Competitive Information Disclosure by Multiple Senders

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October 6, 2017

Abstract

We analyze a model of competition in Bayesian persuasion in which multiple symmetric senders vie for the patronage of a receiver by disclosing information about their respective proposal qualities. We show that a symmetric equilibrium exists and is unique. We then show that as the number of senders increases, each sender discloses information more aggressively; and full disclosure by each sender arises in the limit of infinitely many senders.

Keywords Information Transmission, Bayesian Persuasion, Multiple Senders

JEL Codes D83

1 Introduction

Consider two pharmaceutical companies $i = 1, 2$, each of which produces a drug and competes for demand by a unit mass of consumers.[(1)] The efficacy of a drug is either high or low, and is distributed identically and independently across consumers and drugs. The commonly-known prior is that drug i' is of high efficacy with probability p, which takes the value of either 0.2, or 0.8. All consumers prefer a drug of high efficacy and are otherwise indifferent between the drugs.

Suppose that each firm i simultaneously chooses one of two clinical trials: a completely uninformative trial (null), or a fully informative trial (full) that allows each consumer to learn the efficacy of drug i. The goal of firms is to maximize the number of consumers buying their drugs. This normal-form game can be tabulated as follows:

	null	full
null	$0.5, 0.5$	$1-p, p$
full	$p, 1-p$	$0.5, 0.5$

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[‡]The second author greatly acknowledges the financial support from UNSW Sydney and Australian Research Council.

[(1)]This game is a variant of an example in Gentzkow and Kamenica (2017b).

The unique equilibrium outcome is that both firms choose "null" when the drugs are believed to be low-quality, i.e., $p = 0.2$, and "full" when drugs are believed to be high-quality, i.e., $p = 0.8$. Therefore, competition between firms may not increase information revelation, especially if it is commonly believed that the drugs' efficacy is likely to be low.[(2)]

Things could be quite different if the design of their clinical trials is more flexible so that firms can only partially reveal their drug information. An increase in the flexibility of trial design may affect the degree of information revelation in an ambiguous way. To see this, first suppose that $p = 0.2$. If firm 2 chooses "null," then firm 1 can increase its payoff by deviating to an imperfectly informative clinical trial such that posterior 0.25 is induced with probability 0.8, and 0 with probability 0.2. In this sense, an increase in the flexibility in designing clinical trials can increase the amount of information revealed. Next, suppose that $p = 0.8$. If firm 2 chooses "full," then instead of responding by playing "full," firm 1 can increase its payoff by deviating to an imperfectly informative clinical trial such that posterior 1 is induced with probability 0.75 and posterior 0.2 is induced with probability 0.25. In this case, an increase in the flexibility in designing clinical trials reduces the amount of information revealed.

Though the view that competition increases the amount of information revealed seems intuitive, in light of the observations above, should we expect more information being disclosed under more intense competition, especially when senders have flexibility in their choice of disclosure policies? To answer this question, we analyze a model of competition in Bayesian persuasion. In our model, multiple symmetric senders vie for the patronage of a receiver by disclosing information about their respective proposals. There is no ex-ante information asymmetry, and proposals' qualities are independently and identically distributed according to a commonly known prior.

We assume that each sender can only directly control the disclosure of information regarding his own proposal's quality, but he has full flexibility in choosing his disclosure policy concerning his own proposal's quality. The flexibility in their choice of disclosure policies in turn implies that we can reformulate the problem of each sender as choosing a distribution over marginal posterior beliefs that respects Bayes' rule, as pointed out by Kamenica and Gentzkow (2011). Moreover, optimizing the distribution over posterior distributions is equivalent to finding the concave closure of the sender's payoff as a function of realized posteriors. This reformulation leads us to the observation that a strategy profile is an equilibrium if and only if the induced payoff functions (of posterior distributions) exhibit a particular linear structure. This observation allows us to construct the unique symmetric equilibrium.

We begin our analysis with the binary-quality case. We show that an increase in the number of senders

[(2)]This result resonates well with observations that firms that are believed have higher-quality products implement fit-revelation activities, e.g., distribution of free samples, in a greater intensity than firms that offer low-quality products. See e.g., Gu and Xie (2013) would intensify competition because each sender finds that a more transparent disclosure policy is necessary to stand a good chance in persuading the receiver. More specifically, fixing the other senders' strategies, an increase in the number of senders "convexifies" a sender's payoff function in posteriors, which gives him an incentive to disclose more information. Consequently, in the (unique) symmetric equilibrium with more senders, each sender adopts a more informative disclosure policy. Moreover, we find that as the number of senders approaches infinity, each sender's strategy converges to full disclosure.

We then generalize the model to allow for an arbitrary number of possible quality realizations. Unlike the binary case, each posterior belief is multi-dimensional, so a sender's payoff function over posteriors is no longer homeomorphic to his payoff function over expected qualities induced by respective posteriors. Consequently, the linearity of equilibrium payoff function of posteriors does not translate into the linearity of equilibrium payoff function of expected qualities in a straightforward manner. Nevertheless, if we recast the problem to one of choosing a distribution over expected qualities (rather than choosing a distribution over posteriors), the linearity of equilibrium payoff function remains valid locally, with a possibility of upward kinks only at interim qualities induced by "boundary" posteriors.

We establish that this property of payoff functions over expected qualities, together with other related properties, are necessary and sufficient conditions for an equilibrium. We then use these properties to develop a simple algorithm that constructs the unique symmetric equilibrium. Finally, the algorithm allows us to show that the equilibrium strategy approaches full disclosure as the number of senders goes to infinity.

1.1 Related Literature

As discussed above, the technique developed by Kamenica and Gentzkow (2011) plays a key role in our analysis. Their article has stimulated an active literature on information disclosure game in which the sender(s) can commit to the disclosure mechanism. Below, we discuss a number of articles from the literature that study competition among senders. Ostrovsky and Schwarz (2010) consider a model in which schools disclose information about the ability of their students, with the objective of maximizing their students' overall placement. They find that the distribution of students' abilities across the schools does not affect the equilibrium level of disclosure. An implication is that fixing the prize structure and the distribution of students' abilities, increasing the number of schools has no impact on the equilibrium disclosure. Whereas some of our results concern the characterization of the equilibrium disclosure policies, we are primarily interested in the effect of changes in the number of senders. In particular, we find that an increase in the number of senders leads to more aggressive disclosure by every sender.(3)

(3)Also, in Ostrovsky and Schwarz (2010), there exist some prize structures and distributions of students' abilities such that full disclosure is an equilibrium outcome. In contrast, full disclosure is never an equilibrium in our game

Boleslavsky and Cotton (2016) analyze a Bayesian-persuasion game with two senders and a binary state space, using the observation that the incentive structure facing a sender is similar to that facing a bidder in a complete-information all-pay auction. In contrast, our approach builds upon the linear structure of payoff functions. Our approach is more versatile as it allows us to tackle the more general setting with multiple senders, and study the effect of changes in the number of senders on equilibrium disclosure policies. [4] Furthermore, our analysis highlights an important difference between competitive Bayesian persuasion games and all-pay auctions. Whereas a bid is one-dimensional, a posterior distribution is a multi-dimensional object if there are more than two states. With many states, the equilibrium strategies in our competitive Bayesian persuasion game display a linear structure very different from those of all-pay auctions. In particular, the equilibrium distributions of expected qualities in our game typically feature upward kinks at intermediate utilities, whereas no analogous features exist in all-pay auctions.

Hoffmann et al. (2014) study a persuasion game in which multiple persuaders compete by first acquiring information and then disclose it to a receiver. Their game is different from Bayesian persuasion settings as senders may be privately informed about the state and the senders can only effectively choose between two disclosure policies. They show a limit result that when the number of competing senders is sufficiently large, the unique equilibrium involves all senders adopting the most informative feasible policy. On the other hand, in our setting, the flexibility in the senders' choice of disclosure mechanism allows us to show that the informativeness of the equilibrium disclosure mechanism is strictly increasing in the number of senders.

Most other studies on competitive Bayesian persuasion assume that the senders share a common state of the world, and each sender can independently disclose information about the common state to the receiver. Our setting is different from these studies as senders in our model are endowed with independent proposals, and each can only reveal information concerning his own proposal. Gentzkow and Kamenica (2017a) analyze the effect of competition in a Bayesian persuasion game in which all senders share a common state. They find that adding more senders never makes the set of equilibrium outcomes less informative. However, with equilibrium multiplicity, they also note that the set of outcomes with more competition may not be comparable to those with less competition. On the other hand, our setting has a unique symmetric equilibrium, which allows us to obtain a sharp result regarding the effect of competition on information revelation. In a sequential setting, Li and Norman (2017) provide an example that if only (conditionally) independent mechanisms are feasible for each sender, the equilibrium outcome can be less informative with an additional sender. Au (2015) analyzes a dynamic disclosure setting with a single sender, who effectively faces competition with his future selves if he is unable to commit to a dynamic

[4] Au and Kawai (2017) show that the linear structure of payoff functions also fully characterizes the equilibrium of a game in which two senders' proposal qualities are positively correlated.

dislosure policy.

Board and Lu (2017) consider a search setting in which a receiver, at a positive search cost, sequentially samples senders who provide information concerning a common state. They show that if the receiver's belief is private, and the senders' disclosure mechanisms are independent (conditional on state), then full disclosure is a limit equilibrium as search cost becomes infinitesimally small.(5) On the other hand, in our setting, the senders have independent proposals, and they make disclosure simultaneously. There are two notable differences between our results and those of Board and Lu (2017). First, we find that equilibrium disclosure gets strictly more informative with extra senders, *regardless of the number of existing senders*. On the other hand, Board and Lu (2017)'s result concerns only the limiting case of infinitesimally small search cost. Second, full disclosure is the unique limit equilibrium in our game; whereas in the setting of Board and Lu (2017) with binary states, no disclosure by every sender is always an equilibrium if the prior of the favorable state is relatively high.(6)

There are a number of studies on information transmission with multiple senders using frameworks different from Bayesian persuasion, and these studies have identified various mechanisms through which competition can increase information disclosure. For example, Milgrom and Roberts (1986) considers persuasion game with unsophisticated receiver. Morgan and Krishna (2001) extend Crawford and Sobel (1982) to study a cheap talk game in which two senders have opposing bias, and identify a full-revelation equilibrium. Battaglini (2002) shows the existence of a full-revelation equilibrium in a cheap-talk setting with two senders and a multidimensional state space. Kawai (2015) generalizes the finding of Morgan and Krishna (2001) to multi-dimensional state space.

2 Model

There are N risk-neutral (male) senders, each of whom is endowed with a proposal. They engage in competition for the endorsement of a single (female) receiver. The receiver is an expected-utility-maximizer. The quality of proposal by sender i is denoted by $U_i \in \Omega \equiv \{u_0, u_1, \cdots, u_{M-1}\}$, where $u_{k-1} < u_k$ for all $k = 1, 2, \cdots, M-1$. For each i, U_i is independently and identically distributed with a commonly known prior distribution, denoted by $\pi \in int\,(\triangle\Omega)$, i.e., $\Pr\left(U_i = u_k\right) = \pi\left(u_k\right)$. Though π is commonly known, the realizations U_is are unknown to any player at the beginning of the game.

(5)The intuition is as follows. Suppose every sender provides some information. As the search cost gets very small, the receiver can sample a large number of senders at a small total cost. Then the fact that senders use conditionally independent disclosure mechanisms implies that the receiver has the option of becoming almost fully-informed at a low cost. Therefore, the first sender, upon being sampled, would lose the receiver's patronage if he did not reveal sufficiently informative signals, as the receiver's search is without recall.

(6)See Corollary 2 of Board and Lu (2017). The reason is similar to the Diamond paradox in a standard search setting.

A sender's objective is to maximize the probability that the receiver chooses his proposal. Without loss of generality, we normalize sender i's payoffs to one if the receiver accepts his proposal, and zero otherwise. Each sender i simultaneously chooses an information disclosure mechanism on U_i, which consists of a message space M_i and a conditional distribution $\Phi_i : \Omega \times M_i \rightarrow [0,1]$. The choices of disclosure mechanisms are known to the receiver before she makes her decision. After observing the disclosure mechanisms and realized messages of all senders, the receiver decides which sender's proposal to adopt. Given the information learned, the receiver chooses the sender with the most favorable proposal. In case of a tie, she randomly picks a sender among those with the most favorable proposals with equal probabilities.

By Proposition 1 of Kamenica and Gentzkow (2011), it is without loss of generality to focus on the game of information disclosure played between the senders, in which the set of pure strategies of sender i consists of all Bayes-plausible (marginal) distributions over posterior distributions about sender i's proposal quality. Furthermore, for each mixed strategy, there exists a pure strategy that preserves the expected payoffs of all players. Therefore, without loss of generality, we restrict our attention to pure-strategy Nash equilibria of the game described above.

