Competition in Information Disclosure

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Abstract

We analyze a model of competition in Bayesian persuasion in which two or more senders vie for the patronage of a receiver by disclosing information about their respective proposals. We first establish equilibrium existence for the model of a general state space. We then focus on a binary state space, i.e., each sender's proposal gives the receiver either a high or low utility. With two (possibly asymmetric) senders, we fully characterize the generically unique equilibrium, and show that it has a simple linear structure. We find that a sender who faces a stronger opponent engages in more aggressive disclosure in terms of second-order stochastic dominance. With multiple symmetric senders, we fully characterize the unique symmetric equilibrium. We then show that all senders engage in full disclosure in the limit as the number of senders goes to infinity. Finally, we show that the finding that equilibrium strategy must exhibit a linear structure remains valid locally for any finite state space.

Keywords Information Transmission, Bayesian Persuasion, Multiple Senders

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1 Introduction

We analyze the competition in information disclosure among two or more senders vying for the patronage of a receiver. Suppose that each sender is endowed with an independent proposal, the outcome of which is uncertain, and the receiver can endorse at most one proposal. The senders then try to persuade the receiver by simultaneously disclosing information about their respective proposals. Specifically, each sender chooses the information structure through which the receiver can learn about his payoff for endorsing the sender's proposal. From an individual sender's perspective, each signal generated by the

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information structure is associated with a probability of winning. Naturally, a better signal gives a higher probability of winning. However, holding the other signals of the information structure fixed, the better signal is less likely to arise. Therefore, each sender's optimal disclosure has to balance this trade-off. Facing this trade-off, what disclosure policies would be adopted in equilibrium? What happens if the competition gets more intense, as the number of senders increases, or some sender is endowed with a proposal of better prospect?

There are numerous applications that fit into this framework. First, a procurement agency chooses from a number of independent contractors who supply differentiated products. Whereas no one knows the exact match quality ex-ante, each contractor can persuade the procurement agency to choose its own product by revealing information about the attributes of that product.[(1)] Second, a number of universities try to persuade a potential employer to hire their students. Each university can influence the employer's decision through the information conveyed in the transcript of its students. Although universities can control what will appear on students' transcripts, they commit to their grading policies before they know the actual qualities of their students.[(2)] Third, when a firm with a limited budget can support at most one project, its project leaders may compete by providing the firm's executive evidence about the prospect of their respective projects. The evidence can be the results of their designed information acquisition processes such as product tests and market studies.

In this paper, we analyze a model of competition in Bayesian persuasion among n senders. The space of the underlying state of the world, Ω^n, is an n-dimensional product space, with the i-th dimension representing the receiver's payoff of choosing sender i's proposal. We focus on the class of priors over Ω^n such that each dimension is independently (but not necessarily identically) distributed. We assume each sender i can only control the disclosure of information regarding the i-th dimension. Specifically, sender i's strategy is an information structure over the i-th dimension of Ω^n. The information disclosed by sender i thus informs the receiver only about the payoff of choosing sender i's proposal.

As pointed out by Kamenica and Gentzkow (2011), the choice problem of an information structure is equivalent to one of choosing a distribution over posterior beliefs that respects Bayes' rule. Thus, the basic trade-off that senders face can be illustrated as follows. Suppose, the posterior distribution of each sender can only have a binary support, i.e., it discloses either a good news or a bad news, the probability distribution over this news must respect Bayes' rule. A more transparent disclosure policy corresponds to a support with more extreme posteriors, i.e., it discloses either very good news or very bad

[(1)]This application is related to the model of product differentiation of Perloff and Salop (1985), except that we shut down price competition and focus only on competition in information disclosure.

[(2)]Kolotilin (2015) analyzes the situation where there is only one university, but a potential employer can acquire information not only from the university but also from other sources. He derives the necessary and sufficient conditions for full and no information revelation.

news. Increasing the high posterior (i.e., making a good news better) while fixing the low posterior (bad news) involves the following trade-off. The cost is that the probability of realization of the high posterior (good news) is lowered, as is required by the Bayes' rule. However, the benefit is that, conditional on the realization of the good news, the sender is more likely to beat other senders in the competition.

First, we establish that in the general model, a pure-strategy equilibrium exists. We then specialize to the case of the binary state space, i.e., the situation where each sender's proposal gives the receiver either a high or low utility. With two (possibly asymmetric) senders, the equilibrium is fully characterized, and is shown to be generically unique. Herein, we make use of another insight of Kamenica and Gentzkow (2011): optimizing the posterior distribution is equivalent to finding the concave closure of the sender's payoff as a function of realized posteriors. This insight allows us to show that the equilibrium strategy necessarily exhibits a linear structure: the distribution is uniform in the intermediate posteriors, and possibly has atoms at certain posteriors (i.e., posterior that the product gives high or low utility with certainty).

With the unique equilibrium explicitly constructed, we conduct comparative statics with respect to the strength of each sender's prior. A sender is strong if the prior belief that he has a high-utility proposal is large. Intuitively, a stronger sender is more likely to disclose good news about his proposal. Moreover, facing such a stronger opponent, the other sender would respond by more aggressive disclosure. We formalize this intuition by showing that a strengthening of the sender's own prior results in a weakly higher posterior distribution in terms of first-order stochastic dominance. Moreover, a strengthening of the opponent's prior results in a weakly higher posterior distribution in terms of second-order stochastic dominance.

Next, we analyze the scenario with multiple symmetric senders. Using the observation that the equilibrium must exhibit a linear structure similar to the two-sender case, we show that the symmetric equilibrium is unique, and fully characterize it. Naturally, competition becomes more intense with more senders, and each sender finds more transparent disclosure policy necessary to stand a chance to persuade the receiver. We show that as the number of senders increases, each sender uses a more informative disclosure policy in equilibrium. Moreover, as the number of senders approaches infinity, each sender's strategy converges to full disclosure.

Finally, we analyze the case of a general state space, and provide necessary conditions that equilibrium strategy profile must satisfy. We show that the finding that the equilibrium strategy must respect a linear structure remains valid locally. In particular, the equilibrium strategy must give a "piecewise uniform" distribution over expected utilities. To illustrate the usefulness of this finding, we provide a full characterization of the symmetric equilibrium in the case of two symmetric senders and that the state space has three elements. The symmetric equilibrium is unique up to the distribution of expected utility. A novel feature of the equilibrium is that contrary to the binary state case, for some priors, the equilibrium distribution over expected utility has an upward kink at an intermediate utility.

In sum, this article makes three contributions. First, we establish the existence of pure-strategy equilibrium in a natural setting of competition in Bayesian persuasion. Specializing to a binary state space, we further obtain the uniqueness of an equilibrium in the case of two senders, as well as the uniqueness of a symmetric equilibrium the case of multiple symmetric senders. Second, we show that the equilibrium strategy necessarily exhibits a piecewise linear structure, a finding that tremendously simplifies the search for equilibrium in this class of models. Finally, thanks to the equilibrium uniqueness, we are able to obtain sharp and intuitive comparative statics results regarding the effect of competition on information disclosure.

As discussed above, the technique developed by Kamenica and Gentzkow (2011) plays a key role in our analysis. Their article has stimulated an active literature on information disclosure game in which the sender(s) can commit to the disclosure mechanism. Below, we discuss a number of articles from the literature that study competition among senders. Ostrovsky and Schwarz (2010) considers a model setup similar to ours. In their model, schools disclose information about the ability of their students, with the objective of maximizing the students' overall placement. Their focus is whether the aggregate information disclosed in equilibrium depends on the distribution of students' abilities across the school. In contrast, we focus on equilibrium characterization, and the effect of competition on individual disclosure. Other articles that study competitive Bayesian persuasion assume that the senders share a common state of the world, and each one can independently disclose information on the common state to a single receiver. Allowing each sender to adopt a mechanism that is arbitrarily correlated with each other, Gentzkow and Kamenica (2015a) provide a simple equilibrium characterization. Furthermore, Gentzkow and Kamenica (2015b) identify a necessary and sufficient condition on the set of feasible disclosure mechanisms under which the equilibrium outcome is more informative with an additional sender (regardless of preferences). The game we analyze does not satisfy their condition, so their result is not applicable in this scenario. Li and Norman (2015) provide an example that if only (conditionally) independent mechanisms are feasible for each sender, the equilibrium outcome can be less informative with an additional sender. Board and Lu (2015) consider a search environment in which a buyer (receiver) sequentially learns from senders of a homogeneous product about its attributes. Restricting to (conditionally) independent mechanisms, they show that if the buyer's search history is private, full disclosure is the unique equilibrium outcome as the search cost vanishes. We obtain a somewhat similar result, but in the context of simultaneous disclosure about differentiated products. Au (2015) analyzes a dynamic disclosure setting with a single sender. In the absence of commitment power, the sender faces competition with his future selves. Our paper differs from these aforementioned articles in that we assume that each sender can only disclose information about his proposal and that it is infeasible for him to reveal any information about other senders' proposal.

