rule, we have

$$E_G[u | u \in [u', u'']] = \frac{\sum_{j=I_\kappa}^{I_{\kappa+1}} \beta_j \pi_j u_j}{\sum_{j=I_\kappa}^{I_{\kappa+1}} \beta_j \pi_j}. \tag{25}$$

For notational simplicity, let $\chi \equiv F_G(u')^{N-1}$ and $\zeta \equiv F_G(u'')^{N-1}$. Notice that for $u \in [u', u'']$, the piecewise linearity of $\Pi(u)$ implies $\Pi(u) = \chi + \frac{\zeta - \chi}{u'' - u'}(u - u')$. This implies that on the interval $[u', u'']$, we can write $F_G(u) = \left(\chi + \frac{\zeta - \chi}{u'' - u'}(u - u')\right)^{\frac{1}{N-1}}$, which in turn allows us to compute the conditional expectation as follows:

$$\begin{aligned} E_G[u | u \in [u', u'']] &= \frac{\int_{u'}^{u''} u dF_G(u)}{F_G(u'') - F_G(u')} \\ &= \frac{\left(u'' \zeta^{\frac{1}{N-1}} - u' \chi^{\frac{1}{N-1}}\right) - \frac{N-1}{N} \frac{\zeta^{\frac{N}{N-1}} - \chi^{\frac{N}{N-1}}}{\zeta - \chi}(u'' - u')}{\zeta^{\frac{1}{N-1}} - \chi^{\frac{1}{N-1}}}. \end{aligned} \tag{26}$$

Recall that $\zeta^{\frac{1}{N-1}} = F_G(u') + \sum_{j=I_\kappa}^{I_{\kappa+1}} \beta_j \pi_j$, and $\chi^{\frac{1}{N-1}} = F_G(u')$. Substituting these into equation (26) and equating the subsequent expression with (25) gives equation (24) after straightforward algebra. ■

Lemma 6 *Suppose G is an equilibrium strategy and its induced payoff function $\Pi(u)$ has an upward kink at $u_{I_\kappa} \in \{u_0, \cdots, u_{M-2}\}$. Suppose further that F_G satisfies Property-ℓ, for some $\ell \in \{I_\kappa + 1, \cdots, M+1\}$. Then $s_\ell^{\kappa+1} < \infty$, and ℓ is the largest element of* $\arg\min_{j \in \{I_\kappa+1, \cdots, M+1\}} \left\{s_j^{\kappa+1}\right\}$.

Proof. We first prove that $s_\ell^{\kappa+1} \le s_{\ell'}^{\kappa+1}$ for all $\ell' \in \{I_\kappa + 1, \cdots \ell - 1, \ell + 1, ..., M+1\}$ when G satisfies Property-ℓ.

(Case 1:) Suppose $I_{\kappa+1} \in \{I_\kappa + 1, \cdots, M-2\}$. We show that $s_m^{\kappa+1} \ge s_{I_{\kappa+1}}^{\kappa+1}$ for $m \in \{I_\kappa + 1, \cdots, I_{\kappa+1} - 1\}$. Let $F_G(u_{I_\kappa}) = \chi_\kappa^{\frac{1}{N-1}}$. By applying Lemma 5 on $[u_{I_\kappa}, u_m]$, we know that there exist $\beta_j \in [0,1]$, $j =$

$\{I_\kappa, \cdots, I_{\kappa+1}\}$ such that $F_G(u_m) = \chi_\kappa^{\frac{1}{N-1}} + \sum_{j=I_\kappa}^{I_{\kappa+1}} \beta_j \pi_j$, and

$$\frac{N-1}{N} \frac{F_G(u_m)^N - \chi_\kappa^{\frac{N}{N-1}}}{F_G(u_m)^{N-1} - \chi_\kappa} = \chi_\kappa^{\frac{1}{N-1}} + \beta_{I_\kappa} \pi_{I_\kappa} + \sum_{j=I_\kappa+1}^{I_{\kappa+1}} \beta_j \pi_j \frac{u_m - u_j}{u_m - u_{I_\kappa}} \tag{27}$$