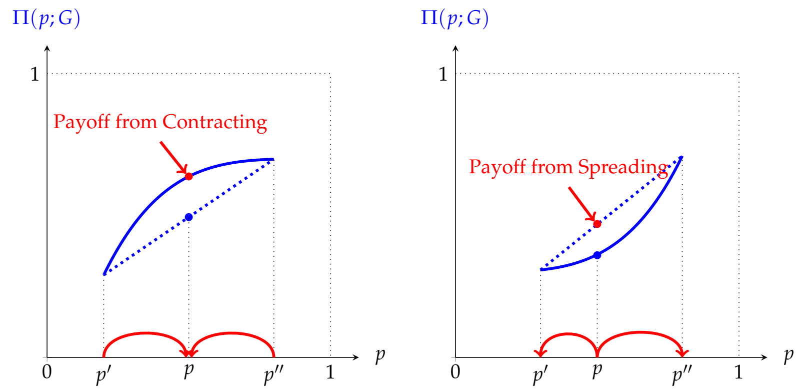
More specifically, denote a generic element of $\Delta(\Omega)$ by $p = (p_0, \cdots, p_{M-1})$. A sender's strategy space is the set of Bayes-plausible distributions over posteriors, $\Lambda \equiv \left\{G \in \triangle(\triangle\Omega) : \int p dG(p) = \pi\right\}$. Since the receiver is an expected-utility-maximizer, given a profile of realized posteriors $\left(p^1, \cdots, p^N\right)$ by the senders, she chooses sender i with probability one if $E_{p^i}[U_i] > E_{p^j}\left[U_j\right]$ for all $j \neq i$, where $E_{p^i}[U_i] = \sum_{l=0}^{M-1} p_l^i u_l$; and with probability $1/\tilde{n}$ if $E_{p^i}[U_i] \geq E_{p^j}\left[U_j\right]$ for all j, and $\tilde{n} = \#\left\{j : E_{p^i}[U_i] = E_{p^j}\left[U_j\right]\right\}$.

The solution concept we use is Bayesian Nash equilibrium. As all senders are ex-ante symmetric, our analysis will focus on symmetric equilibria.

3 Binary Qualities

3.1 Linear Structure of Equilibrium Payoff Function

We begin our analysis with the binary-quality case, i.e., $M = 2$. Denote a generic element of $\Delta(\{u_0, u_1\})$ by $p \in [0,1]$, standing for the probability that $U_i = u_1$, and sender i's strategy as a element of $\Lambda_i \equiv \{G : [0,1] \rightarrow [0,1] : E_G[p] = \pi\}$. We now describe sender i's expected payoff when all other senders use strategy $G(p)$. Notice that if sender i induces a signal p, the receiver chooses him with probability $\frac{1}{k+1}$ if p is the highest signal among all senders and there are k other senders with signal p. Therefore, his expected



(a) concavity/ non-upward-kink

(b) convexity/ upward-kink

Figure 1: Incentives for Spreading and Contracting

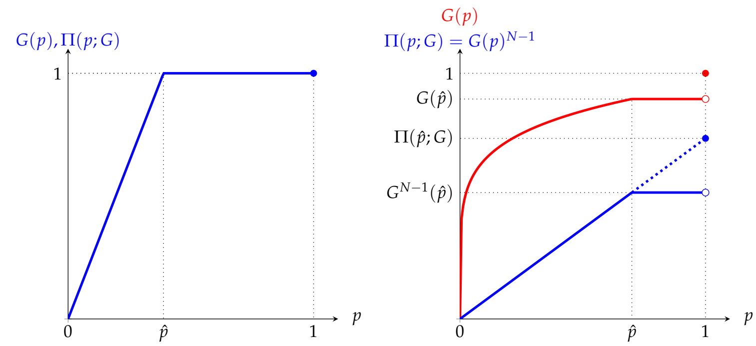
payoff of inducing signal p is

$$\Pi(p,G) \equiv \lim_{p' \to p^-} \sum_{k=0}^{N-1} \frac{1}{k+1} \frac{(N-1)!}{k!\,(N-k-1)!} \left(G(p) - G(p')\right)^k \left(G(p')\right)^{N-k-1}$$
$$= \lim_{p' \to p^-} \frac{\left(G(p)\right)^N - \left(G(p')\right)^N}{N\left(G(p) - G(p')\right)}.$$

If G is continuous at p, then $\Pi(p,G) = G(p)^{N-1}$.

A sender's problem of finding the best response to other senders adopting strategy G is equivalent to finding the concave closure of the payoff function $\Pi(p,G)$, an insight pointed out by Kamenica and Gentzkow (2011). In particular, if an interval of posteriors $(p', p'') \subset (0,1)$ belong to the support of an individual sender's best response, it is necessary that $\Pi(\cdot, G)$ coincides with its concave closure over the interval (p', p''). An implication of this observation is that if interval (p', p'') belongs to the support of a symmetric equilibrium strategy, then it is necessary that $\Pi(p,G)$ is linear over the interval. To see this, suppose $\Pi(p,G)$ is strictly concave over the interval, then each individual sender would find it strictly suboptimal to assign positive probability weights over the whole interval, as his payoff can be increased by concentrating the weights at a single posterior (see Figure 1-(a)). Conversely, suppose $\Pi(p,G)$ is strictly convex over the interval, then again, each sender would have a profitable deviation from the candidate strategy G: this time they would strictly prefer to spread the weights towards both ends of the interval (see Figure 1-(b)).

By a similar logic, a symmetric equilibrium strategy G must have no atom (except possibly at the posterior 1), and no gap (again except possibly at the top). If G has an atom at some posterior $p' < 1$, then the payoff function would differ from its concave closure at p', contradicting that p' is on the support of G. Similarly, suppose the support of G has a gap, say (p', p'') where $p'' < 1$ is on the support of G, then



(a) $N = 2$

(b) $N > 2$

Figure 2: Linear Structure

$\Pi(p, G)$ would differ from its concave closure at p'', which is again a contradiction.

Summarizing the observations above, a necessary condition for the symmetric equilibrium is a linear structure of the payoff function formally defined as follows.

Definition 1 *We say a payoff function exhibits a linear structure if there exist a $\hat{p} \in [0,1]$ and a linear function $\bar{\Pi}$ such that (i) $\bar{\Pi}(p) \geq \Pi(p; G)$ for all $p \in [0,1]$; and (ii)* $\operatorname{supp} G = \operatorname{cl}(\{p \in [0,1] : \bar{\Pi}(p) \geq \Pi(p; G)\}) \in \{[0, \hat{p}], [0, \hat{p}] \cup \{1\}\}$.

Figure 2 illustrates the relationship between a equilibrium strategy and its induced payoff function with the linear structure. The theorem below states that this linear structure of payoff function is the necessary and sufficient condition for the unique symmetric equilibrium.

Theorem 1 *A Bayes-plausible distribution G is a symmetric equilibrium if and only if G induces a payoff function that has the linear structure. Moreover, the symmetric equilibrium exists and is unique.*

3.2 Comparative Statics

In this subsection, we analyze how a change in number of senders affects information disclosure in the unique symmetric equilibrium. Intuitively, as the number of senders increases, the competition for the receiver becomes more intense. To see this, let G_N be a symmetric equilibrium strategy when there are N senders. Then the equilibrium payoff function $\Pi(p, G_N)$ is linear on some interval $[0, \hat{p}_N]$. Now suppose that the number of senders increases to $N' > N$, but $N' - 1$ senders still adopt strategy G_N. Then the payoff function of a sender who faces such $N' - 1$ senders is $(G_N(p))^{N'-1}$, which is convex on $[0, \hat{p}_N]$. Consequently, he benefits from more information disclosure. In response, other senders also engage in more aggressive disclosure. As the number of senders becomes very large, it is extremely likely that an individual sender stands a chance to be chosen by the receiver only if he reveals the most favorable news, i.e., $p = 1$. Consequently, each sender finds it optimal to engage in almost full disclosure, which maximizes the chance that signal $p = 1$ is generated.

Formally, we rank the level of information disclosure of strategies as follows. A strategy G discloses more information than a strategy G', which we denote by $G \succ G'$, if G is a "clockwise rotation" of G'. Specifically, for some $p_L \in (0, 1)$, we have $G(p) > G'(p)$ for all $p \in (0, p_L)$; $G(p_L) = G'(p_L)$; and $G(p) \leq G'(p)$ for all $p \in (p_L, 1]$. It is clear that a clockwise rotation implies second-order stochastic domination, but not vice versa. A strategy G corresponds to full disclosure if and only if $G(0) = 1 - \pi$, which we denote by G_F

Theorem 2 *If the number of senders increases, each sender discloses more information. Moreover, as the number of senders goes to infinity, each sender adopts full disclosure in the limit equilibrium. More formally, let G_N be the unique symmetric equilibrium strategy when there are N senders. Then, $N' > N$ implies $G_{N'} \succ G_N$; and G_N converges to G_F in distribution as $N \to \infty$.*

Theorem 2 confirms the intuition that an increase in the intensity of competition leads to more aggressive disclosure. This result, that having more senders strictly increases disclosure in a unique symmetric equilibrium, contributes to the existing literature in competitive information revelation by moving beyond limit results, while avoiding the issue of equilibrium multiplicity.

Perloff and Salop (1985) analyze competition in setting prices among symmetric sellers of differentiated products, all of which adopt a full disclosure policy. They show that there exists a unique symmetric equilibrium price, which converges to the marginal cost of production as the number of sellers approaches infinity.[7] Our results can, therefore, be viewed as counterparts to theirs in the context of competition in information disclosure.

4 Generalization

4.1 Linear Structure of Payoff Functions

In this subsection, we consider a more general set of feasible proposal qualities Ω, which contains $M (\geq 3)$ distinct elements. In the subsequent analysis, we first show that the equilibrium payoff function necessarily exhibits a linear structure, which is counterpart to that of the binary state-space case analyzed above. By exploiting the equilibrium linear structure, we first show the uniqueness of the symmetric equilib-

[7] The convergence result in their setting requires the tail of the preference distribution to be not too fat. In particular, it holds with a finite support, the case we examine here.

rium. We then develop an algorithm that constructs an equilibrium to show that the unique symmetric equilibrium converges to full disclosure as the number of competing senders approaches infinity.

We begin with a few preliminary observations. First, a sender's strategy $G \in \Delta(\Delta\Omega)$ induces a distribution over ex-post expected qualities; and the receiver selects a sender that gives her the highest expected quality according to the induced distribution. Therefore, a sender's payoff depends on his realized posterior only through the value of expected quality it induces. More specifically, if the other $N-1$ senders use strategy G that induces the distribution F_G over expected qualities, then the expected payoff of inducing posterior p is

$$\Pi_{post}(p;G) \equiv \lim_{u' \to E_p[U_i]^-} \frac{\left(F_G\left(E_p[U_i]\right)\right)^N - \left(F_G(u')\right)^N}{N\left(F_G\left(E_p[U_i]\right) - F_G(u')\right)}. \tag{1}$$

In an equilibrium, a sender neither benefits from more information disclosure, i.e., spreading of posteriors he induces; nor less information disclosure, i.e., contraction of posterior he induces. Therefore, following the same arguments in the previous sections, it is not difficult to see that a Bayes-plausible distribution G over posterior distributions is a symmetric equilibrium strategy if and only if it induces the payoff function $\Pi_{post}(p;G)$ with the linear structure: there exist a $\hat{u} \in (u_0, u_{M-1}]$, and a linear function $\overline{\Pi}_{post}(\cdot) : \Delta\Omega \to \mathbb{R}$ such that (i) $\overline{\Pi}_{post}(p) \geq \Pi_{post}(p;G)$ for all $p \in \Delta\Omega$; and (ii) $\left\{E_p[U] : p \in \operatorname{supp} G\right\} = \left\{E_p[U] : p \in P\right\} \in \left\{[0,\hat{u}], [0,\hat{u}] \cup \{u_{M-1}\}\right\}$, where $P \equiv \operatorname{cl}\left(\left\{p : \Pi_{post}(p;G) = \overline{\Pi}_{post}(p)\right\}\right)$.