Information transmission with multiple senders has been studied using frameworks different from Bayesian persuasion. For example, Milgrom and Roberts (1986) studied a multi-senders persuasion game in which the receiver is unsophisticated. They identify sufficient conditions on senders' preference for the unique equilibrium to be fully revealing. In contrast, we consider fully sophisticated receiver, endogenous senders' information, and comparative statics on the effect of competition. Dewatripont and Tirole (1999) consider costly information acquisition and show that hiring advocates with opposed interests generates information more effectively. In contrast, we abstract away from the moral hazard cost involved in information acquisition and focus on the aforementioned trade-off involved in the design of information structure. Finally, there is a large literature that examines the conflict of interests among senders in the cheap-talk settings. For example, Morgan and Krishna (2001) extends Crawford and Sobel (1982) to a setting with two senders and show that a full-revelation equilibrium exists if the senders have opposing bias. In addition, Battaglini (2002) shows that with two senders and a multidimensional state space, a full-revelation equilibrium generically exists. In contrast, we allow for more than two senders and investigate the effect of the intensity of competition on information disclosed.

The study of all-pay auction with complete information has identified mixed-strategy equilibria with a linear structure similar to those we find in our model of competitive Bayesian persuasion.(3) However, there are a number of notable differences in the nature of the two settings: First, our setting admits a pure-strategy equilibrium, whereas randomization over bids is an essential feature of all-pay auctions. Second, Bayes' rule imposes restrictions on the class of distributions each sender can choose, whereas no analogous restriction is in place in all-pay auctions. Finally, each sender in our setting can guarantee himself a positive ex-ante expected payoff by full disclosure, whereas the weaker bidders in the all-pay auction have zero equilibrium payoff.

The outline of this paper is as follows: The model is set up in Section 2. In Section 3, we analyze the case of binary state space and two possibly asymmetric senders. In Section 4, we consider multiple symmetric senders. The case of state space being arbitrarily finite is explored in Section 5. The final section concludes. Lengthy proofs are relegated to the appendix.

2 Model

There are n risk-neutral (male) senders, each of which is endowed with a proposal. They engage in competition for the endorsement of a single (female) receiver, who can approve at most one proposal. The receiver is an expected-utility-maximizer. Her utility of choosing proposal $i \in \{1, ..., n\}$ is denoted by

(3)See for example, Baye *et al.* (1996), and Hillman and Riley (1989).

$U_i \in \Omega \equiv \{u_0, u_1, ..., u_{m-1}\}$, where $u_{k-1} < u_k$ for all $k = 1, 2, ..., m-1$. For each i, U_i is independently distributed according to probability mass function π_i, i.e., $\Pr(U_i = u_k) = \pi_i(u_k)$. Assume $\pi_i \in int(\triangle\Omega)$ for all $i \in \{1, ..., n\}$. Though $\{\pi_i\}_{i=1}^n$ is commonly known, the realizations U_i's are unknown to any player at the beginning of the game.

Senders compete in information disclosure. Each sender i simultaneously choose an information disclosure mechanism on U_i, which consists of a signal space S_i and a conditional distribution function $\Phi_i : \Omega \times S_i \to [0, 1]$. The choices of disclosure mechanisms are known to the receiver before she makes her endorsement decision.

After observing the disclosure mechanisms and signal realizations of all senders, the receiver decides which sender's proposal to accept, or whether to reject all proposals. The receiver gets a payoff U_i by choosing sender i's proposal, and an outside option $\underline{u}$ if she rejects all proposals. Sender i's payoffs are normalized to one if the receiver accepts his proposal, and zero otherwise. The strategy of the receiver is straightforward. Given sender i's strategy (S_i, Φ_i) and a realization of signal s_i, the receiver forms posterior $p_i = (p_{i,0}, \cdots, p_{i,m-1}) \in \triangle\Omega$ over U_i, where $p_{i,k} = \frac{\pi_i(u_k)\Phi_i(s_i|u_k)}{\sum_{l=0}^{m-1}\pi_i(u_l)\Phi_i(s_i|u_l)}$. The receiver accepts sender i's proposal if the expected utility under posterior p_i is the highest, i.e., $E_{p_i}[U_i] > \max\{\underline{u}, \max_{j \neq i} E_{p_i}[U_j]\}$, where $E_{p_i}[U_i] \equiv \sum_{j=0}^{m-1} p_{i,k} u_k$. In the case of a tie, we assume that the receiver randomly select one sender, with equal probabilities, from the set $\{i : E_{p_i}[U_i] \geq \max\{\underline{u}, \max_{j \neq i} E_{p_i}[U_j]\}\}$.

A distribution of posterior beliefs is an element of $\triangle(\triangle\Omega)$, the set of Borel probability measures over $\triangle\Omega$. Proposition 1 of Kamenica and Gentzkow (2011) shows that there is a one-to-one correspondence between the set of feasible disclosure mechanisms and Bayes-plausible distributions of posterior beliefs over Ω. Formally, a distribution of posterior beliefs $G \in \triangle(\triangle\Omega)$ is Bayes-plausible for sender i if and only if it is an element of $\Lambda_i \equiv \{G \in \triangle(\triangle\Omega) : \int p dG(p) = \pi_i\}$. We focus our analysis on the game of information disclosure played among the senders in which the set of pure strategies of sender i is Λ_i.

The following observations are useful: First, it is without loss of generality to restrict attention to pure-strategy Nash equilibria. The reason is that for each mixed strategy, there exists a pure strategy that preserves the expected payoffs of all players. To see this, note first that the collection of all subsets of Λ_i, 2^{Λ_i}, is a sigma-algebra. A mixed strategy of sender i, σ_i, is a probability measure on $(\Lambda_i, 2^{\Lambda_i})$, so it induces a distribution of posterior beliefs over U_i. Specifically, $G^{\sigma_i}(p) \equiv \int G(p) d\sigma_i(G)$. The induced distribution G^{σ_i} satisfies Bayes-plausibility, as $\int p dG^{\sigma_i}(p) = \int\int p dG(p) d\sigma_i(G) = \pi_i$. As each sender's winning probability depends only on the induced distribution of posterior beliefs, every players' expected payoff remains unchanged if sender i plays the pure strategy $G^{\sigma_i}(\cdot)$ instead of the mixed strategy σ_i.

Second, despite the discontinuity of a sender's payoff in the strategy profile, the following proposition shows that a pure-strategy Nash equilibrium exists:

Proposition 1 *(i) An equilibrium exists. (ii) If all senders' proposals have an identical prior, then a symmetric* *equilibrium exists.*

Proof. In the Appendix. ■

We would like to remark that the assumption of a common state space Ω for all senders is crucial for establishing equilibrium existence. An example on the nonexistence of equilibrium if different senders have different state spaces is deferred to Section 3.3 (see Example 1), at which point the necessary conditions for an equilibrium are established.

3 Binary State Space and Two Senders

In this section, we assume that the state space is binary, i.e., $\Omega \equiv \{u_0, u_1\}$ and $\triangle\Omega = [0,1]$. Our goal is to fully characterize the equilibrium, and conduct comparative statics about the effect of prior on the equilibrium. Below, we first state a preliminary result by Kamenica and Gentzkow (2011) that simplifies the characterization of an optimal posterior distribution. In Section 3.1, we identify necessary and sufficient conditions on the pair of senders' priors under which one or both senders fully disclose his state in equilibrium. In Section 3.2, we analyze the equilibrium in which both senders partially disclose and show that the equilibrium strategy necessarily has a linear structure. This allows us to identify necessary and sufficient conditions on the prior pair for the existence of partial-disclosure equilibrium. In Section 3.3, we establish that the equilibrium exists and is generically unique using the results from the above two subsections. Finally, in Section 3.4, we investigate the equilibrium effect of an improvement in a sender's prior.