Similarly, there exist $\alpha_{I_\kappa} \in (0,1)$ and α_m such that

$$\frac{N-1}{N} \frac{\tilde{F}_G^N - \chi_\kappa^{\frac{N}{N-1}}}{\tilde{F}_G^{N-1} - \chi_\kappa} = \chi_\kappa^{\frac{1}{N-1}} + (1-\alpha_{I_\kappa}) \pi_{I_\kappa} + \sum_{j=I_\kappa+1}^{m-1} \pi_j \frac{u_m - u_j}{u_m - u_{I_\kappa}}, \tag{28}$$

where $\tilde{F}_G = \chi_\kappa^{\frac{1}{N-1}} + (1-\alpha_{I_\kappa}) \pi_{I_\kappa} + \sum_{j=I_\kappa}^{m-1} \pi_j + \alpha_m \pi_m$.

We show that $F_G^{N-1}(u_m) \leq \tilde{F}_G^{N-1}$. First, notice that $\frac{\xi^N - \chi^{\frac{N}{N-1}}}{\xi^{N-1} - \chi}$ is increasing in ξ for all $\xi > \chi^{\frac{1}{N-1}}$. Therefore by (27) and (28), $F_G^{N-1}(u_m) \leq \tilde{F}_G^{N-1}$ if and only if

$$\beta_{I_\kappa} \pi_{I_\kappa} + \sum_{j=I_\kappa+1}^{I_{\kappa+1}} \beta_j \pi_j \frac{u_m - u_j}{u_m - u_{I_\kappa}} \leq (1-\alpha_{I_\kappa}) \pi_{I_\kappa} + \sum_{j=I_\kappa+1}^{m-1} \pi_j \frac{u_m - u_j}{u_m - u_{I_\kappa}}.$$

Observe that $\beta_{I_\kappa} \leq 1 - \alpha_{I_\kappa}$. Also, since $u_{I_{\kappa+1}} > u_m$, $\sum_{j=m+1}^{I_{\kappa+1}} \beta_j \pi_j \frac{u_m - u_j}{u_m - u_{I_\kappa}} \leq 0$. Therefore, $\sum_{j=I_\kappa+1}^{I_{\kappa+1}} \beta_j \pi_j \frac{u_m - u_j}{u_m - u_{I_\kappa}} \leq \sum_{j=I_\kappa+1}^{m-1} \pi_j \frac{u_m - u_j}{u_m - u_{I_\kappa}}$, i.e., $F_G^{N-1}(u_m) \leq \tilde{F}_G^{N-1}$. Thus, by the definition of $s_{I_{\kappa+1}}^{\kappa+1}$ and $s_m^{\kappa+1}$, we have $s_{I_{\kappa+1}}^{\kappa+1} \leq s_m^{\kappa+1}$.

(Case 2:) Suppose $I_{\kappa+1} \in \{I_\kappa + 1, \cdots, M-2\}$. We show that $s_m^{\kappa+1} > s_{I_{\kappa+1}}^{\kappa+1}$ for $m \in \{I_{\kappa+1}+1, \cdots, k-2\}$. Suppose that F_G^{N-1} has an upward-kink at u_I, $I \in \{I_{\kappa+1}, \cdots, M-2\}$ and no upward-kink at $u_j \in \{u_{I+1}, \cdots, u_{m-1}\}$; and $s_m^{\kappa+1} < \infty$. For notational simplicity, define $\tilde{\chi}_I \equiv \chi_\kappa + s_m^{\kappa+1}(u_I - u_{I_\kappa})$ and $\chi_I \equiv F_G^{N-1}(u_I)$.