Theorem 3 *G is a symmetric equilibrium strategy if and only if it induces $\Pi_{post}(p;G)$ with the linear structure.*

As a sender's payoff depends on his realized posterior only through the value of expected quality induced by the posterior, we can also define the expected payoff of inducing a certain expected quality $u \in [u_0, u_{M-1}]$. The payoff of inducing expected quality of $u \in [u_0, u_{M-1}]$ when the other $N-1$ senders use strategy G is

$$\Pi(u;G) \equiv \lim_{u' \to u^-} \frac{\left(F_G(u)\right)^N - \left(F_G(u')\right)^N}{N\left(F_G(u) - F_G(u')\right)}. \tag{2}$$

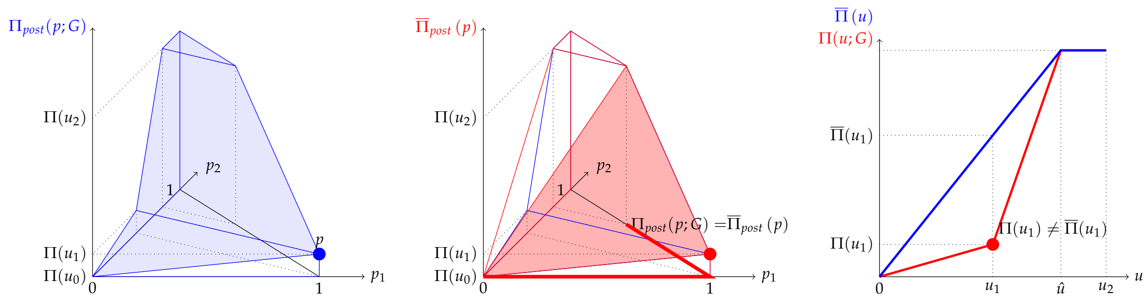
When there is no confusion, we omit G from $\Pi(u;G)$ for expositional simplicity.

One may conjecture that the linear structure of $\Pi_{post}(p;G)$ implies the linear structure of $\Pi(u;G)$ defined by (2).(8) However, this conjecture turns out to be false. The reason is that whereas interior posterior can always be spread when the quality is binary, in the current environment, an interior expected quality $u \in (u_0, u_{M-1})$ may be induced by a degenerate posterior, which cannot be spread.(9)

Figure 3 illustrates this point for the case where $N = 2$ and $M = 3$. Figure 3-(a) and (b) are drawn on the simplex $\left\{(p_1, p_2) \in [0,1]^2 : p_1 + p_2 \leq 1\right\}$. Figure 3-(a) illustrates the payoff function $\Pi_{post}(p;G)$

(8)That is, there exist a $\hat{u} \in (u_0, u_{M-1}]$ and a linear function $\overline{\Pi}$ defined on $[u_0, u_{M-1}]$ such that $\overline{\Pi}(u) \geq \Pi(u;G)$ for all $u \in [u_0, u_{M-1}]$; and $\operatorname{supp} F_G = U \in \left\{[0,\hat{u}], [0,\hat{u}] \cup \{u_{M-1}\}\right\}$, where $U \equiv \operatorname{cl}\left(\left\{u : \Pi(u;G) = \overline{\Pi}(u)\right\}\right)$.

(9)A posterior $p \in \triangle\Omega$ is degenerate if $p_m = 1$ for some m and $p_{m'} = 0$ for all $m' \neq m$.



(a) $\Pi_{post}(p;G)$

(b) $\overline{\Pi}_{post}(p)$ and supp G

(c) $\Pi(u;G)$ and $\overline{\Pi}(u)$

Figure 3: $N = 2$ and $M = 3$

of sender 1 when sender 2 uses the following strategy G: The support of the strategy is

$$\left\{(p_1, p_2) \in [0,1]^2 : p_2 = 0\right\} \cup \left\{(p_1, p_2) \in [0,1]^2 : p_0 = 0 \text{ and } p_1 u_1 + p_2 u_2 \leq \hat{u}\right\},$$

whic isdepicted by the thick red lines on the simplex in 3-(b). This strategy assigns conditionally uniform weights to the respective intervals. The red plane in Figure 3-(b) illustrates a linear function $\overline{\Pi}_{post}(p)$ on the convex hull of supp G. It satisfies $\overline{\Pi}_{post}(p) \geq \Pi_{post}(p;G)$ for all possible distributions. Furthermore, the set of posterior distributions such that $\overline{\Pi}_{post}(p) = \Pi_{post}(p;G)$ coincides with supp G. According to Theorem 3, this strategy G constitutes a symmetric equilibrium.

Notice that G induces the payoff function over expected quality $\Pi(u;G)$ depicted by the red curve in Figure 3-(c). Sender 1's payoff function with respect to induced expected quality $\Pi(u;G)$ exhibits an upward-kink at $u = u_1$ and hence $\Pi(u_1;G) \neq \overline{\Pi}(u_1)$ for any linear function $\overline{\Pi}(u)$ such that $\overline{\Pi}(u) \geq \Pi(u;G)$ for all $u \in [u_0, u_{M-1}]$. This is because over the support of G, expected quality u_1 is only induced by a degenerate posterior distribution p such that $p_1 = 1$. Consequently, even though sender 1's payoff function $\Pi(u;G)$ exhibits an upward-kink at $u = u_1$, he cannot spread the posterior distribution at $u = u_1$ to increase his payoff. Thus, the linear structure of $\Pi(u;G)$ is not a necessary condition for an equilibrium.

In the subsequent analysis, we show that the linear structure of $\Pi_{post}(p;G)$ in an equilibrium, as identified in Theorem 3, has a number of implications on the structure of $\Pi(u;G)$. These implications turn out to provide a complete characterization of the unique equilibrium distribution of expected qualities. This characterization gives us a simple algorithm for constructing the unique equilibrium distribution of expected qualities.

4.2 Necessity of Linear Structure

The linear structure of $\Pi_{post}(p)$ has the following immediate implications on the corresponding payoff function in expected quality $\Pi(u)$. First, as $\Pi_{post}(p)$ has no atom except possibly at the degenerate posterior with $p_{M-1}=1$, $\Pi(u)$ is continuous on the interval $[u_0, u_{M-1})$. Second, there is an $\hat{u} \in [u_0, u_{M-1}]$ such that $\Pi(u)$ is increasing on the interval $[u_0, \hat{u}]$ and constant on the interval $[\hat{u}, u_{M-1})$. Moreover, for $u_m < \hat{u}$, $\Pi(u)$ is linear on the interval $[u_m, \min\{u_{m+1}, \hat{u}\}]$. Intuitively, each $u \in (u_m, \min\{u_{m+1}, \hat{u}\})$ is necessarily induced by a non-degenerate posterior, u can always be "spread" into u_m and $\min\{u_{m+1}, \hat{u}\}$. The linearity of $\Pi(u)$ over the interval $[u_m, \min\{u_{m+1}, \hat{u}\}]$ then ensures such deviation is not profitable. These necessary equilibrium conditions are summarized by the piecewise-linearity condition below.

Definition 2 (Piecewise-linearity Condition) *A payoff function $\Pi(u)$ is piecewise-linear with $\hat{u} \in (u_0, u_{M-1}]$ if (i) it is continuous on the interval $[u_0, u_{M-1})$ with $\Pi(u_0)=0$; and (ii) it is linear on the interval $[u_m, \min\{u_{m+1}, \hat{u}\}]$ for each $m = 0, \cdots, M-2$, and constant on the interval $[\hat{u}, u_{M-1})$.*

For a piecewise-linear payoff function $\Pi(u)$, define s_m as the slope of $\Pi(u)$ on the interval $[u_m, \min\{u_{m+1}, \hat{u}\}]$. Also, define $s^-(\hat{u})$ as the slope of $\Pi(u)$ on $\left[u_{\tilde{\imath}(\hat{u})}, \hat{u}\right]$, where $\tilde{\imath}(\hat{u}) \equiv \arg\max\{m : u_m < \hat{u}\}$, and define $s^+(\hat{u})$ as the slope of the line that connects $(\hat{u}, \Pi(\hat{u}))$ and $(u_{M-1}, \Pi(u_{M-1}))$ on the graph of Π.

We now look at the property at the "top" of the equilibrium payoff function Π. There are three possibilities. One possibility is that F_G does not have an atom at u_{M-1}; this possibility is covered by case (i) in the definition below. See Figure 4-(d). If F_G has an atom at u_{M-1}, then there are two possibilities. One possibility is that $s^-(\hat{u}) < s^+(\hat{u})$ as in Figure 4-(b). This happens only when the size atom of F_G at u_{M-1} is exactly π_{M-1}, i.e., $\Pi(\hat{u}) = (1-\pi_{M-1})^{N-1}$.[(10)] This possibility is covered by case (ii) in the definition below. The last possibility is that $s^-(\hat{u}) = s^+(\hat{u})$, which is covered by case (iii) in the definition below. Also see Figure 4-(c).

Definition 3 (Atom Condition) *A piecewise-linear payoff function $\Pi(u)$ satisfies the atom condition with $\hat{u} \in (u_0, u_{M-1})$ if either (i) $\Pi(\hat{u}) = 1$; (ii) $\Pi(\hat{u}) = (1-\pi_{M-1})^{N-1}$ and $s^-(\hat{u}) < s^+(\hat{u})$; or (iii) $\Pi(\hat{u}) \in \left[(1-\pi_{M-1})^{N-1}, 1\right)$ and $s^-(\hat{u}) = s^+(\hat{u})$.*

As we have seen in the previous subsection, the linear structure of $\Pi_{post}(p)$ does not necessarily imply $s_m = s_{m+1}$ because u_{m+1} may be induced by a degenerate posterior. The linear structure of $\Pi_{post}(p)$, however, rules out the possibility that $s_m > s_{m+1}$. This is because if $s_m > s_{m+1}$, then a sender will benefit from contracting the weights on the expected qualities in the neighborhood of u_{m+1} onto u_{m+1}. Furthermore, if $s_m < s_{m+1}$, i.e., $\Pi(u)$ exhibits an upward-kink at u_{m+1}, then each $u \in \text{supp}(F_G)$ is *not*

[(10)]Otherwise, a positive measure of $u \le \hat{u}$ is induced by posteriors p with $p_{M-1} > 0$ under G, and facing such a payoff function $\Pi(u)$, the sender can profit from "spreading" the induced utilities to u_{M-1} and some lower utilities.

induced by a convex combination of $u_k \in \{u_{m+2}, \cdots, u_{M-1}\}$ and $u_{k'} \in \{u_0, \cdots, u_m\}$. For otherwise, the upward-kink of $\Pi(u)$ at u_{m+1} implies that a sender can benefit from spreading such a u to some pair of expected qualities $\{u', u''\}$ with $u' < u_{m+1}$ and $u'' > u_{m+1}$.

Definition 4 (Upward-Kink Condition) *A piecewise linear payoff function $\Pi(u)$ satisfies the upward-kink condition if both conditions below hold. First, $s_m \leq s_{m+1}$ for all $m \in \{0, 1, ..., \tilde{\imath}(\hat{u}) - 1\}$. Second, if $\Pi(u)$ has an upward kink at u_{m+1}, then for each $p \in \operatorname{supp}(G)$, $p_k p_{k'} = 0$ for all $k \in \{0, \cdots, m\}$ and $k' \in \{m+2, \cdots, M-1\}$.*

If a Bayes-plausible distribution G is a symmetric equilibrium strategy, then it necessarily induces a payoff function that satisfies all conditions above.

Definition 5 (Generalized Linear Structure) *A Bayes-plausible strategy G induces a payoff function $\Pi(u)$ with the generalized linear structure if there exists a $\hat{u} \in (u_0, u_{M-1})$ such that $\Pi(u)$ satisfies the piecewise-linearity condition with $\hat{u}$, the atom condition with $\hat{u}$, and the upward-kink condition.*

In sum, a necessary condition for a strategy to constitute a symmetric equilibrium is that it induces a payoff function with the generalized linear structure. Below, by offering a simple algorithm that constructs a payoff function with the generalized linear structure, we show that the generalized linear structure is also a sufficient condition for an equilibrium. As we will see in the next subsection, if $\Pi(u; G)$ satisfies the generalized linear structure is also a sufficient condition, then $\Pi_{post}(p; G)$ satisfies the linear structure, and hence G is a symmetric equilibrium strategy.