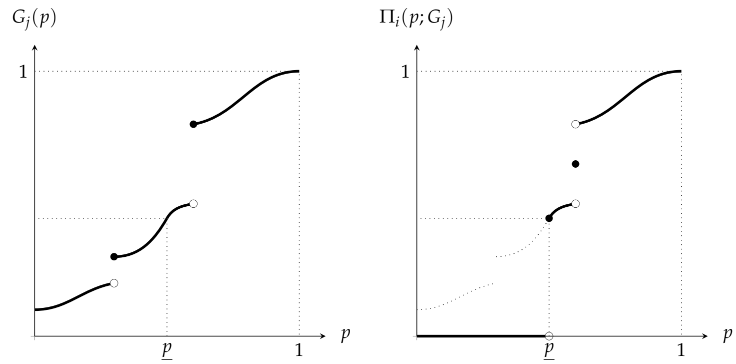
Denote a generic element of $\triangle\Omega$ by $p \in [0,1]$, standing for the probability that $U_i = u_1$. Similarly, the prior of sender i, denoted by $\pi_i \in (0,1)$, stands for the prior probability that $U_i = u_1$. As the receiver has an outside option of $\underline{u}$, he rejects all proposals if all posterior beliefs on U_i are too low. Specifically, the minimum posterior on U_i needed to induce acceptance is $\underline{p} \equiv \max\left\{0, \frac{\underline{u}-u_0}{u_1-u_0}\right\}$.

Suppose, $G_j(p)$ is the strategy used by sender j. As illustrated in Figure 1, sender i's payoff as a function of the realized posterior p is given by

$$\Pi_i(p; G_j) = \begin{cases} \frac{1}{2}\left(G_j(p) + \lim_{p' \to p^-} G_j(p')\right) & p \geq \underline{p} \\ 0 & p < \underline{p} \end{cases}, \tag{1}$$

A strategy pair $(G_1(p), G_2(p))$ constitutes an equilibrium if and only if for each $i = 1,2$ and $j \neq i$, $G_i(p)$ is a best response to $G_j(p)$, i.e., it solves $\max_{G_i \in \Lambda_i} E_{G_i}\Pi_i(p; G_j)$. As shown in Kamenica and Gentzkow (2011), the maximization problem can be solved by looking for the concave closure of Π_i (with respect to p). The concave closure of Π_i is defined as the smallest concave function that is everywhere weakly greater than Π_i:

$$\text{con}[\Pi_i](p) \equiv \sup\{z | (p,z) \in co(\Pi_i)\}, \tag{2}$$



(a) $G_j(p)$ (b) $\Pi_i(p;G_j)$

Figure 1: $G_j(p)$ and $\Pi_i(p;G_j)$

where $co\,(\Pi_i)$ stands for the convex hull of the graph of Π_i. We say that a posterior $p \in \triangle\Omega$ is on $\text{con}\,[\Pi_i]$ if and only if $\Pi_i\left(p;G_j\right) = \text{con}\,[\Pi_i]\,(p)$, and denote the set of such posteriors by $C\,(\Pi_i) \equiv \{p \in \triangle\Omega : \Pi_i\,(p) = \text{con}\,[\Pi_i]\,(p)\}$. Also, denote by $supp\,\{G_i\} \subset \triangle\Omega$ the support of a distribution G_i. The following lemma, a direct implication of Corollary 2 of Kamenica and Gentzkow (2011), will be used extensively for our equilibrium characterization.

Lemma 1 *(i) Strategy $G_i \in \Lambda_i$ is a best response to $G_j \in \Lambda_j$ if and only if $E_{G_i}\Pi_i\left(p;G_j\right) = \text{con}\,[\Pi_i]\,(\pi_i)$. (ii) If $G_i \in \Lambda_i$ is a best response to $G_j \in \Lambda_j$, then $\int_{C(\Pi_i)} dG_i\,(p) = 1$, that is, $\text{con}\,[\Pi_i]\,(p) = \Pi_i\,(p)$ for all $p \in supp\,\{G_i\}$, except possibly for a zero measure under G_i.*

An intuition for part (ii) is as follows: Suppose $\Pi_i\left(p;G_j\right)$ is locally convex on $[p', p'']$ and sender i assigns a point mass at some $p \in (p', p'')$. Conditional on the realization of the point mass, his payoff is $\Pi_i\left(p;G_j\right)$. However, he can strictly increase his payoff by replacing the point mass with a mean-preserving spread with support $\{p', p''\}$, as implied by the local convexity of the payoff function: $\alpha\Pi_i\left(p';G_j\right) + (1-\alpha)\,\Pi_i\left(p'';G_j\right) > \Pi_i\left(p;G_j\right)$, where $\alpha p' + (1-\alpha)\,p'' = p$.

3.1 Equilibrium with Full Disclosure

In this subsection, we look for equilibria in which one or both senders adopt full disclosure. Let us begin with a simple observation that if $\underline{p} = 0$, i.e., $\underline{u} \leq u_0$ then there is no equilibrium in which any sender fully discloses. To see this, suppose sender j adopts full disclosure, i.e.,

$$G_j\,(p) = G_j^F\,(p) \equiv \begin{cases} 1-\pi_j & \text{if } p \in [0,1) \\ 1 & \text{if } p = 1 \end{cases}.$$

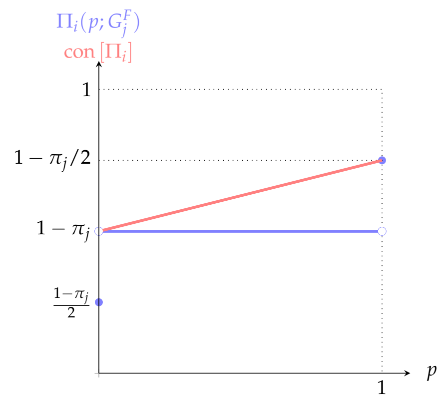


Figure 2: $\underline{p} = 0$

Using (1), as shown in Figure 2, sender i's payoff as a function of posterior is given by

$$\Pi_i\left(p; G_j^F\right) = \begin{cases} \frac{1-\pi_j}{2} & \text{if } p = 0 \\ 1-\pi_j & \text{if } p \in (0,1) \\ 1-\frac{\pi_j}{2} & \text{if } p = 1 \end{cases}.$$

As Π_i exhibits a jump at $p = 0$, we have $0 \notin C(\Pi_i)$. Consequently, there is no best response for sender i; for any G_i, there exists an alternative disclosure policy G_i^ε with support $\{\varepsilon, 1\}$ that gives a strictly higher payoff.(4)

However, if $\underline{p} > 0$, then there exists an equilibrium in which sender i discloses fully if and only if π_j is sufficiently high. First, we identify the condition under which full disclosure is a best response for sender i if sender j discloses fully. As in Figure 3, sender i's payoff function if sender j fully discloses is given by

$$\Pi_i\left(p; G_j^F\right) = \begin{cases} 0 & \text{if } p \in \left[0, \underline{p}\right) \\ 1-\pi_j & \text{if } p \in \left[\underline{p}, 1\right) \\ 1-\frac{\pi_j}{2} & \text{if } p = 1 \end{cases}. \quad (3)$$

A best response of sender i is also full disclosure if and only if $\text{con}\,[\Pi_i]\left(\underline{p}\right) = \underline{p}\left(1-\frac{\pi_j}{2}\right)$, or equivalently, $\pi_j \geq 2\frac{1-\underline{p}}{2-\underline{p}}$ (see Figure 3-(a)). In particular, if $\pi_j > 2\frac{1-\underline{p}}{2-\underline{p}}$, then full disclosure is sender i's unique best response, as $C(\Pi_i) = \{0,1\}$. In contrast, if $\pi_j = 2\frac{1-\underline{p}}{2-\underline{p}}$, then $C(\Pi_i) = \left\{0, \underline{p}, 1\right\}$ and every Bayes-plausible distribution with support $\left\{0, \underline{p}, 1\right\}$ is a best response for sender i. However, in order that full disclosure is optimal for sender j, it is necessary that G_i puts a high enough weight on $p = 1$, which in turn requires π_i to be sufficiently large.