By applying Lemma 5 on $[u_I, u_m]$, we know that there exist $\beta_I^G \in (0,1)$ and $\beta_j^G \in [0,1]$, $j \in \{I+1, ..., M-1\}$ such that $F_G(u_m) - F_G(u_I) = \sum_{j=I_{\kappa+1}}^{M-1} \beta_j^G \pi_j$ and

$$\frac{N-1}{N} \frac{F_G(u_m)^N - \chi_I^{\frac{N}{N-1}}}{F_G(u_m)^{N-1} - \chi_I} = F_G(u_I) + \sum_{j=I}^{M-1} \beta_j^G \pi_j \frac{u_m - u_j}{u_m - u_I}. \tag{29}$$

Similarly, since $s_m^{\kappa+1} < \infty$, there there exist $\alpha_{I_\kappa}^m, \alpha_m^m \in [0,1]$, $\gamma_m^m \in [0, \alpha_m^m]$, and $\beta_j^m \in [0,1]$ for each $j = I_\kappa, ..., m-1$ such that,

$$\frac{N-1}{N} \frac{\left(\sum_{j=0}^{m-1} \pi_j + \alpha_m^m \pi_m\right)^N - \tilde{\chi}_I^{\frac{N}{N-1}}}{\left(\sum_{j=0}^{m-1} \pi_j + \alpha_m^m \pi_m\right)^{N-1} - \tilde{\chi}_I} = \tilde{\chi}_I^{\frac{1}{N-1}} + \sum_{j=I_\kappa}^{m-1} \beta_j^m \pi_j \frac{u_m - u_j}{u_m - u_I}, \tag{30}$$

where $\tilde{\chi}_I^{\frac{1}{N-1}} = \sum_{j=0}^{I_\kappa - 1} \pi_j + \alpha_{I_\kappa}^m \pi_{I_\kappa} + \sum_{j=I_\kappa}^{m-1} \left(1 - \beta_j^m\right) \pi_j + \gamma_m^m \pi_m$.

We now argue that the left-hand side of (29) is strictly smaller than that of (30). Let Δ be the difference between the right-hand side of (30) minus the right-hand side of (29), and ϖ_j be the coefficient of π_j, $j \in \{0, 1, ..., M-1\}$, of Δ. We argue that $\varpi_j \geq 0$ for all j with strict inequality for at least one j. (i) If

$j \in \{0, \cdots, I_\kappa - 1\}$, then $\varpi_j = 0$. (ii) If $j = I_\kappa$, then $\varpi_{I_\kappa} = \alpha^m_{I_\kappa} + \left(1 - \beta^m_{I_\kappa}\right) + \beta^m_{I_\kappa} \frac{u_m - u_{I_\kappa}}{u_m - u_I} - 1 > 0$ because $\frac{u_m - u_{I_\kappa}}{u_m - u_I} > 1$. (iii) If $j \in \{I_\kappa + 1, \cdots, I - 1\}$, then $\varpi_j = \left(1 - \beta^m_j\right) + \beta^m_j \frac{u_m - u_j}{u_m - u_I} - 1 \geq 0$. (iv) If $j = I$, then $\varpi_I = 1 - \beta^G_I \geq 0$. (v) If $j \in \{I + 1, \cdots, m - 1\}$, then $\varpi_j = \left(1 - \beta^m_j\right) + \beta^m_j \frac{u_m - u_j}{u_m - u_I} - \beta^G_j \frac{u_m - u_j}{u_m - u_I} \geq 0$ because $\frac{u_m - u_j}{u_m - u_I} \in (0, 1)$. (vi) for $j = k$, $\varpi_j = 1 - \beta^m_m + \gamma^m_m \geq 0$. (vii) For $j \in \{k + 1, \cdots, M - 1\}$, $\varpi_j = -\beta^G_j \frac{u_m - u_j}{u_m - u_I} \geq 0$.