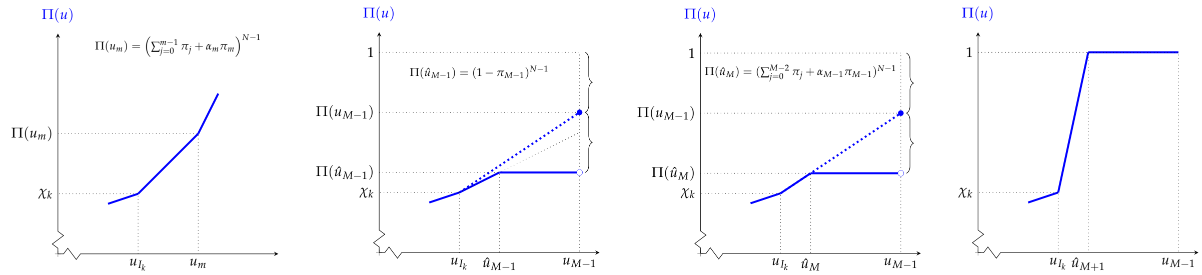
4.3 Algorithm and Sufficiency

In this subsection, we briefly describe an algorithm that constructs a payoff function satisfying the generalized linear structure defined above. As the algorithm yields a unique output, the equilibrium distribution of expected utilities (if exists) is unique.

Suppose the equilibrium symmetric strategy G induces a distribution of expected utilities F_G such that the κ-th upward kink occurs at u_{I_κ}. As the equilibrium payoff function $\Pi(u)$ satisfies the generalized linear structure, exactly one of the following properties, as illustrated by Figure 4, holds.

Property-m : The first upward kink after u_{I_κ} occurs at $u_m \in \{u_{I_\kappa+1}, \cdots, u_{M-2}\}$, i.e., $\Pi(u)$ is linear on the interval $[u_{I_\kappa}, u_m]$ and has an upward kink at u_m.

Property-$M-1$: $\Pi(u)$ does not have any upward kink at $u_i \in \{u_{I_\kappa+1}, \cdots, u_{M-2}\}$. Moreover, there exists a $\hat{u}_{M-1} \in (u_{I_\kappa}, u_{M-1})$ such that $F_G(\hat{u}_{M-1}) = 1 - \pi_{M-1}$ and $\frac{\Pi(\hat{u}_{M-1}) - \Pi(u_{I_\kappa})}{\hat{u}_{M-1} - u_{I_\kappa}} < \frac{\Pi(u_{M-1}) - \Pi(\hat{u}_{M-1})}{u_{M-1} - \hat{u}_{M-1}}$.



(a) Property-m (b) Property-$M-1$ (c) Property-M (d) Property-$M+1$

Figure 4: Properties $m, M-1, M, M+1$

Property-$M:$ $\Pi(u)$ does not have any upward-kink at $u_i \in \{u_{I_\kappa+1}, \cdots, u_{M-2}\}$. Moreover, there exists a $\hat{u}_M \in (u_{I_\kappa}, u_{M-1})$ such that $\frac{\Pi(\hat{u}_M)-\Pi(u_{I_\kappa})}{\hat{u}_M - u_{I_\kappa}} = \frac{\Pi(u_{M-1})-\Pi(\hat{u}_M)}{u_{M-1}-\hat{u}_M}$.

Property-$M+1:$ $\Pi(u)$ does not have any upward-kink at $u_i \in \{u_{I_\kappa+1}, \cdots, u_{M-2}\}$. Moreover, $F_G(\hat{u}_{M+1}) = 1$ for some $\hat{u}_{M+1} \in (u_{I_\kappa}, u_{M-1}]$.

The key of the algorithm is to identify which property above holds, given the κ-th upward kink occurs at u_{I_κ}. For simplicity of exposition, we illustrate the case of $N = 2$ here. See Appendix A for the formal description that covers any finite number of senders.

We first define a sequence of *potential slopes*, $s_m^{\kappa+1}$ for each $m \in \{I_\kappa, I_\kappa+1, ..., M-1\}$ when Π has the $(\kappa+1)$-th upward kink at u_m. Specifically, it is defined by equating the two expressions for $E_G[u|u \in [u_{I_\kappa}, u_m]]$ discussed below. First, as $\Pi(u) = F_G(u)$ is linear on the interval $[u_{I_\kappa}, u_m]$, $E_G[u|u \in [u_{I_\kappa}, u_m]] = \frac{1}{2}(u_{I_\kappa} + u_m)$. Second, by the upward-kink condition at u_m, there exists an $\alpha_m \in [0,1]$ such that $F_G(u_m) = \sum_{j=0}^{m-1} \pi_j + \alpha_m \pi_m$ and

$$E_G[u|u \in [u_{I_\kappa}, u_m]] = \frac{\left(\sum_{j=0}^{I_\kappa} \pi_j - F_G(u_{I_\kappa})\right) \pi_{I_\kappa} u_{I_\kappa} + \sum_{j=I_\kappa+1}^{m-1} \pi_j u_j + \alpha_m \pi_m u_m}{\left(\sum_{j=0}^{I_\kappa} \pi_j - F_G(u_{I_\kappa})\right) \pi_{I_\kappa} + \sum_{j=I_\kappa+1}^{m-1} \pi_j + \alpha_m \pi_m}.$$

If there exists an $\alpha_m \in (0,1)$ that equates the two expressions for $E_G[u|u \in [u_{I_\kappa}, u_m]]$, then the slope $s_m^{\kappa+1}$ is defined to be $\frac{\sum_{j=0}^{m-1} \pi_j + \alpha_m \pi_m - F_G(u_{I_\kappa})}{u_m - u_{I_\kappa}}$. If no such α_m exists, then define $s_m^{\kappa+1} \equiv \infty$. We define $s_\ell^{\kappa+1}, \ell \in \{M-1, M, M+1\}$ in a similar manner.

If $s_\ell^{\kappa+1} = \infty$, then it is clear that Π does not satisfy Property-ℓ. The converse is not necessarily true, i.e., even if $s_\ell^{\kappa+1} < \infty$, Π may not satisfy Property-ℓ. However, as we formally prove in the appendix (Lemma 2), if $s_\ell^{\kappa+1} < s_{\ell'}^{\kappa+1} < \infty$ for some ℓ and ℓ', or $s_\ell = s_{\ell'}$ and $\ell' < \ell$, then Π does not satisfy Property-ℓ'. The intuition is that keeping the value of $\Pi(u)$ as low as possible slacks the constraints imposed by the upward-kink condition and the atom condition at the top on the higher values of u_m. Using this result, we can identify the property that Π satisfies by finding the index $\ell \in \{I_\kappa, I_\kappa + 1, ..., M+1\}$ that minimizes $s_\ell^{\kappa+1}$.[11] Therefore, by initiating the algorithm with $I_0 = 0$, we can construct a payoff function by identifying the locations of all upward kinks.

The theorem below establishes the existence and uniqueness of a symmetric equilibrium. As a result, the algorithm described above necessarily identifies a payoff function satisfying the generalized linear structure. Moreover, as is clear from the description of the algorithm, it yields at most one output. The uniqueness of the algorithm's output in turn implies that the generalized linear structure of induced payoff function is also a sufficient condition for an equilibrium.

Theorem 4 *A Bayes-plausible strategy G is an equilibrium if and only if the induced payoff function $\Pi(u; G)$ has the generalized linear structure. A symmetric equilibrium exists and is unique up to the induced distribution of expected qualities. The algorithm described above constructs the unique equilibrium distribution of expected-quality. If $N = 2$, then the distribution of expected qualities is necessarily symmetric in equilibrium.*

Note that whereas the distribution of expected qualities is unique in equilibrium, there may, in general, be multiple posterior distributions that induce it.

Finally, using the algorithm, we can show that the symmetric equilibrium approaches full disclosure in the limit as the number of senders goes to infinity. More formally, let $F_{Full}(u)$ be the expected-quality distribution that corresponds to full disclosure, i.e., $F_{Full}(u) = \sum_{j=0}^{m} \pi_j$ for all $u \in [u_m, u_{m+1})$, and $m = 0, \cdots, M-2$.

Theorem 5 *Let $F_{G,N}$ be the unique symmetric equilibrium expected-quality distribution if there are N symmetric senders. Then $F_{G,N}$ converges to F_{Full} in distribution as the number of senders approaches infinity.*

The intuition of the result is as follows. When a sender is facing a large number of competing senders, he understands that with a very high probability, some other senders would generate a signal with expected quality very close to the maximum equilibrium value. This creates a strong incentives for each sender to maximize the probability of generating the most favorable equilibrium signal, so when the number of senders is sufficiently large, each sender would almost fully reveal the state u_{M-1}. In a similar manner, for any $m \in \{1, \cdots, M-2\}$, conditional on the highest signal among the other senders being no higher than u_m, each sender understands that with a very high probability, some other senders would generate a signal with expected quality very close to the maximum equilibrium value conditional on it being no higher than u_m. Therefore, each sender almost fully reveals u_m.

[11] In the case of a tie, pick the largest index.

5 Conclusion

In this paper, we study competitive Bayesian persuasion with multiple symmetric senders. We find that in the unique symmetric equilibrium, each sender engages in more aggressive disclosure when he competes with a larger number of senders. Moreover, full disclosure arises as the limit equilibrium outcome as the number of senders approaches infinity. In showing these results, we provide an algorithm that computes the unique symmetric equilibrium for an arbitrary finite state space.

There are a number of extensions deserving further investigation. First, one can introduce ex-ante asymmetry between senders. We believe that the linear structure of payoff function could still play a key role in equilibrium characterization. We note that the sufficiency of this finding holds true in general. However, a potential complication that may arise is that the equilibrium strategy of some sender may have an atom at some degenerate posterior.

Another interesting direction is to consider competition in persuading multiple heterogenous receivers, as in the context of voting (Alonso and Câmara (2016)) and takeover contest between raiders and incumbent management (Dekel and Wolinsky (2011)). Insofar as the median voter or shareholder is pivotal, our analysis could be directly applicable as all senders would target persuading the median receiver. However, the analysis could be much more complicated if different senders would target different coalitions of receivers in equilibrium. Finally, competition in dynamic persuasion would be another exciting avenue for future research.(12) Even if senders are ex-ante symmetric, as more information are revealed over time, they become heterogenous as persuasion goes on. Analysis of the first extension above would thus pave a way for studying competition in dynamic persuasion.

Appendix A: Formal Description of Algorithm

Output of Algorithm:

Define $I_0 = 0, \sigma_0 \equiv 0$ and $\chi_0 \equiv 0$. By the end of κ-th step, the algorithm generates $\left(\{I_j\}_{j=0}^{\kappa}, \{\sigma_j\}_{j=0}^{\kappa}, \{\chi_j\}_{j=0}^{\kappa}\right)$, where I_j is the j-th upward kink of Π, σ_j is the right-derivative of Π at u_{I_j}, and $\chi_j = \Pi\left(u_{I_j}\right)$.

Description of Algorithm:

The $\kappa + 1$-th step of the algorithm proceeds as follows:

1. Calculate $\left\{s_j^{\kappa+1}\right\}_{j=I_{\kappa+1}}^{M+1}$ and $\left\{\hat{u}_j\right\}_{j=M-1}^{M+1}$ defined by (3), (4), (6), and (9).
2. Define $I_{\kappa+1} \equiv \max\left\{\arg\min_{\ell \in \{I_\kappa+1,\cdots,M+1\}} \left\{s_\ell^{\kappa+1}\right\}\right\}$.

(12) Au (2015) and Ely (2017) study the problem of single-sender dynamic persuasion.

3. If $I_{\kappa+1} = M-1, M$, or $M+1$, then the algorithm constructs F_G by

$$F_G(u) = \begin{cases} \left(\sum_{j=0}^{\kappa} \sigma_j \max\left\{0, \left(\min\left\{u_{I_j}, u\right\} - u_{I_{j-1}}\right)\right\}\right)^{\frac{1}{N-1}} & u \in [u_0, u_{I_\kappa}] \\ \left(\sum_{j=0}^{\kappa} \sigma_j \left(u_{I_j} - u_{I_{j-1}}\right) + s_{I_{\kappa+1}}^{\kappa+1} \left(\min\left\{u, \hat{u}_{I_{\kappa+1}}\right\} - u_{I_\kappa}\right)\right)^{\frac{1}{N-1}} & u \in (u_{I_\kappa}, u_{M-1}) \\ 1 & u = u_{M-1} \end{cases}.$$

4. If $I_{\kappa+1} \in \{I_\kappa + 1, \cdots, M-2\}$, then define $\sigma_{\kappa+1} \equiv \min_{\ell \in \{I_\kappa+1,\cdots,M+1\}} \left\{s_\ell^{\kappa+1}\right\}$ and $\chi_{\kappa+1} \equiv \sum_{j=0}^{\kappa+1} \sigma_j \left(u_{I_j} - u_{I_{j-1}}\right)$. The algorithm proceeds to $\kappa+2$-th step with $\left(\left\{I_j\right\}_{j=0}^{\kappa+1}, \left\{\sigma_j\right\}_{j=0}^{\kappa+1}, \left\{\chi_j\right\}_{j=0}^{\kappa+1}\right)$.