(4)To see this, any policy G_i gives a strictly lower payoff than $\text{con}\,[\Pi_i](\pi_i) = 1-\pi_j+\pi_i\frac{\pi_j}{2}$. Policy G_i^ε gives an expected payoff of $\frac{1-\pi_i}{1-\varepsilon}\Pi_i\left(\varepsilon; G_j^F\right) + \frac{\pi_i-\varepsilon}{1-\varepsilon}\Pi_i\left(1; G_j^F\right)$, which approaches $\text{con}\,[\Pi_i](\pi_i)$ as $\varepsilon \to 0$.

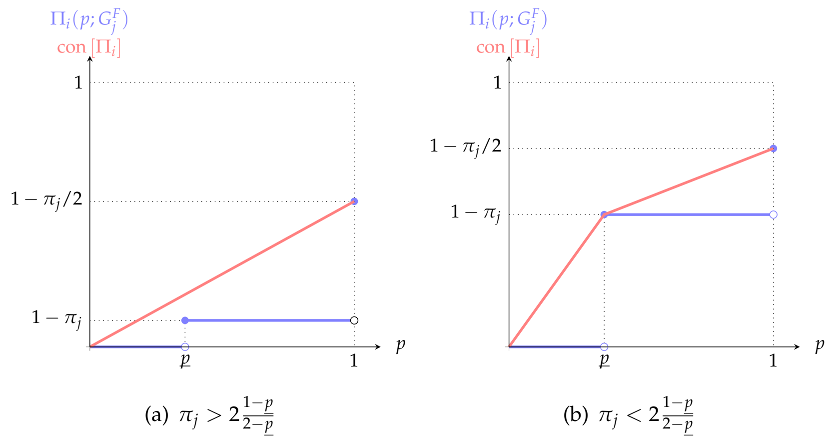


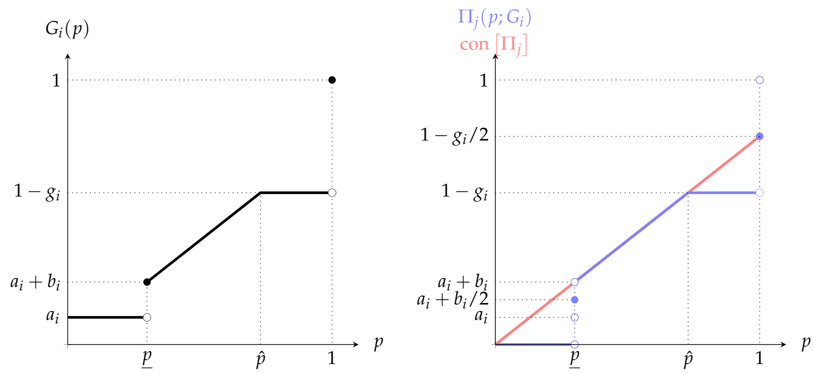
Figure 3: Cases (a) and (b)

Finally, if $\pi_j < 2\frac{1-\underline{p}}{2-\underline{p}}$, as in Figure 3-(b), then $C(\Pi_i) = \left\{0, \underline{p}, 1\right\}$. Therefore, the best response of sender i is either (i) a mechanism with support $\left\{\underline{p}, 1\right\}$ (if $\pi_i > \underline{p}$), or (ii) one with support $\left\{0, \underline{p}\right\}$ (if $\pi_i < \underline{p}$), or (iii) no disclosure (if $\pi_i = \underline{p}$). In the latter two cases, full disclosure is never a best response for sender j, so we do not have an equilibrium. In the first case, full disclosure can be a best response for sender j, provided that π_i, thus the weight of G_i on $p = 1$, is sufficiently large. To see why sender i does not disclose fully even if sender j does, note that as π_j is small, the probability that sender j reveals a bad signal (posterior zero) is large, in which case sender i can win the competition by inducing posterior $\underline{p}$. Consequently, she is happy to induce $\underline{p}$ rather than posterior zero, even at the cost of a lower probability of inducing posterior 1. However, from sender j's perspective, his opponent is strong and has a high chance of inducing posterior 1. Therefore, the chance that he can win by inducing posterior $\underline{p}$ is very low and does not justify the cost of lowering the probability of inducing posterior 1. Consequently, sender j opts for full disclosure.

The following lemma summarizes the above discussion and states the precise necessary and sufficient conditions for the existence of equilibrium in which one or both senders fully disclose.

Lemma 2 *(i) Full disclosure by both senders is an equilibrium if and only if $\pi_i, \pi_j \geq 2\frac{1-\underline{p}}{2-\underline{p}}$. (ii) Full disclosure by sender j and partial disclosure by sender i is an equilibrium if and only if either (a) $\pi_j < 2\frac{1-\underline{p}}{2-\underline{p}}$ and $\pi_i \geq \frac{\underline{p}^2+2(1-\underline{p})}{2-\underline{p}}$, or (b) $\pi_j = 2\frac{1-\underline{p}}{2-\underline{p}}$ and $\pi_i \geq \frac{2(1-\underline{p})}{2-\underline{p}}$.*

Proof. In the Appendix. ■



(a) $G_i(p)$ (b) $\Pi_j(p;G_i)$

Figure 4: Theorem 1

3.2 Equilibrium with Partial Disclosure

In this subsection, we focus on equilibrium in which both senders engage in partial disclosure. The main result is that every strategy in a partial-disclosure equilibrium takes a simple form as depicted in Figure 4.

Theorem 1 *Suppose (G_1, G_2) is an equilibrium strategy profile in which both senders engage in partial disclosure. Then, there exists a common $\hat{p} \in \left[\underline{p}, 1\right]$, $a_i \in [0, 1-\pi_i]$, $g_i \in [0, \pi_i]$, and $b_i \in [0, 1-a_i-g_i]$ for each $i = 1, 2$ such that*

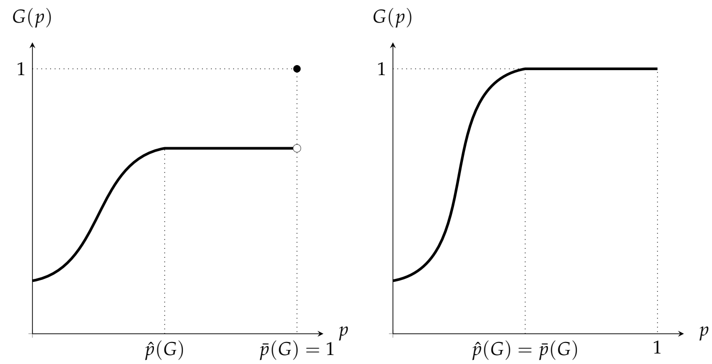
$$G_i(p) = \begin{cases} a_i & \text{if } p \in [0, \underline{p}) \\ a_i + b_i & \text{if } p = \underline{p} \\ a_i + b_i + \frac{1-g_i-(a_i+b_i)}{\hat{p}-\underline{p}}\left(p - \underline{p}\right) & \text{if } p \in (\underline{p}, \hat{p}] \\ 1-g_i & \text{if } p \in (\hat{p}, 1) \\ 1 & \text{if } p = 1 \end{cases}. \tag{4}$$

Consequently, the payoff function $\Pi_j(p, G_i)$ of sender j takes the form:

$$\Pi_j(p; G_i) = \begin{cases} 0 & \text{if } p \in [0, \underline{p}) \\ a_i + \frac{b_i}{2} & \text{if } p = \underline{p} \\ a_i + b_i + \frac{1-g_i-(a_i+b_i)}{\hat{p}-\underline{p}}\left(p - \underline{p}\right) & \text{if } p \in (\underline{p}, \hat{p}] \\ 1-g_i & \text{if } p \in (\hat{p}, 1) \\ 1-\frac{g_i}{2} & \text{if } p = 1 \end{cases}. \tag{5}$$

Moreover, G_i is a Bayes-plausible distribution.

$$\underline{p}b_i + \frac{1}{2}\left(1-g_i-(a_i+b_i)\right)\left(\hat{p}+\underline{p}\right) + g_i = \pi_i. \tag{6}$$