Lastly, we show that $s^{\kappa+1}_{I_{\kappa+1}} < s^{\kappa+1}_k$. To see this notice that if $s^{\kappa+1}_{I_{\kappa+1}} \geq s^{\kappa+1}_m$, then the upward-kink condition implies $\tilde{\chi}_I \leq \chi_I$ and $\sum_{j=0}^{m-1} \pi_j + \alpha^m_m \pi_m \leq F_G(u_m)$. However, $\frac{\xi^N - \chi^{\frac{N}{N-1}}}{\xi^{N-1} - \chi}$ is increasing both in ξ and χ. Therefore, that the left-hand side of (29) is strictly smaller than that of (30) implies $F_G(u_m) < \sum_{j=0}^{m-1} \pi_j + \alpha^m_m \pi_m$, a contradiction.

(Case 3:) Suppose $I_{\kappa+1} \in \{I_\kappa + 1, \cdots, M - 2\}$. We show that $s^{\kappa+1}_j > s^{\kappa+1}_{I_{\kappa+1}}$ for all $j \in \{M - 1, M, M + 1\}$. Suppose $\hat{u}_j \leq u_{I_{\kappa+1}}$. Then it is straightforward from the definitions of $s^{\kappa+1}_j$ and $s^{\kappa+1}_{I_{\kappa+1}}$ that $s^{\kappa+1}_j > s^{\kappa+1}_{I_{\kappa+1}}$. Suppose next that $\hat{u}_j > u_{I_{\kappa+1}}$. Then replacing u_m with $\hat{u}_j$ in the analysis of Case 2 above leads to $s^{\kappa+1}_j > s^{\kappa+1}_{I_{\kappa+1}}$.

(Case 4:) Suppose $I_{\kappa+1} \in \{M - 1, M, M + 1\}$. It is straightforward to see that $s^{\kappa+1}_j = \infty$ for $j \in \{M - 1, M, M + 1\} \setminus \{I_{\kappa+1}\}$. For $k \in \{I_\kappa + 1, ..., M - 2\}$, $s^{\kappa+1}_{I_{\kappa+1}} \leq s^{\kappa+1}_k$ follows from an argument identical to that in Case 1 above.

Lastly, suppose that there exist ℓ and $\ell' > \ell$ such that $s^{\kappa+1}_\ell = s^{\kappa+1}_{\ell'}$ for some κ. If $\ell' \in \{I_\kappa + 1, ..., M - 2\}$, then the proof of Case 2 shows that F_G does not satisfy Property-ℓ. If $\ell \in \{I_\kappa + 1, \cdots, M - 2\}$, then the proof of Case 3 shows that F_G does not satisfy Property-ℓ. By Case 4, if $\ell = M - 1$ or M, then $s^{\kappa+1}_{\ell'} = \infty$ for all $\ell' \in \{M, M + 1\} \setminus \{\ell\}$. Therefore, if $s^{\kappa+1}_\ell = s^{\kappa+1}_{\ell'}$ and $\ell < \ell'$, then F_G does not satisfy Property-ℓ. ■

We now prove Theorem 8. We have already argued the "only if" part in the text. To see the "if" part, observe that as an immediate corollary of Lemma 6, if a symmetric equilibrium exists, the equilibrium distribution of expected qualities is unique. Furthermore, the existence of a symmetric equilibrium follows from Corollary 4.3 of Reny (1999).[35] Lemma 6 implies that the algorithm constructs the unique equilibrium distribution of expected qualities.

Finally, the game is symmetric and zero-sum. Therefore, if $N = 2$, the interchangeability property of zero-sum games implies that if there exist multiple equilibria or an asymmetric equilibrium, then there exists multiple symmetric equilibria, which is a contradiction.Q.E.D.

Proof of Theorem 9

We show by induction that there exist N_k, for each $k = 1, \cdots, M - 2$ such that (i) $N > N_k$ implies $I_{k,N} = k$ and (ii) $N' > N > N_k$ implies $F_{G,N'}(u_k) < F_{G,N}(u_k)$ and $\lim_{N \to \infty} F_{G,N}(u_k) = \sum_{l=0}^{k-1} \pi_l$.