Definitions of $\left\{s_j^{\kappa+1}\right\}_{j=I_{\kappa+1}}^{M+1}$ and $\left\{\hat{u}_j\right\}_{j=M-1}^{M+1}$:

- $s_m^{\kappa+1}$: For each $m \in \{I_\kappa + 1, \cdots, M-1\}$, define

$$s_m^{\kappa+1} \equiv \begin{cases} \frac{\left(\sum_{j=0}^{m-1} \pi_j + \tilde{\alpha}_m \pi_m\right)^{N-1} - \chi_\kappa}{u_m - u_{I_\kappa}} & \text{if } \tilde{\alpha}_m \in (0,1) \\ \infty & \text{otherwise} \end{cases}, \tag{3}$$

where $\tilde{\alpha}_m$ solves

$$\frac{N-1}{N} \frac{\left(\sum_{j=0}^{m-1} \pi_j + \tilde{\alpha}_m \pi_m\right)^N - \chi_\kappa^{\frac{N}{N-1}}}{\left(\sum_{j=0}^{i-1} \pi_j + \tilde{\alpha}_m \pi_m\right)^{N-1} - \chi_\kappa} = \sum_{j=0}^{I_\kappa} \pi_j + \sum_{j=I_\kappa+1}^{m-1} \pi_j \frac{u_m - u_j}{u_m - u_{I_\kappa}}.$$

- $\left(s_{M-1}^{\kappa+1}, \hat{u}_{M-1}\right)$:

$$\left(s_{M-1}^{\kappa+1}, \hat{u}_{M-1}\right) \equiv \begin{cases} \left(\frac{\left(\sum_{j=0}^{M-2} \pi_j\right)^{N-1} - \chi_\kappa}{\hat{u}_{M-1} - u_{I_\kappa}}, \hat{u}_{M-1}\right) & \text{if } \hat{u}_{M-1} \in (u_{I_\kappa}, u_{M-1}) \text{ and } I_\kappa \in \{0, \cdots, M-3\} \\ (\infty, \infty) & \text{otherwise} \end{cases}, \tag{4}$$

where $\hat{u}_{M-1}$ solves

$$\frac{N-1}{N} \frac{(1-\pi_{M-1})^N - \chi_\kappa^{\frac{N}{N-1}}}{(1-\pi_{M-1})^{N-1} - \chi_\kappa} = \sum_{j=0}^{I_\kappa} \pi_j + \sum_{j=I_\kappa+1}^{M-2} \pi_j \frac{\hat{u}_{M-1} - u_j}{\hat{u}_{M-1} - u_{I_\kappa}}. \tag{5}$$

- $\left(s_M^{\kappa+1}, \hat{u}_M\right)$:

$$\left(s_M^{\kappa+1}, \hat{u}_M\right) \equiv \begin{cases} \left(\frac{\left(\sum_{j=0}^{M-2} \pi_j + \tilde{\alpha}_M \pi_{M-1}\right)^{N-1} - \chi_\kappa}{\hat{u}_M - u_{I_\kappa}}, \hat{u}_M\right) & \text{if } \hat{u}_M \in (u_{I_\kappa}, u_{M-1}) \text{ and } \tilde{\alpha}_M \in [0,1) \\ (\infty, \infty) & \text{otherwise} \end{cases}. \tag{6}$$

where $(\tilde{\alpha}_M, \hat{u}_M)$ is the solution to the system of equations

$$\frac{N-1}{N}\frac{\left(\sum_{j=0}^{M-2}\pi_j+\tilde{\alpha}_M\pi_{M-1}\right)^N-\chi_\kappa^{\frac{N}{N-1}}}{\left(\sum_{j=0}^{M-2}\pi_j+\tilde{\alpha}_M\pi_{M-1}\right)^{N-1}-\chi_\kappa}$$
$$=\sum_{j=0}^{I_\kappa}\pi_j+\sum_{j=I_\kappa+1}^{M-2}\pi_j\frac{\hat{u}_M-u_j}{\hat{u}_M-u_{I_\kappa}}+\tilde{\alpha}_M\pi_{M-1}\frac{\hat{u}_M-u_{M-1}}{\hat{u}_M-u_{I_\kappa}}. \tag{7}$$

and

$$\frac{\left(\sum_{j=0}^{M-2}\pi_j+\alpha_M\pi_{M-1}\right)^{N-1}-\chi_\kappa}{\hat{u}_M-u_{I_\kappa}}$$
$$=\frac{1}{u_{M-1}-\hat{u}_M}\left(\frac{1-\left(\sum_{j=0}^{M-2}\pi_j+\alpha_M\pi_{M-1}\right)^N}{N\left(1-\left(\sum_{j=0}^{M-2}\pi_j+\alpha_M\pi_{M-1}\right)\right)}-\left(\sum_{j=0}^{M-2}\pi_j+\alpha_M\pi_{M-1}\right)^{N-1}\right). \tag{8}$$

- $\left(s_{M+1}^{\kappa+1}, \hat{u}_{M+1}\right)$:

$$\left(s_{M+1}^{\kappa+1}, \hat{u}_{M+1}\right) \equiv \begin{cases} \left(\frac{1-\chi_\kappa}{\hat{u}_{M+1}-u_{I_\kappa}}, \tilde{u}_{M+1}\right) & \text{if } \hat{u}_{M+1} \in (u_{I_\kappa}, u_{M-1}] \\ (\infty,\infty) & \text{otherwise} \end{cases}, \tag{9}$$

where $\tilde{u}_{M+1}$ is the solution to

$$\frac{N-1}{N}\frac{1-\chi_\kappa^{\frac{N}{N-1}}}{1-\chi_\kappa}=\sum_{j=0}^{I_\kappa}\pi_j+\sum_{j=I_\kappa+1}^{M-1}\pi_j\frac{\hat{u}_{M+1}-u_j}{\hat{u}_{M+1}-u_{I_\kappa}}. \tag{10}$$

Appendix B: Proofs

Proof of Theorem 1

The necessity and the sufficiency of the linear structure of the payoff function for binary quality is a special case of Theorem 3, and thus omitted.

We first establish the existence of a symmetric equilibrium by showing that there exists a strategy G such that the induced payoff function has the linear structure defined in Definition 1. Recall the linear structure of payoff function is characterized by a cutoff postrior $\hat{p}$ and whether or not it has a jump at posterior 1. Note that $\Pi(p, G) = G^{N-1}(p)$ for all $p \in [0, 1)$, and $\Pi(1, G) = \frac{1-G^N(\hat{p})}{N(1-G(\hat{p}))}$. Therefore, for $p \in [0, \hat{p}]$, we have $G(p) = \left(\frac{p}{\hat{p}}G^{N-1}(\hat{p})\right)^{\frac{1}{N-1}}$. The Bayes-plausibility condition implies

$$\frac{\hat{p}}{N}G(\hat{p})+1-G(\hat{p})-\pi=0. \tag{11}$$

If $\Pi(p, G)$ does not jump at posterior 1, then it is necessary that $G(\hat{p}) = 1$, and equation (11) simplifies to $\hat{p} = N\pi$. Thus, if $N\pi \leq 1$, the existence of a symmetric equilibrium is guaranteed.

Next, if $\Pi(p, G)$ has an upward jump at posterior 1, then the linear structure of Π requires that $G(\hat{p}) < 1$ and $\frac{1-G^N(\hat{p})}{N(1-G(\hat{p}))} = \frac{G^{N-1}(\hat{p})}{\hat{p}}$. The latter equality can be rewritten as

$$\hat{p} = \frac{NG^{N-1}(\hat{p})(1 - G(\hat{p}))}{1 - G^N(\hat{p})}. \tag{12}$$

The existence of a symmetric equilibrium is guaranteed if there exists a pair $(\hat{p}, G(\hat{p})) \in [0,1)^2$ that simultaneously solves equations (11) and (12). Substituting equation (12) into equation (11) and rearranging gives

$$\frac{1 - G(\hat{p})}{1 - G^N(\hat{p})} = \pi. \tag{13}$$

As the left-hand side of equation (13) is decreasing in $G(\hat{p})$, equals 1 when $G(\hat{p}) = 0$, and has a limit $\frac{1}{N}$ at $G(\hat{p}) = 1$, a unique solution in $G(\hat{p}) \in [0,1)$ exists if and only if $N\pi > 1$. Moreover, the convexity of the left-hand side of equation (13) in $G(\hat{p})$ implies that the solution satisfies $G(\hat{p}) < \frac{1-\pi}{1-N^{-1}}$. This in turn implies that the solution to the system of equations (11) and (12) has $\hat{p} < 1$.

Finally, we establish the uniqueness of the symmetric equilibrium. First, if $N\pi \leq 1$, the derivation above implies that the system of equations (11) and (12) has no solution, and thus all symmetric equilibria has no atom at posterior 1. Consequently, the symmetric equilibrium is necessarily unique as equation (11) uniquely pins down the cutoff value $\hat{p}$. Similarly, if $N\pi > 1$, equation (11) admits no solution with $G(\hat{p}) = 1$, so all symmetric equilibria has an atom at posterior 1. Consequently, the symmetric equilibrium is necessarily unique as the system of equations (11) and (12) uniquely pins down the equilibrium values of $\hat{p}$ and $G(\hat{p})$. Q.E.D.

Proof of Theorem 2

Consider a pair of integers N_1 and $N_2 > N_1$. Suppose first $N_1 > 1/\pi$. In this case, there exists a $\hat{p}_N \in (0,1)$ such that $G_N^{N-1}(p)$, $N = N_1, N_2$, is linear on $p \in [0, \hat{p}_N]$, and has an atom at posterior 1. For notational simplicity, let $g_N \equiv G_N(\hat{p}_N)$.

We first show that $g_{N_1} > g_{N_2}$ and $\hat{p}_{N_1} > \hat{p}_{N_2}$. Recall equation (13) reads $\frac{1-g_N}{1-g_N^N} = \pi$. Since $\frac{1-x}{1-x^N}$ is decreasing in both x and N, we have $g_{N_1} > g_{N_2}$. Furthermore, observe that $\frac{Nx^{N-1}(1-x)}{1-x^N}$ is decreasing in N and increasing in x. Therefore, the finding that g_N is decreasing in N and equation (12) together imply $\hat{p}_{N_1} > \hat{p}_{N_2}$.[13]

[13]Since $\frac{\partial(1-x^n+n\ln x)}{\partial x} = n\frac{1-x^n}{x} > 0$ for $x \in (0,1]$, $\frac{\partial\left(\frac{nx^{n-1}(1-x)}{1-x^n}\right)}{\partial n} = (1-x)\,x^{n-1}\frac{1-x^n+n\ln x}{(1-x^n)^2} < 0$. Furthermore, $\frac{\partial(n(1-x)-(1-x^n))}{\partial x} = -n\left(1-x^{n-1}\right) < 0$. Therefore, $\frac{\partial\left(\frac{nx^{n-1}(1-x)}{1-x^n}\right)}{\partial x} = \frac{nx^{n-2}}{(1-x^n)^2}\left(n(1-x) - (1-x^n)\right) > 0$.

Note that on the interval $\left[0, \hat{p}_{N_1}\right]$, while $G_{N_1}^{N_1-1}(p)$ is linear, $G_{N_2}^{N_1-1}(p)$ is concave. If $G_{N_1}^{N_1-1}(p) > G_{N_2}^{N_1-1}(p)$ for all $p \in \left(0, \hat{p}_{N_2}\right)$, then G_{N_2} first-order stochastically dominates G_{N_1}, which is a contradiction. Together with the fact that $G_{N_1}(0) = G_{N_2}(0) = 0$ and $\hat{p}_{N_2} < \hat{p}_{N_1}$, G_{N_2} must cross G_{N_1} once from above. Therefore, G_{N_2} discloses more information than G_{N_1}.

The cases where $N_2 \leq 1/\pi$ and $N_1 \leq 1/\pi < N_2$ can be shown in a similar manner, and hence omitted.

Lastly, for $N > 1/\pi$, g_N is decreasing in N and $\pi = \frac{1-g_N}{1-g_N^N}$. Therefore, $\lim_{N\to\infty} g_N = 1-\pi$. Then by the Bayes-plausibility condition, we have $\lim_{N\to\infty} G_N(p) = 1-\pi$ for all $p \in (0,1)$. As a result, G_N converges to full disclosure in distribution. Q.E.D.