(a) $\bar{p}(G) \neq \hat{p}(G)$ (b) $\bar{p}(G) = \hat{p}(G)$

Figure 5: $\bar{p}(G)$ and $\hat{p}(G)$

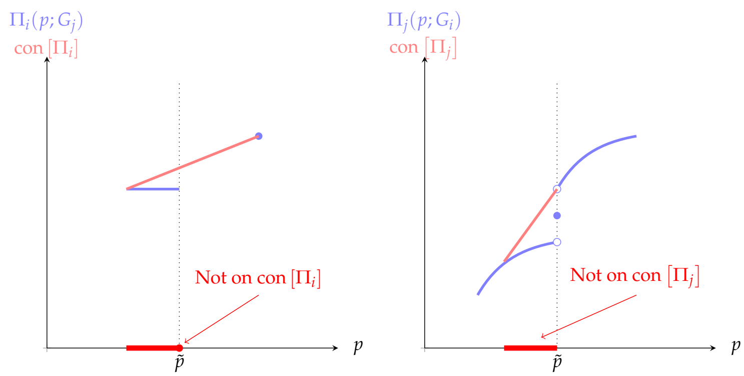


Figure 6: Lemma 3-3

Theorem 1, a necessary condition on the equilibrium strategies, follows from a series of observations. To state these formally, we first introduce a couple of notations. For each distribution $G \in \triangle(\triangle\Omega)$, denote $\bar{p}(G) \equiv \max supp(G)$ and $\hat{p}(G) \equiv \sup(supp(G) \cap (0,1))$. Here, $\bar{p}(G)$ is the upper bound on the support of G, whereas $\hat{p}(G)$ is the upper bound of G's support in $int(\triangle\Omega)$. These definitions are illustrated in Figure 5. By definitions, we have $\bar{p}(G) \geq \hat{p}(G)$. Note that $\bar{p}(G) > \hat{p}(G)$ if and only if G is flat on some interval $(1-\varepsilon, 1)$, where $\varepsilon > 0$, and has an atom at $p = 1$.

Lemma 3 *Suppose (G_i, G_j) is an equilibrium strategy profile in which both senders engage in partial disclosure. For $i = 1, 2$, and $j \neq i$, G_i and G_j satisfy the following properties:*

1. *Sender i does not induce any posterior in $\left(0, \underline{p}\right)$, that is, $G_i(p) = G_i(0)$ for all $p \in \left(0, \underline{p}\right)$.*
2. *The upper bound on the support of each senders' strategy is equal, that is, $\bar{p}(G_i) = \bar{p}(G_j)$.*

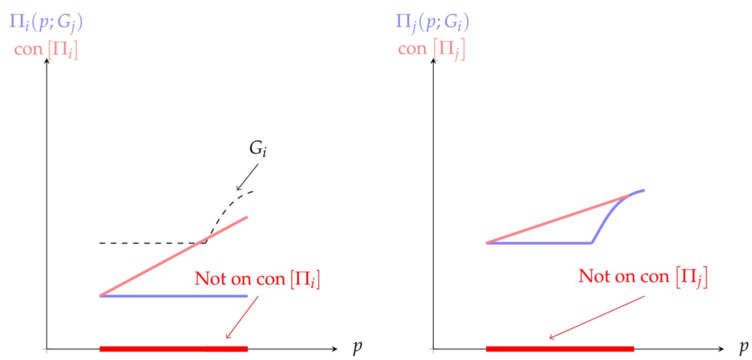


Figure 7: Lemma 3-4

3. *G_i does not contain any atom except possibly at $p = 0, \underline{p}$, or 1.*

4. *G_i is strictly increasing on $\left[\underline{p}, \hat{p}\left(G_i\right)\right]$.*

5. *$\hat{p}\left(G_i\right) = \hat{p}\left(G_j\right)$.*

6. *$G_i\left(p\right)$ is linear on $\left[\underline{p}, \hat{p}\left(G_i\right)\right]$.*

Proof. In the Appendix. ■

We discuss each part of the lemma below. Lemma 3-1 states that no sender induces posterior $p \in \left(0, \underline{p}\right)$. This is because for all $p \in \left(0, \underline{p}\right)$, sender i payoff of inducing p is necessarily zero. However, he can secure a positive payoff by inducing posterior 1. Consequently, sender i can increase his payoff by "spreading" the posterior to 0 and 1. To see Lemma 3-2, suppose $\bar{p}\left(G_i\right) > \bar{p}\left(G_j\right) \geq \pi_i$. Then $\Pi_i\left(p; G_j\right) = 1$ for all $p > \bar{p}\left(G_j\right)$. Posteriors in the interval $\left(\bar{p}\left(G_j\right), \bar{p}\left(G_i\right)\right]$ are "overkill": sender i can guarantee successful persuasion by inducing posteriors marginally above $\bar{p}\left(G_j\right)$. Sender i can therefore strictly improve his payoff by "moving" the probability of realization from interval $\left(\bar{p}\left(G_j\right) + \varepsilon, \bar{p}\left(G_i\right)\right]$ to $\left(\bar{p}\left(G_j\right), \bar{p}\left(G_j\right) + \varepsilon\right)$.

The intuition for Lemma 3-3 is as follows: Suppose, sender i puts an atom at some $\tilde{p} \in \left(\underline{p}, 1\right)$. As in Figure 6, the best response of sender j assigns no probability to an interval slightly below $\tilde{p}$, as this interval does not belong to $C\left(\Pi_j\right)$. This in turn implies that Π_i is flat on this interval. Consequently, $\tilde{p} \notin C\left(\Pi_i\right)$, and sender i should not put a positive probability on $\tilde{p}$, a contradiction.

The intuition for Lemma 3-4 is as follows. Suppose, I is a maximal interval on which G_i is flat,[5] as is depicted in Figure 7. Then Π_j is flat on I, and so no posteriors on I are in $C\left(\Pi_j\right)$. Together with the fact that G_j has no atom at $\sup I$, we know that $\sup I + \varepsilon \notin C\left(\Pi_i\right)$ for some small $\varepsilon > 0$. Consequently, G_i is

[5] Formally, if $I' \supseteq I$ is also an interval such that G_i is flat on I', then $I' = I$.

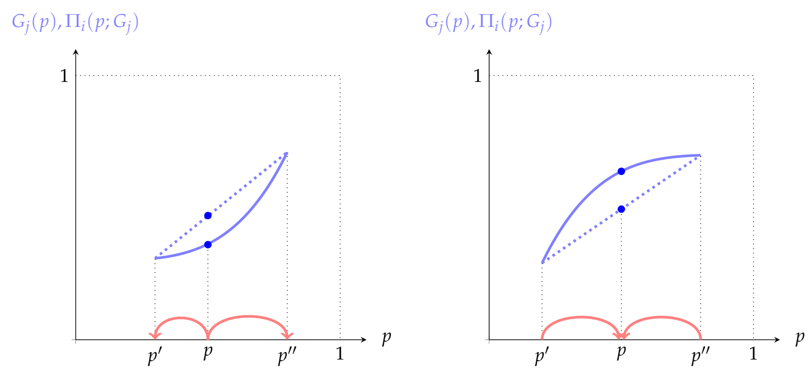


Figure 8: Lemma 3-6

flat on $[\sup I, \sup I + \varepsilon)$, contradicting the maximality of I. The intuition behind Lemma 3-5 is similar to Lemma 3-2.

Now we are ready to discuss Lemma 3-6, which is key to our equilibrium construction. Parts 4 and 5 imply that there is a common posterior $\hat{p} \equiv \hat{p}(G_i) = \hat{p}(G_j)$ such that $\left[\underline{p}, \hat{p}\right] \subset supp\{G_i\}$, and $\left[\underline{p}, \hat{p}\right] \subset supp\{G_j\}$. Lemma 3-6 states that both distributions are conditionally uniform on $\left[\underline{p}, \hat{p}\right]$. That G_j cannot be locally convex follows from Lemma 1. Specifically, if G_j is locally convex on some interval $[p', p''] \subseteq \left[\underline{p}, \hat{p}\right]$, then sender i never induces posterior $p \in (p', p'')$. Otherwise, sender i can increase his payoff by spreading $p \in (p', p'')$ to p' and p''. On the other hand, G_j cannot be locally concave on $[p', p''] \subseteq \left[\underline{p}, \hat{p}\right]$. To see this, take a pair of small positive numbers, ε' and ε'', such that the intervals $[p', p' + \varepsilon']$, $[p'' - \varepsilon'', p''] \subset supp\{G_i\}$. Replace the weights that G_i assigns to the intervals $[p', p' + \varepsilon']$ and $[p'' - \varepsilon'', p'']$ with weights on a single posterior

$$\frac{1}{[G_i(p'') - G_i(p'' - \varepsilon'') + G_i(p' + \varepsilon') - G_i(p')]} \left[\int_{p'}^{p'+\varepsilon'} x dG_i(x) + \int_{p''-\varepsilon''}^{p''} x dG_i(x)\right] \in (p', p'').$$

This probability weight reallocation, preserving Bayes-plausibility of G_i, increases sender i's payoff because of the local concavity of G_j and hence Π_i.