We start with $k = 1$. That is, we show that for a sufficiently large N, Property-m, $m \in \{2, \cdots, M - 2\}$,

[35]Let $V_i(G_i, G_{-i})$ be the sender i's payoff when the strategy profile is (G_i, G_{-i}). Since $V_i(G, ..., G) = 1/n$, the game is quasi-symmetric, compact, diagonally quasiconcave, diagonally payoff secure, and $V_i(G, ..., G)$ is upper semicontinuous with respect to G.

does not hold at u_0. This is because if G satisfies Property-m, then there exists an $\alpha_m \in (0,1)$ such that

$$\frac{N-1}{N}\left(\sum_{j=0}^{m-1}\pi_j+\alpha_m\pi_m\right)=\left(\pi_0+\sum_{j=1}^{m-1}\pi_j\frac{u_m-u_j}{u_m-u_0}\right). \tag{31}$$

and $F_{G,N}(u_m)=\sum_{j=0}^{m-1}\pi_j+\alpha_m\pi_m$. Notice that the left-hand side of (31) implies that α_m is strictly decreasing in N. Furthermore, for a sufficiently large N, $\frac{u_m-u_j}{u_m-u_0}<\frac{N-1}{N}$ for all $j\in\{1,\cdots,m-1\}$. Therefore, for a sufficiently large N, α_m that solves (31) has to be negative. To see that Property-$M-1$ does not hold at u_0, notice that (10) simplifies to $\frac{N-1}{N}\sum_{j=0}^{M-2}\pi_j=\sum_{j=1}^{M-2}\pi_j\frac{\hat{u}_{M-1}-u_j}{\hat{u}_{M-1}-u_0}$. Notice $\frac{\hat{u}_{M-1}-u_j}{\hat{u}_{M-1}-u_0}<\frac{u_{M-1}-u_j}{u_{M-1}-u_0}<1$ for each $j=1,\cdots,M-2$. Therefore, for a sufficiently large N, there exists no $\hat{u}_{M-1}<u_{M-1}$ such that $\frac{N-1}{N}\sum_{j=0}^{M-2}\pi_j=\sum_{j=1}^{M-2}\pi_j\frac{\hat{u}_{M-1}-u_j}{\hat{u}_{M-1}-u_0}$. Similarly, by (12), and (15), we can show that for a sufficiently large N, neither Properties-M, nor $M+1$ holds. This proves that for a sufficiently large N, $I_{1,N}=1$ and $F_{G,N}(u_1)=\pi_0+\frac{\pi_0}{(N-1)\pi_1}$.

Suppose the induction hypothesis holds for all $k=1,\cdots,l$, where $l\leq M-3$. We show that Property-$m, m\in\{l+2,\cdots,M-2\}$ does not hold for a sufficiently large N. Suppose otherwise. Then, there exists an $\alpha_m\in(0,1)$ such that $F_{G,N}(u_m)=\sum_{j=0}^{m-1}\pi_j+\alpha_m\pi_m$ and

$$\frac{N-1}{N}\frac{\left(\sum_{j=0}^{m-1}\pi_j+\alpha_m\pi_m\right)^N-\left(F_{G,N}(u_l)\right)^N}{\left(\sum_{j=0}^{m-1}\pi_j+\alpha_m\pi_m\right)^{N-1}-\left(F_{G,N}(u_l)\right)^{N-1}}=\sum_{j=0}^{l}\pi_j+\sum_{j=l+1}^{m-1}\pi_j\frac{u_m-u_j}{u_m-u_l}. \tag{32}$$