Proof of Theorem 3

The sufficiency is straightforward. We thus only show the necessity. We first introduce a few notations. For $u \in [u_0, u_{M-1}]$, define $P(u) \equiv \left\{p \in \Delta\Omega : E_p[U_i] = u\right\}$, and $G(A) \equiv \int_A dG$ for any set $A \subset \Delta\Omega$. We use $\tilde{\pi}_m \in \Delta\Omega$ to denote a degenerate posterior p such that $p_m = 1$. We also define $\hat{u} = \sup\left\{u : u = E_p[U_i], p \in (\operatorname{supp} G) \setminus \tilde{\pi}_{M-1}\right\}$; and $\bar{u} = \sup\left\{u : u = E_p[U_i], p \in \operatorname{supp} G\right\}$.

By Corollary 2 of Kamenica and Gentzkow (2011): (a) a strategy G is a best response to payoff function $\Pi_{post}(p; G)$ if and only if $E_G\left[\Pi_{post}(p; G)\right] = C\left(\Pi_{post}(\pi; G)\right)$, where $C\left(\Pi_{post}(p; G)\right)$ is the concave closure of Π. (b) If G is a best response to $\Pi_{post}(p; G)$, then G assigns a zero measure to the set x$\left\{p : C\left(\Pi_{post}(p; G)\right) > \Pi_{post}(p; G)\right\}$.

First, we show that F_G does not have atom at any $u \in [u_0, u_{M-1})$. Suppose F_G has an atom at some $u \in [u_0, u_{M-1})$. Then $\Pi_{post}(p; G) < C\left(\Pi_{post}(p; G)\right)$ for all $p \in P(u)$. This contradicts that G assigns an atom at some $p \in P(u)$.

Next, there exists an $\hat{u} \leq u_{M-1}$ such that $F_G(u)$ is increasing on $[u_0, \hat{u})$. Let (u_l, u_h), where $u_h < \hat{u}$, be a maximal open interval in $[u_0, u_{M-1}]$ such that $F_G(u_h) - F_G(u_l) = 0$. Let $\varepsilon > 0$ and we construct a strategy G_ε as follows. Define $x(\varepsilon)$ by the solution to the following equation:

$$u_l = \frac{\int_{u_l - x(\varepsilon)}^{u_l} u dF_G(u) + \int_{u_h}^{u_h+\varepsilon} u dF_G(u)}{F_G(u_h+\varepsilon) - F_G(u_h) + F_G(u_l) - F_G(u_l - x(\varepsilon))}. \tag{14}$$

Strategy G_ε modifies the equilibrium strategy G by combining weights on the intervals $[u_h, u_h+\varepsilon]$ and $[u_l - x(\varepsilon), u_l]$ to form an atom at u_l. The profit of adopting strategy G_ε exceeds that of G by at least

$$\begin{aligned}
&\left(1 - \frac{1}{N}\right)\left[F_G(u_l) - F_G(u_l - x(\varepsilon))\right]\left[F_G^{N-1}(u_l) - F_G^{N-1}(u_l - x(\varepsilon))\right] \\
&- \frac{1}{N}\left[F_G(u_h+\varepsilon) - F_G(u_h)\right]\left[F_G^{N-1}(u_h+\varepsilon) - F_G^{N-1}(u_h)\right].
\end{aligned} \tag{15}$$

The reason is that the two strategies yields different payoffs only if both the sender's induced expected utility, as well as that of the highest among the other $N-1$ senders, lie in the intervals $[u_l - x(\varepsilon), u_l]$

and $[u_h, u_h + \varepsilon]$. Conditional on the highest expected utility of other $N-1$ senders lie in the interval $[u_h, u_h + \varepsilon]$, strategy G' lowers the sender's probability of winning by $\frac{1}{N}\left[F_G\left(u_h+\varepsilon\right)-F_G\left(u_h\right)\right]$. On the other hand, conditional on the highest expected utility of other $N-1$ senders lie in the interval $[u_l - x(\varepsilon), u_l]$, strategy G' raises the sender's probability of winning by $\left(1-\frac{1}{N}\right)\left[F_G\left(u_l\right)-F_G\left(u_l-x\left(\varepsilon\right)\right)\right]$. It suffices to show that expression (15) is positive for some $\varepsilon > 0$, i.e.,

$$\frac{F_G\left(u_l\right)-F_G\left(u_l-x\left(\varepsilon\right)\right)}{F_G\left(u_h+\varepsilon\right)-F_G\left(u_h\right)}\frac{F_G^{N-1}\left(u_l\right)-F_G^{N-1}\left(u_l-x\left(\varepsilon\right)\right)}{F_G^{N-1}\left(u_h+\varepsilon\right)-F_G^{N-1}\left(u_h\right)}>\frac{1}{N-1} \tag{16}$$

holds for some $\varepsilon > 0$.

As $u_h, u_l \in \operatorname{supp}(F_G)$, the definition of $x(\cdot)$ in equation (14) guarantees that it is locally differentiable at 0. Differentiating equation (14) with respect to ε and rearranging, we get

$$x'\left(\varepsilon\right)=\frac{\left(u_h-u_l+\varepsilon\right)f_G\left(u_h+\varepsilon\right)}{x\left(\varepsilon\right)f_G\left(u_l-x\left(\varepsilon\right)\right)},$$

where f_G is the density function of F_G. As $x(0) = 0$, we have $\lim_{\varepsilon\to 0} x'(\varepsilon) = \infty$. Now the limiting value of the left-hand side of inequality (16) is given by

$$\begin{aligned}
&\lim_{\varepsilon\to 0}\frac{F_G\left(u_l\right)-F_G\left(u_l-x\left(\varepsilon\right)\right)}{F_G\left(u_h+\varepsilon\right)-F_G\left(u_h\right)}\frac{F_G^{N-1}\left(u_l\right)-F_G^{N-1}\left(u_l-x\left(\varepsilon\right)\right)}{F_G^{N-1}\left(u_h+\varepsilon\right)-F_G^{N-1}\left(u_h\right)}\\
&=\lim_{\varepsilon\to 0}\frac{f_G\left(u_l-x\left(\varepsilon\right)\right)x'\left(\varepsilon\right)}{f_G\left(u_h+\varepsilon\right)}\lim_{\varepsilon\to 0}\frac{-F_G^{N-2}\left(u_l-x\left(\varepsilon\right)\right)f_G\left(u_l-x\left(\varepsilon\right)\right)x'\left(\varepsilon\right)}{F_G^{N-2}\left(u_h+\varepsilon\right)f_G\left(u_h+\varepsilon\right)}\\
&=\infty.
\end{aligned}$$

We now establish that $C\left(\Pi_{post}\left(p;G\right)\right)$ is linear on the convex hull $\operatorname{co}(\operatorname{supp} G)$ of $\operatorname{supp} G$. By the Minkowski-Caratheodory theorem, for any $p \in \operatorname{co}(\operatorname{supp} G)$ there exists a set of posteriors $Q \subset \operatorname{supp} G$ such that $|Q| \leq M$, and a set of weights $\left\{\alpha_p(q) \in [0,1] : q \in Q\right\}$ such that $p = \sum_{q\in Q}\alpha_p(q)\, q$ and $\sum_{q\in Q}\alpha_p(q) = 1$. Therefore, if $C\left(\Pi_{post}\left(p;G\right)\right)$ is not linear on $\operatorname{co}(\operatorname{supp} G)$, then there exists a $p \in \operatorname{co}(\operatorname{supp} G)$ such that $C\left(\Pi_{post}\left(p;G\right)\right) > \sum_{q\in Q}\alpha_p(q)\,\Pi_{post}(q;G)$. For each $p' \in \operatorname{co}(\operatorname{supp} Q)$, there exists a set of weights $\left\{\alpha_{p'}(q) \in [0,1] : q \in Q\right\}$ such that $p' = \sum_{q\in Q}\alpha_{p'}(q)\, q$. If $\Pi_{post}(p';G) = \sum_{q\in Q}\alpha_{p'}(q)\,\Pi_{post}(q;G)$ for all $p' \in \operatorname{co}(\operatorname{supp} Q)$, then $C\left(\Pi_{post}\left(p;G\right)\right) = \sum_{q\in Q}\alpha_p(q)\, C\left(\Pi_{post}\left(q;G\right)\right) = \sum_{q\in Q}\alpha_p(q)\,\Pi_{post}(q;G)$. Therefore, $C\left(\Pi_{post}\left(p;G\right)\right) > \sum_{q\in Q}\alpha_p(q)\,\Pi_{post}(q;G)$ implies that there exists a $p' \in \operatorname{co}(\operatorname{supp} Q)$ such that $\Pi_{post}(p';G) > \sum_{q\in Q}\alpha_{p'}(q)\,\Pi_{post}(q;G)$. Since $\Pi_{post}(\cdot;G)$ is continuous on $\left\{\tilde{p} : E_{\tilde{p}}[U_i] \in [u_0, u_{M-1})\right\}$, this implies that there exists a profitable deviation G' such that $Q \not\subset \operatorname{supp} G'$, a contradiction.

This proves that $C\left(\Pi_{post}\left(p;G\right)\right)$ is linear on $\operatorname{co}(\operatorname{supp} G)$. Therefore, there exists a linear function $\overline{\Pi}_{post}(p)$ with the following properties. For all p, $\overline{\Pi}_{post}(p) \geq C\left(\Pi_{post}\left(p;G\right)\right)$, and $\overline{\Pi}_{post}(p) = C\left(\Pi_{post}\left(p;G\right)\right)$ if and only if $p \in \operatorname{co}(\operatorname{supp} G)$. Moreover, if $F_G(\hat{u}) = 1$, then $\overline{\Pi}_{post}(p) > C\left(\Pi_{post}\left(p;G\right)\right)$ for all p such that $E_p[U_i] \in (\hat{u}, u_{M-1}]$. If $F_G(\hat{u}) < 1$, then $C\left(\Pi_{post}\left(p;G\right)\right) > \Pi_{post}(p;G)$ for all p such that $E_p[U_i] \in (\hat{u}, u_{M-1})$. Q.E.D.

Proof of Theorem 4

We first prove the following two lemmata.

Lemma 1 *Suppose G induces $\Pi(u)$ with the generalized linear structure, and $\Pi(u)$ has a κ-th upward kink at u_{I_κ}. Fix a pair of expected qualities, u' and u'', such that $u_{I_\kappa} \leq u' < u'' \leq u_{I_{\kappa+1}}$. Then there exist $\beta_j \in [0,1], j = I_\kappa, \cdots, I_{\kappa+1}$ such that $F_G(u') = F_G(u) + \sum_{j=I_\kappa}^{I_{\kappa+1}} \beta_j \pi_j$; and*

$$\frac{N-1}{N} \frac{(F_G(u''))^N - (F_G(u'))^N}{(F_G(u''))^{N-1} - (F_G(u'))^{N-1}} = F_G(u') + \sum_{j=I_\kappa}^{I_{\kappa+1}} \beta_j \pi_j \frac{u''-u_j}{u''-u'}. \tag{17}$$

Proof. Suppose $u_{I_\kappa} \leq u' < u'' \leq u_{I_{\kappa+1}}$. The upward-kink condition implies that $u \in [u', u'']$ is induced only by $u_{I_\kappa}, \cdots, u_{I_{\kappa+1}}$. This implies that there exist $\beta_j \in [0,1]$, $j = I_\kappa, \cdots, I_{\kappa+1}$ such that $F_G(u'') = F_G(u') + \sum_{j=I_\kappa}^{I_{\kappa+1}} \beta_j \pi_j$. Therefore, by Bayes' rule, we have

$$E_G[u|u \in [u', u'']] = \frac{\sum_{j=I_\kappa}^{I_{\kappa+1}} \beta_j \pi_j u_j}{\sum_{j=I_\kappa}^{I_{\kappa+1}} \beta_j \pi_j}. \tag{18}$$

For notational simplicity, let $\chi \equiv F_G(u')^{N-1}$ and $\zeta \equiv F_G(u'')^{N-1}$. Notice that for $u \in [u', u'']$, the piecewise linearity of $\Pi(u)$ implies $\Pi(u) = \chi + \frac{\zeta-\chi}{u''-u'}(u-u')$. This implies that on the interval $[u', u'']$, we can write $F_G(u) = \left(\chi + \frac{\zeta-\chi}{u''-u'}(u-u')\right)^{\frac{1}{N-1}}$, which in turn allows us to compute the conditional expectation as follows:

$$\begin{aligned} E_G[u|u \in [u', u'']] &= \frac{\int_{u'}^{u''} u dF_G(u)}{F_G(u'') - F_G(u')} \\ &= \frac{\left(u''\zeta^{\frac{1}{N-1}} - u'\chi^{\frac{1}{N-1}}\right) - \frac{N-1}{N} \frac{\zeta^{\frac{N}{N-1}} - \chi^{\frac{N}{N-1}}}{\zeta - \chi}(u''-u')}{\zeta^{\frac{1}{N-1}} - \chi^{\frac{1}{N-1}}}. \end{aligned} \tag{19}$$

Recall that $\zeta^{\frac{1}{N-1}} = F_G(u') + \sum_{j=I_\kappa}^{I_{\kappa+1}} \beta_j \pi_j$, and $\chi^{\frac{1}{N-1}} = F_G(u')$. Substituting these into equation (19) and equating the subsequent expression with (18) gives equation (17) after straightforward algebra. ■

Lemma 2 *Suppose G is an equilibrium strategy and its induced payoff function $\Pi(u)$ has an upward kink at $u_{I_\kappa} \in \{u_0, \cdots, u_{M-2}\}$. Suppose further that F_G satisfies Property-ℓ, for some $\ell \in \{I_\kappa + 1, \cdots, M+1\}$. Then $s_\ell^{\kappa+1} < \infty$, and ℓ is the largest element of* $\arg\min_{j \in \{I_\kappa+1, \cdots, M+1\}} \left\{s_j^{\kappa+1}\right\}$.