Lemma 3 informs us that the equilibrium strategy is linear in some intermediate region and may possibly have atoms at posteriors 0, $\underline{p}$, and 1. Consequently, the equilibrium strategy takes the form of (4). Finally, equation (6) is the Bayes-plausibility condition.

Lemma 1 imposes further requirements on an equilibrium with both senders engaging in partial disclosure.

Lemma 4 *Following the notations in Theorem 1, the following properties hold. (i) If $\underline{p} > 0$, then at most one of b_i and b_j is positive. If $\underline{p} = 0$, then at most one of a_i and a_j is positive; (ii) If $\underline{p} > 0$, then $a_i + b_i > 0$. (iii) Either $g_i = g_j = 0$, or $g_i, g_j > 0$.*

Proof. In the Appendix. ■

Theorem 1 and Lemma 4 impose a rather strict structure on the equilibrium strategies. In the next subsection, we illustrate that by combining with the sufficient condition in Lemma 1, we can fully characterize the equilibrium.

3.3 Equilibrium Uniqueness and Characterization

In this subsection, we establish equilibrium uniqueness by scanning over all possible forms of equilibria permitted by Theorem 1 and Lemma 4. Observe first that by Lemma 4, it is without loss of generality to set $b_j = 0$, and impose $a_j \geq 0$. In the case of $\underline{p} > 0$, there are four possibilities, depending on whether there is an atom at the top or not, and whether both senders assign an atom at the bottom. Specifically, the four possible forms of equilibria are as follows: (1) $g_i = g_j = 0$ and $a_i > 0$; (2) $g_i = g_j = 0$ and $a_i = 0$; (3) $g_i, g_j > 0$ and $a_i > 0$; and (4) $g_i, g_j > 0$ and $a_i = 0$. On the other hand, if $\underline{p} = 0$, only cases (2) and (4) are relevant.

We only cover case (3) with $\underline{p} > 0$ using Figure 4 here, and a complete analysis can be found in the proof of Proposition 2.

In case (3), the support of G_j is $\{0,1\} \cup \left[\underline{p}, \hat{p}\right]$, with no atom in the interval $\left[\underline{p}, \hat{p}\right]$. By Lemma 1, a necessary condition for G_j being a best response is thus $\{0,1\} \cup \left(\underline{p}, \hat{p}\right) \subset C\left(\Pi_j\right)$. By Theorem 1, this is equivalent to requiring the following slope conditions (see Figure 4):

$$1 - \frac{g_i}{2} = \frac{1 - g_i}{\hat{p}} = \frac{a_i + b_i}{\underline{p}}. \tag{7}$$

If these slope conditions are satisfied, then con $\left[\Pi_j\right](\cdot)$ is linear on $\triangle\Omega$, so every Bayes-plausible posterior distribution that assigns positive weights only on $C\left(\Pi_j\right) = \{0,1\} \cup (\underline{p}, \hat{p}]$ is optimal. Therefore, (7) is also sufficient for the optimality of G_j.

In a similar vein, the support of G_i is $\{0,1\} \cup [\underline{p}, \hat{p}]$, possibly with an atom at $\underline{p}$. By Lemma 1, the necessary condition for G_i being a best response is thus $\{0,1\} \cup \left(\underline{p}, \hat{p}\right) \subset C\left(\Pi_i\right)$. This is equivalent to requiring the following slope conditions:

$$\frac{1 - \frac{g_j}{2} - a_j}{1 - \underline{p}} = \frac{1 - g_j - a_j}{\hat{p} - \underline{p}} = \frac{a_j}{\underline{p}}. \tag{8}$$

Again, if these slope conditions hold, then con $[\Pi_i](\cdot)$ is linear on $\triangle\Omega$, so every Bayes-plausible posterior distribution that assigns positive weights only on $C\left(\Pi_i\right) = \{0,1\} \cup [\underline{p}, \hat{p}]$ is optimal. Therefore, (8) is also sufficient for the optimality of G_i. In sum, slope conditions for equations (7) and (8) are necessary and sufficient conditions for an equilibrium with form (3).

The system of equations (6), (7) and (8) has six equations in six unknowns $\left(\hat{p}, a_i, b_i, a_j, g_i, g_j\right)$. Straightforward algebra shows that a solution (and hence an equilibrium of this form) exists if and only if the prior

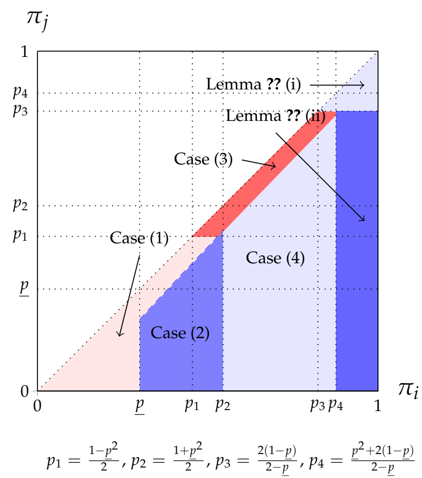


Figure 9: Proposition 2

pair (π_i, π_j) satisfies the following conditions:[(6)]

$$\frac{1-\underline{p}^2}{2} < \pi_j < \min\left\{\frac{2\left(1-\underline{p}\right)}{2-\underline{p}}, \pi_i\right\}; \text{ and}$$

$$\text{either } \pi_j + \sqrt{\pi_j^2 + \underline{p}^2} > \pi_i + \sqrt{\pi_i^2 - \underline{p}^2}, \text{ or } \pi_i < \underline{p}.$$

This region of prior pairs is identified by case (3) in Figure 9. Following a similar analysis for the other cases, we can work out the corresponding regions of prior pairs, all of which are depicted in Figure 9. The observation that these regions do not overlap gives equilibrium uniqueness.

Proposition 2 *The equilibrium is unique except in the nongeneric case that* $\pi_j = 2\frac{1-\underline{p}}{2-\underline{p}}$ *and* $\pi_i \geq \frac{2(1-\underline{p})}{2-\underline{p}}$ *for some* $i = 1, 2$, *and* $j \neq i$ *(case ii.b of Lemma 2).*

Proof. In the Appendix. ■

In the proof, we show that with $\underline{p} > 0$, the four cases of partial-disclosure equilibrium, together with the full-disclosure equilibrium computed in Lemma 2, partition the set of all prior pair (π_i, π_j). With $\underline{p} = 0$, there is no full disclosure equilibrium. It is shown that cases 2 and 4 of partial disclosure partition the set of all prior pairs.

We conclude the discussion on equilibrium disclosure with two remarks. First, the equilibrium characterized in Theorem 1 features a uniform distribution over interior posteriors, with the possibility of atoms at extreme posteriors. This resembles the mixed-strategies equilibria identified in the study of all-pay auction with complete information. In an all-pay auction, each bidder must be indifferent between the bids

[(6)]The exact solutions to the system can be found in the proof of Proposition 2.

on the support of his equilibrium strategy. With monetary transfers entering linearly in the utility function, it is therefore necessary that a bidder's probability of winning is linear in his bid. On the other hand, in our setting of competition in Bayesian persuasion with binary states, in order that a sender is willing to put positive probability on every posterior on the support of his equilibrium strategy, it is necessary that the concave closure of his payoff function Π_i is linear on the support (recall part (vi) of Lemma 3). Despite their similarities, there are two major differences in the resulting equilibrium strategies between our model and the all-pay auctions. First, our equilibria often feature an atom at the top, that is, $p = 1$ realizes with a strictly positive probability. However, the equilibria in all-pay auction typically features only an atom at the bottom of the feasible bids, but no atom at the top. Second, in our analysis of general state space in Section 5, we find that whereas the equilibrium strategy remains "locally linear",(7) it often features an upward kink at intermediate utilities. However, this feature is absent in the all-pay auction models.