For any ε, there exists an $\tilde{N}$ such that $N>\tilde{N}$ implies that the left-hand side of (32) is bounded from below by $\sum_{j=0}^{m-1}\pi_j-\varepsilon$.[36] However, the right-hand side of (32) is bounded from above by $\sum_{j=0}^{m-1}\pi_j$, which is a contradiction. Similar arguments shows that none of Properties-$M-1, M, M+1$ holds at u_l. Therefore, for a sufficiently large N, $I_{l+1,N}=l+1$, and there exists an $\alpha_{l+1,N}\in(0,1)$ such that $F_{G,N}(u_{l+1})=\sum_{j=0}^{l}\pi_j+\alpha_{l+1,N}\pi_{l+1}$ and

$$\frac{N-1}{N}\frac{\left(\sum_{j=0}^{l}\pi_j+\alpha_{l+1,N}\pi_{l+1}\right)^N-\left(F_{G,N}(u_l)\right)^N}{\left(\sum_{j=0}^{l}\pi_j+\alpha_{l+1,N}\pi_{l+1}\right)^{N-1}-\left(F_{G,N}(u_l)\right)^{N-1}}=\sum_{j=0}^{l}\pi_j.$$

[36] This is because $\frac{N-1}{N}\frac{x^{N-1}-y^N}{x^{N-1}-y^{N-1}}$ is strictly increasing in $N\in\mathbb{N}$. Notice that $\frac{N-1}{N}\frac{x^N-y^N}{x^{N-1}-y^{N-1}}=x\times\frac{N-1}{N}\times\frac{1-z^N}{1-z^{N-1}}$, where $z=y/x$, and

$$\begin{aligned}\frac{N-1}{N}\frac{1-z^N}{1-z^{N-1}}-\frac{N-2}{N-1}\frac{1-z^{N-1}}{1-z^{N-2}}&=\frac{(N-1)^2\left(1-z^N\right)\left(1-z^{N-2}\right)-N(N-2)\left(1-z^{N-1}\right)^2}{N(N-1)\left(1-z^{N-1}\right)\left(1-z^{N-2}\right)}\\&=\frac{\left(1+z^{2N-2}\right)+z^{N-2}\left((N-1)^2\left(1+z^2\right)-N(N-2)(2z)\right)}{N(N-1)\left(1-z^{N-1}\right)\left(1-z^{N-2}\right)}\\&\geq\frac{\left(1+z^{2N-2}\right)+(N-1)^2z^{N-2}(1-z)^2}{N(N-1)\left(1-z^{N-1}\right)\left(1-z^{N-2}\right)}>0.\end{aligned}$$

Notice $\frac{N-1}{N}\frac{x^{N-1}-y^N}{x^{N-1}-y^{N-1}}$ is strictly increasing in $N \in \mathbb{N}$, y, and $x-y$, when $y \in (0,1)$, and $x \in (y,1)$.[37] Therefore, $\left(\sum_{j=0}^{l}\pi_j + \alpha_{l+1,N}\pi_{l+1}\right) - F_{G,N}(u_l)$ is strictly decreasing in N. Since $F_{G,N}(u_l)$ is strictly decreasing in N by the induction hypothesis, we have that $\alpha_{l+1,N}$ is strictly decreasing in N, and $F_{G,N}(u_{l+1}) \to \sum_{j=0}^{l}\pi_j$.

We have established that $F_{G,N}(u_k) = \sum_{j=0}^{k-1}\pi_j + o(1)$ for $k = 0, \cdots, M-2$, where $o(1) \to 0$ as $N \to \infty$. Therefore, for any $u \in (u_k, u_{k+1}), k \in \{0, \cdots, M-3\}$,

$$F_G(u) = \left(\frac{\left(\sum_{j=0}^{k-1}\pi_j\right)^{N-1}(u_{k+1}-u) + \left(\sum_{j=0}^{k}\pi_j\right)^{N-1}(u-u_k)}{u_{k+1}-u_k}\right)^{\frac{1}{N-1}} + o(1) \to \sum_{j=0}^{k}\pi_j \text{ as } N \to \infty.$$