Proof. We first prove that $s_\ell^{\kappa+1} \leq s_{\ell'}^{\kappa+1}$ for all $\ell' \in \{I_\kappa + 1, \cdots \ell - 1, \ell + 1, ..., M+1\}$ when G satisfies Property-ℓ.

(Case 1:) Suppose $I_{\kappa+1} \in \{I_\kappa + 1, \cdots, M-2\}$. We show that $s_m^{\kappa+1} \geq s_{I_{\kappa+1}}^{\kappa+1}$ for $m \in \{I_\kappa + 1, \cdots, I_{\kappa+1} - 1\}$. Let $F_G(u_{I_\kappa}) = \chi_\kappa^{\frac{1}{N-1}}$. By applying Lemma 1 on $[u_{I_\kappa}, u_m]$, we know that there exist $\beta_j \in [0,1]$, $j =$

$\{I_\kappa, \cdots, I_{\kappa+1}\}$ such that $F_G(u_m) = \chi_\kappa^{\frac{1}{N-1}} + \sum_{j=I_\kappa}^{I_{\kappa+1}} \beta_j \pi_j$, and

$$\frac{N-1}{N} \frac{F_G(u_m)^N - \chi_\kappa^{\frac{N}{N-1}}}{F_G(u_m)^{N-1} - \chi_\kappa} = \chi_\kappa^{\frac{1}{N-1}} + \beta_{I_\kappa} \pi_{I_\kappa} + \sum_{j=I_\kappa+1}^{I_{\kappa+1}} \beta_j \pi_j \frac{u_m - u_j}{u_m - u_{I_\kappa}} \tag{20}$$

Similarly, there exist $\alpha_{I_\kappa} \in (0,1)$ and α_m such that

$$\frac{N-1}{N} \frac{\tilde{F}_G^N - \chi_\kappa^{\frac{N}{N-1}}}{\tilde{F}_G^{N-1} - \chi_\kappa} = \chi_\kappa^{\frac{1}{N-1}} + (1-\alpha_{I_\kappa}) \pi_{I_\kappa} + \sum_{j=I_\kappa+1}^{m-1} \pi_j \frac{u_m - u_j}{u_m - u_{I_\kappa}}, \tag{21}$$

where $\tilde{F}_G = \chi_\kappa^{\frac{1}{N-1}} + (1-\alpha_{I_\kappa}) \pi_{I_\kappa} + \sum_{j=I_\kappa}^{m-1} \pi_j + \alpha_m \pi_m$.

We show that $F_G^{N-1}(u_m) \leq \tilde{F}_G^{N-1}$. First, notice that $\frac{\xi^N - \chi^{\frac{N}{N-1}}}{\xi^{N-1} - \chi}$ is increasing in ξ for all $\xi > \chi^{\frac{1}{N-1}}$. Therefore by (20) and (21), $F_G^{N-1}(u_m) \leq \tilde{F}_G^{N-1}$ if and only if

$$\beta_{I_\kappa} \pi_{I_\kappa} + \sum_{j=I_\kappa+1}^{I_{\kappa+1}} \beta_j \pi_j \frac{u_m - u_j}{u_m - u_{I_\kappa}} \leq (1-\alpha_{I_\kappa}) \pi_{I_\kappa} + \sum_{j=I_\kappa+1}^{m-1} \pi_j \frac{u_m - u_j}{u_m - u_{I_\kappa}}.$$

Observe that $\beta_{I_\kappa} \leq 1 - \alpha_{I_\kappa}$. Also, since $u_{I_{\kappa+1}} > u_m$, $\sum_{j=m+1}^{I_{\kappa+1}} \beta_j \pi_j \frac{u_m - u_j}{u_m - u_{I_\kappa}} \leq 0$. Therefore, $\sum_{j=I_\kappa+1}^{I_{\kappa+1}} \beta_j \pi_j \frac{u_m - u_j}{u_m - u_{I_\kappa}} \leq \sum_{j=I_\kappa+1}^{m-1} \pi_j \frac{u_m - u_j}{u_m - u_{I_\kappa}}$, i.e., $F_G^{N-1}(u_m) \leq \tilde{F}_G^{N-1}$. Thus, by the definition of $s_{I_{\kappa+1}}^{\kappa+1}$ and $s_m^{\kappa+1}$, we have $s_{I_{\kappa+1}}^{\kappa+1} \leq s_m^{\kappa+1}$.

(Case 2:) Suppose $I_{\kappa+1} \in \{I_\kappa + 1, \cdots, M-2\}$. We show that $s_m^{\kappa+1} > s_{I_{\kappa+1}}^{\kappa+1}$ for $m \in \{I_{\kappa+1} + 1, \cdots, k-2\}$. Suppose that F_G^{N-1} has an upward-kink at u_I, $I \in \{I_{\kappa+1}, \cdots, M-2\}$ and no upward-kink at $u_j \in \{u_{I+1}, \cdots, u_{m-1}\}$; and $s_m^{\kappa+1} < \infty$. For notational simplicity, define $\tilde{\chi}_I \equiv \chi_\kappa + s_m^{\kappa+1}(u_I - u_{I_\kappa})$ and $\chi_I \equiv F_G^{N-1}(u_I)$.

By applying Lemma 1 on $[u_I, u_m]$, we know that there exist $\beta_I^G \in (0,1)$ and $\beta_j^G \in [0,1]$, $j \in \{I+1, ..., M-1\}$ such that $F_G(u_m) - F_G(u_I) = \sum_{j=I_{\kappa+1}}^{M-1} \beta_j^G \pi_j$ and

$$\frac{N-1}{N} \frac{F_G(u_m)^N - \chi_I^{\frac{N}{N-1}}}{F_G(u_m)^{N-1} - \chi_I} = F_G(u_I) + \sum_{j=I}^{M-1} \beta_j^G \pi_j \frac{u_m - u_j}{u_m - u_I}. \tag{22}$$

Similarly, since $s_m^{\kappa+1} < \infty$, there there exist $\alpha_{I_\kappa}^m, \alpha_m^m \in [0,1]$, $\gamma_m^m \in [0, \alpha_m^m]$, and $\beta_j^m \in [0,1]$ for each $j = I_\kappa, ..., m-1$ such that,

$$\frac{N-1}{N} \frac{\left(\sum_{j=0}^{m-1} \pi_j + \alpha_m^m \pi_m\right)^N - \tilde{\chi}_I^{\frac{N}{N-1}}}{\left(\sum_{j=0}^{m-1} \pi_j + \alpha_m^m \pi_m\right)^{N-1} - \tilde{\chi}_I} = \tilde{\chi}_I^{\frac{1}{N-1}} + \sum_{j=I_\kappa}^{m-1} \beta_j^m \pi_j \frac{u_m - u_j}{u_m - u_I}, \tag{23}$$

where $\tilde{\chi}_I^{\frac{1}{N-1}} = \sum_{j=0}^{I_\kappa - 1} \pi_j + \alpha_{I_\kappa}^m \pi_{I_\kappa} + \sum_{j=I_\kappa}^{m-1} \left(1 - \beta_j^m\right) \pi_j + \gamma_m^m \pi_m$.

We now argue that the left-hand side of (22) is strictly smaller than that of (23). Let Δ be the difference between the right-hand side of (23) minus the right-hand side of (22), and ϖ_j be the coefficient of π_j, $j \in \{0, 1, ..., M-1\}$, of Δ. We argue that $\varpi_j \geq 0$ for all j with strict inequality for at least one j. (i) If

$j \in \{0, \cdots, I_\kappa - 1\}$, then $\varpi_j = 0$. (ii) If $j = I_\kappa$, then $\varpi_{I_\kappa} = \alpha^m_{I_\kappa} + \left(1 - \beta^m_{I_\kappa}\right) + \beta^m_{I_\kappa} \frac{u_m - u_{I_\kappa}}{u_m - u_I} - 1 > 0$ because $\frac{u_m - u_{I_\kappa}}{u_m - u_I} > 1$. (iii) If $j \in \{I_\kappa + 1, \cdots, I - 1\}$, then $\varpi_j = \left(1 - \beta^m_j\right) + \beta^m_j \frac{u_m - u_j}{u_m - u_I} - 1 \geq 0$. (iv) If $j = I$, then $\varpi_I = 1 - \beta^G_I \geq 0$. (v) If $j \in \{I + 1, \cdots, m - 1\}$, then $\varpi_j = \left(1 - \beta^m_j\right) + \beta^m_j \frac{u_m - u_j}{u_m - u_I} - \beta^G_j \frac{u_m - u_j}{u_m - u_I} \geq 0$ because $\frac{u_m - u_j}{u_m - u_I} \in (0, 1)$. (vi) for $j = k$, $\varpi_j = 1 - \beta^m_m + \gamma^m_m \geq 0$. (vii) For $j \in \{k + 1, \cdots, M - 1\}$, $\varpi_j = -\beta^G_j \frac{u_m - u_j}{u_m - u_I} \geq 0$.

Lastly, we show that $s^{\kappa+1}_{I_{\kappa+1}} < s^{\kappa+1}_k$. To see this notice that if $s^{\kappa+1}_{I_{\kappa+1}} \geq s^{\kappa+1}_m$, then the upward-kink condition implies $\tilde{\chi}_I \leq \chi_I$ and $\sum_{j=0}^{m-1} \pi_j + \alpha^m_m \pi_m \leq F_G(u_m)$. However, $\frac{\xi^N - \chi^{\frac{N}{N-1}}}{\xi^{N-1} - \chi}$ is increasing both in ξ and χ. Therefore, that the left-hand side of (22) is strictly smaller than that of (23) implies $F_G(u_m) < \sum_{j=0}^{m-1} \pi_j + \alpha^m_m \pi_m$, a contradiction.

(Case 3:) Suppose $I_{\kappa+1} \in \{I_\kappa + 1, \cdots, M - 2\}$. We show that $s^{\kappa+1}_j > s^{\kappa+1}_{I_{\kappa+1}}$ for all $j \in \{M - 1, M, M + 1\}$. Suppose $\hat{u}_j \leq u_{I_{\kappa+1}}$. Then it is straightforward from the definitions of $s^{\kappa+1}_j$ and $s^{\kappa+1}_{I_{\kappa+1}}$ that $s^{\kappa+1}_j > s^{\kappa+1}_{I_{\kappa+1}}$. Suppose next that $\hat{u}_j > u_{I_{\kappa+1}}$. Then replacing u_m with $\hat{u}_j$ in the analysis of Case 2 above leads to $s^{\kappa+1}_j > s^{\kappa+1}_{I_{\kappa+1}}$.

(Case 4:) Suppose $I_{\kappa+1} \in \{M - 1, M, M + 1\}$. It is straightforward to see that $s^{\kappa+1}_j = \infty$ for $j \in \{M - 1, M, M + 1\} \setminus \{I_{\kappa+1}\}$. For $k \in \{I_\kappa + 1, ..., M - 2\}$, $s^{\kappa+1}_{I_{\kappa+1}} \leq s^{\kappa+1}_k$ follows from an argument identical to that in Case 1 above.