Second, we mention in the discussion following Proposition 1 that an equilibrium may not exist if the senders have an uncommon state space. Example 1 below provides an illustration.

Example 1 (Equilibrium Nonexistence for Uncommon State Spaces) *Suppose the support of U_1 is $\{0,1\}$, with a prior $\Pr(U_1 = 1) = 0.7$; whereas the support of U_2 is $\{0,2\}$ with a prior $\Pr(U_2 = 2) = 0.1$. Moreover $\underline{u} = 0$. We use the notation $p_i \in [0,1]$ to stand for the posterior belief that the U_i of sender i is positive.*

There are two types of mechanisms for sender 2: either (i) $supp\{G_2\} \cap (0, 0.5) \neq \emptyset$, or (ii) $supp\{G_2\} \cap (0, 0.5) = \emptyset$. Consider the first type. The argument similar to that of Lemma 3 leads us to conclude that the concave closure of payoff function must be linear on $[0,1]$ for sender 1; and on $[0,0.5]$ for sender 2. Furthermore, argument parallel to Proposition 2 implies that sender 1 assigns an atom at $p_1 = 1$, whereas sender 2 assigns an atom at $p_2 = 0.5$.(8) Then, however, $\Pi_2(p_2; G_1)$ is discontinuous at $p_2 = 0.5$, so sender 2 would not find it optimal to assign an atom at $p_2 = 0.5$, a contradiction.

In the second type of mechanisms, sender 2 must assign an atom at $p_2 = 0$, as otherwise, Bayes-plausibility can never be satisfied. Moreover, sender 2 would never assign a positive measure on $p_2 \in (0.5, 1)$, as any such posterior is an overkill (see part 2 of Lemma 3 for a formal argument). Consequently, the only possible equilibrium mechanism of sender 2 is one that has a support $\{0, 0.5\}$. However, sender 1 would have no best response to this (see Figure 2).

The nonexistence of mutual best responses in the example above arises because the best possible news that sender 1 can generate is $U_1 = 1$. Therefore, whenever sender 1 puts an atom at the top, i.e., $p_1 = 1$, sender 2 would have no best response as she wants to beat the atom by an infinitesimal amount. The

(7)See Lemma 9 in Section 5.

(8)More specifically, given that G_1 must be linear on $[0,1)$, Bayes-plausibility dictates that G_1 must assign an atom at $p_1 = 1$. Then by part (iii) of Lemma 4, G_2 must also assign an atom at $p_2 = 0.5$.

nature of equilibrium nonexistence here is similar to that of first-price auction with complete information and uncommon values.

3.4 Comparative Statics

In this subsection, we analyze the effect of an increase in a sender's prior on the equilibrium disclosure mechanisms. Intuitively, the better the prior of sender i, i.e., the higher the value of π_i, the more favorable is the news that sender i discloses. On the other hand, facing a stronger opponent, sender j will have to respond with more aggressive disclosure to increase his chance of persuading the receiver. This intuition is formalized by the proposition below.

Proposition 3 *Suppose $\pi_i \neq 2\frac{1-\underline{p}}{2-\underline{p}}$ for $i = 1, 2$. Let (G_i, G_j) be the equilibrium if the pair of priors is (π_i, π_j). Similarly, let $\left(G_i', G_j'\right)$ be the equilibrium if the pair of priors is (π_i', π_j), where $\pi_i' > \pi_i$. Then G_i' first-order stochastically dominates G_i. Moreover, either $G_j' = G_j$, or G_j' is a mean-preserving spread of G_j.*

Proof. In the Appendix. ■

Recall that the equilibrium posterior distribution must take a particular form as required by Lemma 1. The first part of proposition states that an increase in sender i's prior must be accompanied by a downward shift of the distribution of G_i. In the proof, it is shown that this is achieved by either (i) an expansion in the support (in cases 1 and 2 of partial disclosure); or (ii) an increase in the atom size at $p = 1$ (in cases 3 and 4 of partial disclosure, as well as case ii.a of Lemma 2). Note that the result does not follow directly from Bayes-plausibility, which only requires that the posterior goes up on average with a stronger prior.

The second part of the proposition states that facing a stronger opponent, sender j would either maintain his initial disclosure mechanism, or she would adopt one that second-order stochastically dominates the initial mechanism. In cases 1 and 2 of partial disclosure, an increase in sender i's prior results in an expansion in the support of G_i. As sender j's mechanism must exhibit the linear structure identified in Lemma 3, and have a common support as G_i, sender j must spread the probability weight towards both ends of $p = 0$ and $p = 1$. On the other hand, suppose sender i increases his atom at $p = 1$, as in cases 3 and 4 of partial disclosure. As the slope condition in equation (7) requires both senders having the same size of the atom, this forces sender j to meet the increase in the atom size of sender i. Moreover, Bayes-plausibility forces sender j to increase the weight on low posterior values.

4 Binary State Space and N-Symmetric Senders

In this section, we consider the disclosure game with n symmetric senders, i.e., $\pi_i = \pi \in (0,1)$ for all $i = 1, 2, ..., n$. We restrict attention to symmetric equilibria, i.e., equilibria in which all senders adopt an identical disclosure mechanism. Our objective is to study the effect of increasing the number of competitors on the equilibrium information disclosure of each individual sender.

We first describe sender i's expected payoff when all other senders use strategy $G(p)$. If sender i induces posterior $p < \underline{p}$, then his payoff is zero. If he induces posterior $p \geq \underline{p}$, then he wins with probability $\frac{1}{k+1}$ if p is the highest posterior among all senders, and there are k other senders offering p. Therefore, his expected payoff of offering posterior $p \geq \underline{p}$ is given by

$$\lim_{p' \to p^-} \sum_{k=0}^{n-1} \frac{1}{k+1} \frac{(n-1)!}{k!(n-k-1)!} \left(G(p) - G(p')\right)^k \left(G(p')\right)^{n-k-1} = \lim_{p' \to p^-} \frac{(G(p))^n - (G(p'))^n}{n(G(p) - G(p'))}.$$

If G is continuous at p, then the payoff is simply $(G(p))^{n-1}$. In sum, sender i's payoff of inducing posterior p when all other senders adopt strategy $G(p)$ is,

$$\Pi(p, G) = \begin{cases} \lim_{p' \to p^-} \frac{(G(p))^n - (G(p'))^n}{n(G(p) - G(p'))} & p \geq \underline{p} \\ 0 & p < \underline{p} \end{cases}.$$

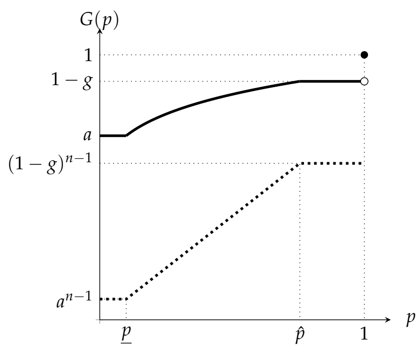
We proceed by a similar approach in Section 3. Necessary and sufficient conditions for full disclosure equilibrium are first identified. Then we study partial disclosure equilibrium. After establishing equilibrium uniqueness, we show that each sender adopts a more transparent disclosure policy with an increase in the number of competing senders, and eventually full disclosure in the limit.

Consider the equilibrium with full disclosure. Suppose that $n-1$ senders are disclosing fully. If $n-1$ is very large, then the probability that at least one of them gives a realized posterior $p = 1$ is close to one. Consequently, the remaining sender's payoff of inducing a posterior $p \in \left[\underline{p}, 1\right)$, given by $\Pi(p) = (1-\pi)^{n-1}$, is sufficiently small in comparison to the payoff of inducing posterior 1, given by $\Pi(1) = \frac{1-(1-\pi)^n}{n\pi}$. Consequently, he finds it optimal to disclose fully in order to maximize the probability of inducing $p = 1$. This intuition is formalized by the following lemma:

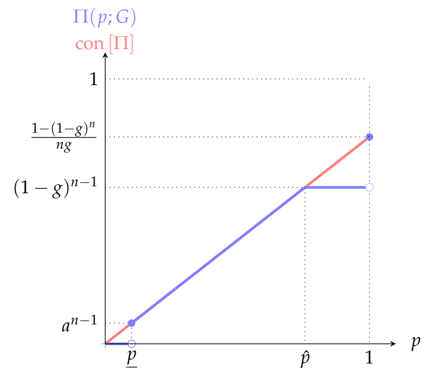
Lemma 5 *There exists an equilibrium with full disclosure if and only if $\underline{p} \geq p^F(n) \equiv \frac{n\pi(1-\pi)^{n-1}}{1-(1-\pi)^n}$. Equivalently, there exists a $n^F\left(\underline{p}\right) \in \mathbb{N} \cup \{\infty\}$ such that a full-disclosure equilibrium exists if and only if $n \geq n^F\left(\underline{p}\right)$.*

Proof. In the Appendix. ■

The cutoff $p^F(n)$ is strictly positive for all $n \in \mathbb{N}$, so there is no full-disclosure equilibrium if $\underline{p} = 0$. On ther other hand, in the case $\underline{p} > 0$, as $p^F(n)$ is decreasing and approaches 0 as $n \to \infty$, a full-disclosure equilibrium always exists if the number of senders is sufficiently large.