Next, we show that $F_{G,N}(u) \to 1-\pi_{M-1}$ as $N \to \infty$ for $u \in (u_{M-2}, u_{M-1})$. For a sufficiently large N, Property-M holds at u_{M-2}. That is, by, (12) and (13), there exist $\hat{u}_{M,N} \in (u_{M-2}, u_{M-1})$ and $\alpha_N \in (0,1)$ such that

$$\frac{N-1}{N}\frac{\left(\sum_{j=0}^{M-2}\pi_j + \alpha_N\pi_{M-1}\right)^N - (F_{G,N}(u_{M-2}))^N}{\left(\sum_{j=0}^{M-2}\pi_j + \alpha_N\pi_{M-1}\right)^{N-1} - (F_{G,N}(u_{M-2}))^{N-1}} = \sum_{j=0}^{M-2}\pi_j + \alpha_N\pi_{M-1}\frac{\hat{u}_{M,N}-u_{M-1}}{\hat{u}_{M,N}-u_{M-2}} \tag{33}$$

and

$$\frac{1-\left(\frac{F_{G,N}(u_{M-2})}{\sum_{j=0}^{M-2}\pi_j+\alpha_N\pi_{M-1}}\right)^{N-1}}{\frac{1-\left(\sum_{j=0}^{M-2}\pi_j+\alpha_N\pi_{M-1}\right)^N}{N\left(\left(\sum_{j=0}^{M-2}\pi_j+\alpha_N\pi_{M-1}\right)^{N-1}-\left(\sum_{j=0}^{M-2}\pi_j+\alpha_N\pi_{M-1}\right)^N\right)}-1} = \frac{\hat{u}_{M,N}-u_{M-2}}{u_{M-1}-\hat{u}_{M,N}}. \tag{34}$$

We show that $\alpha_N \to 0$. Suppose not, then there exists a subsequence $\{N_k\}$ such that $\alpha_{N_k} \to \alpha$, for some $\alpha > 0$, and $\hat{u}_{M,N}$ converges. Then the left-hand side of equation (33) converges to $\sum_{j=0}^{M-2}\pi_j + \alpha\pi_{M-1}$, whereas the limit of right-hand side is bounded from above by $\sum_{j=0}^{M-2}\pi_j$, a contradiction.

As $\alpha_N \to 0$, the left-hand side of equation (34) converges to 0, so $\lim_{N\to\infty}\hat{u}_{M,N} = u_{M-2}$. This proves that for any $u \in (u_{M-2}, u_{M-1})$, $F_{G,N}(u) \to 1-\pi_{M-1}$.

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[37]Notice that $\frac{N-1}{N}\frac{x^N-y^N}{x^{N-1}-y^{N-1}} = x \times \frac{N-1}{N} \times \frac{1-z^N}{1-z^{N-1}}$, where $z = y/x$. Next, siNce $\frac{\partial \frac{1-z^N}{1-z^{N-1}}}{\partial z} = \frac{z^N\left(N(1-z)-\left(1-z^N\right)\right)}{\left(z-z^N\right)^2} > 0$ for all $z < 1$, $\frac{N-1}{N}\frac{x^{N-1}-y^N}{x^{N-1}-y^{N-1}}$ is increasing in y. Lastly, observe that $\frac{\partial \frac{(y+d)^N-y^N}{(y+d)^{N-1}-y^{N-1}}}{\partial d} = \frac{(y+d)^{N-2}\left((y+d)^N-y^N-Ny^{N-1}d\right)}{\left((y+d)^{N-1}-y^{N-1}\right)^2}$ and $\frac{\partial\left((y+d)^N-y^N-Ny^{N-1}d\right)}{\partial d} = N\left((y+d)^{N-1}-y^{N-1}\right) > 0$. Therefore, $\frac{\partial \frac{(y+d)^N-y^N}{(y+d)^{N-1}-y^{N-1}}}{\partial d} \geq 0$.

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