Lastly, suppose that there exist ℓ and $\ell' > \ell$ such that $s^{\kappa+1}_\ell = s^{\kappa+1}_{\ell'}$ for some κ. If $\ell' \in \{I_\kappa + 1, ..., M - 2\}$, then the proof of Case 2 shows that F_G does not satisfy Property-ℓ. If $\ell \in \{I_\kappa + 1, \cdots, M - 2\}$, then the proof of Case 3 shows that F_G does not satisfy Property-ℓ. By Case 4, if $\ell = M - 1$ or M, then $s^{\kappa+1}_{\ell'} = \infty$ for all $\ell' \in \{M, M + 1\} \setminus \{\ell\}$. Therefore, if $s^{\kappa+1}_\ell = s^{\kappa+1}_{\ell'}$ and $\ell < \ell'$, then F_G does not satisfy Property-ℓ. ■

We now prove Theorem 4. We have already argued the "only if" part in the text. To see the "if" part, observe that as an immediate corollary of Lemma 2, if a symmetric equilibrium exists, the equilibrium distribution of expected qualities is unique. Furthermore, the existence of a symmetric equilibrium follows from Corollary 4.3 of Reny (1999).[14] Lemma 2 implies that the algorithm constructs the unique equilibrium distribution of expected qualities.

Finally, the game is symmetric and zero-sum. Therefore, if $N = 2$, the interchangeability property of zero-sum games implies that if there exist multiple equilibria or an asymmetric equilibrium, then there exists multiple symmetric equilibria, which is a contradiction.Q.E.D.

Proof of Theorem 5

We show by induction that there exist N_k, for each $k = 1, \cdots, M - 2$ such that (i) $N > N_k$ implies $I_{k,N} = k$ and (ii) $N' > N > N_k$ implies $F_{G,N'}(u_k) < F_{G,N}(u_k)$ and $\lim_{N \to \infty} F_{G,N}(u_k) = \sum_{l=0}^{k-1} \pi_l$.

We start with $k = 1$. That is, we show that for a sufficiently large N, Property-m, $m \in \{2, \cdots, M - 2\}$,

[14] Let $V_i(G_i, G_{-i})$ be the sender i's payoff when the strategy profile is (G_i, G_{-i}). Since $V_i(G, ..., G) = 1/n$, the game is quasi-symmetric, compact, diagonally quasiconcave, diagonally payoff secure, and $V_i(G, ..., G)$ is upper semicontinuous with respect to G.

does not hold at u_0. This is because if G satisfies Property-m, then there exists an $\alpha_m \in (0,1)$ such that

$$\frac{N-1}{N}\left(\sum_{j=0}^{m-1}\pi_j+\alpha_m\pi_m\right)=\left(\pi_0+\sum_{j=1}^{m-1}\pi_j\frac{u_m-u_j}{u_m-u_0}\right). \tag{24}$$

and $F_{G,N}(u_m)=\sum_{j=0}^{m-1}\pi_j+\alpha_m\pi_m$. Notice that the left-hand side of (24) implies that α_m is strictly decreasing in N. Furthermore, for a sufficiently large N, $\frac{u_m-u_j}{u_m-u_0}<\frac{N-1}{N}$ for all $j\in\{1,\cdots,m-1\}$. Therefore, for a sufficiently large N, α_m that solves (24) has to be negative. To see that Property-$M-1$ does not hold at u_0, notice that (5) simplifies to $\frac{N-1}{N}\sum_{j=0}^{M-2}\pi_j=\sum_{j=1}^{M-2}\pi_j\frac{\hat{u}_{M-1}-u_j}{\hat{u}_{M-1}-u_0}$. Notice $\frac{\hat{u}_{M-1}-u_j}{\hat{u}_{M-1}-u_0}<\frac{u_{M-1}-u_j}{u_{M-1}-u_0}<1$ for each $j=1,\cdots,M-2$. Therefore, for a sufficiently large N, there exists no $\hat{u}_{M-1}<u_{M-1}$ such that $\frac{N-1}{N}\sum_{j=0}^{M-2}\pi_j=\sum_{j=1}^{M-2}\pi_j\frac{\hat{u}_{M-1}-u_j}{\hat{u}_{M-1}-u_0}$. Similarly, by (7), and (10), we can show that for a sufficiently large N, neither Properties-M, nor $M+1$ holds. This proves that for a sufficiently large N, $I_{1,N}=1$ and $F_{G,N}(u_1)=\pi_0+\frac{\pi_0}{(N-1)\pi_1}$.

Suppose the induction hypothesis holds for all $k=1,\cdots,l$, where $l\le M-3$. We show that Property-$m, m\in\{l+2,\cdots,M-2\}$ does not hold for a sufficiently large N. Suppose otherwise. Then, there exists an $\alpha_m\in(0,1)$ such that $F_{G,N}(u_m)=\sum_{j=0}^{m-1}\pi_j+\alpha_m\pi_m$ and

$$\frac{N-1}{N}\frac{\left(\sum_{j=0}^{m-1}\pi_j+\alpha_m\pi_m\right)^N-\left(F_{G,N}(u_l)\right)^N}{\left(\sum_{j=0}^{m-1}\pi_j+\alpha_m\pi_m\right)^{N-1}-\left(F_{G,N}(u_l)\right)^{N-1}}=\sum_{j=0}^{l}\pi_j+\sum_{j=l+1}^{m-1}\pi_j\frac{u_m-u_j}{u_m-u_l}. \tag{25}$$

For any ε, there exists an $\tilde{N}$ such that $N>\tilde{N}$ implies that the left-hand side of (25) is bounded from below by $\sum_{j=0}^{m-1}\pi_j-\varepsilon$.[(15)] However, the right-hand side of (25) is bounded from above by $\sum_{j=0}^{m-1}\pi_j$, which is a contradiction. Similar arguments shows that none of Properties-$M-1, M, M+1$ holds at u_l. Therefore, for a sufficiently large N, $I_{l+1,N}=l+1$, and there exists an $\alpha_{l+1,N}\in(0,1)$ such that $F_{G,N}(u_{l+1})=\sum_{j=0}^{l}\pi_j+\alpha_{l+1,N}\pi_{l+1}$ and

$$\frac{N-1}{N}\frac{\left(\sum_{j=0}^{l}\pi_j+\alpha_{l+1,N}\pi_{l+1}\right)^N-\left(F_{G,N}(u_l)\right)^N}{\left(\sum_{j=0}^{l}\pi_j+\alpha_{l+1,N}\pi_{l+1}\right)^{N-1}-\left(F_{G,N}(u_l)\right)^{N-1}}=\sum_{j=0}^{l}\pi_j.$$

(15) This is because $\frac{N-1}{N}\frac{x^{N-1}-y^N}{x^{N-1}-y^{N-1}}$ is strictly increasing in $N\in\mathbb{N}$. Notice that $\frac{N-1}{N}\frac{x^N-y^N}{x^{N-1}-y^{N-1}}=x\times\frac{N-1}{N}\times\frac{1-z^N}{1-z^{N-1}}$, where $z=y/x$, and

$$\begin{aligned}\frac{N-1}{N}\frac{1-z^N}{1-z^{N-1}}-\frac{N-2}{N-1}\frac{1-z^{N-1}}{1-z^{N-2}}&=\frac{(N-1)^2\left(1-z^N\right)\left(1-z^{N-2}\right)-N(N-2)\left(1-z^{N-1}\right)^2}{N(N-1)\left(1-z^{N-1}\right)\left(1-z^{N-2}\right)}\\&=\frac{\left(1+z^{2N-2}\right)+z^{N-2}\left((N-1)^2\left(1+z^2\right)-N(N-2)(2z)\right)}{N(N-1)\left(1-z^{N-1}\right)\left(1-z^{N-2}\right)}\\&\ge\frac{\left(1+z^{2N-2}\right)+(N-1)^2z^{N-2}(1-z)^2}{N(N-1)\left(1-z^{N-1}\right)\left(1-z^{N-2}\right)}>0.\end{aligned}$$

Notice $\frac{N-1}{N}\frac{x^{N-1}-y^N}{x^{N-1}-y^{N-1}}$ is strictly increasing in $N \in \mathbb{N}$, y, and $x-y$, when $y \in (0,1)$, and $x \in (y,1)$.[16] Therefore, $\left(\sum_{j=0}^{l} \pi_j + \alpha_{l+1,N}\pi_{l+1}\right) - F_{G,N}(u_l)$ is strictly decreasing in N. Since $F_{G,N}(u_l)$ is strictly decreasing in N by the induction hypothesis, we have that $\alpha_{l+1,N}$ is strictly decreasing in N, and $F_{G,N}(u_{l+1}) \to \sum_{j=0}^{l} \pi_j$.

We have established that $F_{G,N}(u_k) = \sum_{j=0}^{k-1} \pi_j + o(1)$ for $k = 0, \cdots, M-2$, where $o(1) \to 0$ as $N \to \infty$. Therefore, for any $u \in (u_k, u_{k+1})$, $k \in \{0, \cdots, M-3\}$,

$$F_G(u) = \left(\frac{\left(\sum_{j=0}^{k-1} \pi_j\right)^{N-1} (u_{k+1}-u) + \left(\sum_{j=0}^{k} \pi_j\right)^{N-1} (u-u_k)}{u_{k+1}-u_k} \right)^{\frac{1}{N-1}} + o(1) \to \sum_{j=0}^{k} \pi_j \text{ as } N \to \infty.$$

Next, we show that $F_{G,N}(u) \to 1-\pi_{M-1}$ as $N \to \infty$ for $u \in (u_{M-2}, u_{M-1})$. For a sufficiently large N, Property-M holds at u_{M-2}. That is, by, (7) and (8), there exist $\hat{u}_{M,N} \in (u_{M-2}, u_{M-1})$ and $\alpha_N \in (0,1)$ such that

$$\frac{N-1}{N} \frac{\left(\sum_{j=0}^{M-2} \pi_j + \alpha_N \pi_{M-1}\right)^N - \left(F_{G,N}(u_{M-2})\right)^N}{\left(\sum_{j=0}^{M-2} \pi_j + \alpha_N \pi_{M-1}\right)^{N-1} - \left(F_{G,N}(u_{M-2})\right)^{N-1}} = \sum_{j=0}^{M-2} \pi_j + \alpha_N \pi_{M-1} \frac{\hat{u}_{M,N}-u_{M-1}}{\hat{u}_{M,N}-u_{M-2}} \tag{26}$$

and

$$\frac{1-\left(\frac{F_{G,N}(u_{M-2})}{\sum_{j=0}^{M-2}\pi_j+\alpha_N\pi_{M-1}}\right)^{N-1}}{\frac{1-\left(\sum_{j=0}^{M-2}\pi_j+\alpha_N\pi_{M-1}\right)^N}{N\left(\left(\sum_{j=0}^{M-2}\pi_j+\alpha_N\pi_{M-1}\right)^{N-1}-\left(\sum_{j=0}^{M-2}\pi_j+\alpha_N\pi_{M-1}\right)^N\right)}-1} = \frac{\hat{u}_{M,N}-u_{M-2}}{u_{M-1}-\hat{u}_{M,N}}. \tag{27}$$

We show that $\alpha_N \to 0$. Suppose not, then there exists a subsequence $\{N_k\}$ such that $\alpha_{N_k} \to \alpha$, for some $\alpha > 0$, and $\hat{u}_{M,N}$ converges. Then the left-hand side of equation (26) converges to $\sum_{j=0}^{M-2} \pi_j + \alpha\pi_{M-1}$, whereas the limit of right-hand side is bounded from above by $\sum_{j=0}^{M-2} \pi_j$, a contradiction.

As $\alpha_N \to 0$, the left-hand side of equation (27) converges to 0, so $\lim_{N\to\infty} \hat{u}_{M,N} = u_{M-2}$. This proves that for any $u \in (u_{M-2}, u_{M-1})$, $F_{G,N}(u) \to 1-\pi_{M-1}$.

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[16] Notice that $\frac{N-1}{N}\frac{x^N-y^N}{x^{N-1}-y^{N-1}} = x \times \frac{N-1}{N} \times \frac{1-z^N}{1-z^{N-1}}$, where $z = y/x$. Next, siNce $\frac{\partial \frac{1-z^N}{1-z^{N-1}}}{\partial z} = \frac{z^N\left(N(1-z)-\left(1-z^N\right)\right)}{\left(z-z^N\right)^2} > 0$ for all $z < 1$, $\frac{N-1}{N}\frac{x^{N-1}-y^N}{x^{N-1}-y^{N-1}}$ is increasing in y. Lastly, observe that $\frac{\partial \frac{(y+d)^N-y^N}{(y+d)^{N-1}-y^{N-1}}}{\partial d} = \frac{(y+d)^{N-2}\left((y+d)^N-y^N-Ny^{N-1}d\right)}{\left((y+d)^{N-1}-y^{N-1}\right)^2}$ and $\frac{\partial\left((y+d)^N-y^N-Ny^{N-1}d\right)}{\partial d} = N\left((y+d)^{N-1}-y^{N-1}\right) > 0$. Therefore, $\frac{\partial \frac{(y+d)^N-y^N}{(y+d)^{N-1}-y^{N-1}}}{\partial d} \geq 0$.

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