(a) $G(p)$



(b) $\Pi_i(p;G)$

Figure 10: Lemma 6

Next we analyze the equilibrium with partial disclosure. The following lemma states the particular form that every symmetric equilibrium strategy must take, which is depicted in Figure 10-(a); it is the counterpart of Theorem 1.

Lemma 6 *Suppose G is the equilibrium strategy in a symmetric equilibrium with partial disclosure. Then there exists* $\hat{p} \in \left[\underline{p}, 1\right]$, $a \in [0, 1-\pi]$, *and* $g \in [0, \pi]$ *such that*

$$G(p) = \begin{cases} a & \text{if } p \in [0, \underline{p}) \\ \left(a^{n-1} + \frac{(1-g)^{n-1} - a^{n-1}}{\hat{p} - \underline{p}} \left(p - \underline{p}\right)\right)^{\frac{1}{n-1}} & \text{if } p \in \left[\underline{p}, \hat{p}\right] \\ 1-g & \text{if } p \in (\hat{p}, 1) \\ 1 & \text{if } p = 1 \end{cases}. \tag{9}$$

Moreover, Bayes plausibility requires

$$\int_{\underline{p}}^{\hat{p}} p dG(p) + g = \pi. \tag{10}$$

Furthermore, $a = 0$ *if and only if* $\underline{p} = 0$.

Proof. In the Appendix. ■

The proof of the Lemma is similar to that of Theorem 1. First, arguments along the line of those in Lemma 3 rule out interior atoms, and flat segments on $\left[\underline{p}, \hat{p}(G)\right]$ in the symmetric equilibrium strategy G. Moreover, $\Pi(p, G)$ is necessarily linear on $\left[\underline{p}, \hat{p}(G)\right]$ for otherwise, a sender is unwilling to assign positive probability on all of those posteriors.

By Lemma 6, our search for partial-disclosure equilibrium can be focused on the following two cases: (1) $g = 0$ and (2) $g > 0$ and $\hat{p} < 1$. Below we illustrate how to construct an equilibrium with $g > 0$ and $\underline{p} > 0$ using Figure 10.

Suppose that every other sender adopts strategy (9). A sender's payoff of inducing posterior p is given by

$$\Pi(p) = \begin{cases} 0 & \text{if } p \in [0, \underline{p}) \\ a^{n-1} + \frac{(1-g)^{n-1} - a^{n-1}}{\hat{p} - \underline{p}} \left(p - \underline{p}\right) & \text{if } p \in \left[\underline{p}, \hat{p}\right] \\ (1-g)^{n-1} & \text{if } p \in (\hat{p}, 1) \\ \frac{1-(1-g)^n}{ng} & \text{if } p = 1 \end{cases} . \tag{11}$$

In order that a sender finds it optimal to adopt strategy (9), it is necessary to ensure that $\{0,1\} \cup \left[\underline{p}, \hat{p}\right] \subset C(\Pi)$. This translate into the following slope conditions:

$$\frac{\Pi\left(\underline{p}\right)}{\underline{p}} = \frac{\Pi(\hat{p})}{\hat{p}} = \Pi(1).$$

If these slope conditions hold, then $\text{con}[\Pi](\cdot)$ is linear on $[0,1]$. Consequently, every Bayes-plausible posterior distribution that assigns positive weights only on $C(\Pi) = \{0,1\} \cup (\underline{p}, \hat{p}]$ is optimal. Therefore, the strategy identified in Lemma 6 is indeed optimal. In sum, the above slope conditions are necessary and sufficient condition for a symmetric equilibrium of this form.

Using (11), the above slope conditions above give

$$a = \left(\underline{p}\frac{1-(1-g)^n}{ng}\right)^{\frac{1}{n-1}}, \text{ and } \hat{p} = \frac{ng(1-g)^{n-1}}{1-(1-g)^n}. \tag{12}$$

Substituting (9) and (12) into (10) and rearranging, we arrive at the following equation in g:

$$\frac{g}{1-(1-g)^n} - \left(\frac{\underline{p}}{n}\right)^{\frac{n}{n-1}} \left(\frac{1-(1-g)^n}{g}\right)^{\frac{1}{n-1}} - \pi = 0. \tag{13}$$

It can be shown that there exists a $n^P\left(\underline{p}\right) \in \mathbb{N}$ such that the equation above admits a unique solution in the interval $[0, \pi]$ if and only if $n \in \left[n^P\left(\underline{p}\right), n^F\left(\underline{p}\right)\right]$. In a similar manner, we can show that a symmetric equilibrium with $g = 0$ exists if and only if $n \leq n^P\left(\underline{p}\right)$. This leads us to the following proposition.

Proposition 4 *A symmetric equilibrium exists, and is unique. When $n \geq n^F\left(\underline{p}\right)$, the unique symmetric equilibrium is with full disclosure. If $n < n^F\left(\underline{p}\right)$ the unique symmetric equilibrium is with partial disclosure.*

Proof. In the Appendix. ■

Intuitively, as the number of senders increases, the competition for the receiver becomes more intense. In response, senders adopt more aggressive information disclosure mechanisms. This intuition is confirmed by the proposition below, which states that an increase in the number of senders results in an equilibrium distribution that is a mean-preserving spread of the initial distribution. Moreover, in the limit as the number of senders increase to infinity, full disclosure by all senders arises in equilibrium.

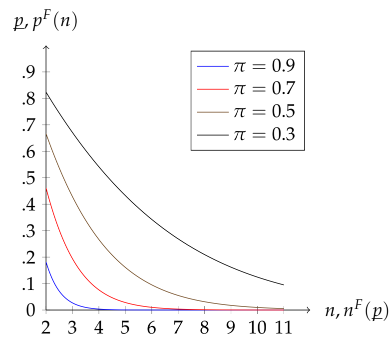


Figure 11: Proposition 4

Proposition 5 *Suppose G_n is the symmetric equilibrium strategy with n senders. Denote by G^F the distribution function corresponding to full disclosure.*[(9)] *(i) If $n_2 > n_1$ and $n_1 \leq n^F\left(\underline{p}\right)$, then G_{n_2} is a mean-preserving spread of G_{n_1}. (ii) If $\underline{p} > 0$, then $G_n = G^F$ for all $n > n^F\left(\underline{p}\right)$. If $\underline{p} = 0$, then G_n converges to G^F pointwise as $n \to \infty$.*

Proof. In the Appendix. ■

Perloff and Salop (1985) analyze competition in setting prices among n symmetric senders, all of which adopt a full disclosure policy. They show that there is a unique symmetric equilibrium, and the price converges to the marginal cost of production as the number of senders approaches infinity.[(10)] Our results can, therefore, be viewed as counterparts to theirs in the context of competition in information disclosure. Figure 11 illustrates the number of senders needed to achieve full disclosure in equilibrium for different combinations of common prior π, and threshold probability $\underline{p}$.

Propositions 3 and 5 confirm the intuition that competition among senders leads to more aggressive disclosure. Gentzkow and Kamenica (2015a) consider the effect of competition in a setting of Bayesian persuasion in which all senders share a common state. They find that adding more senders never makes the set of equilibrium outcomes less informative. However, with equilibrium multiplicity, they also note that the set of outcomes with more competition may not be comparable to those with less competition. On ther other hand, our setting admits a unique symmetric equilibrium, which allows us to obtain a sharp result concerning the effect of competition on information revelation.

Remark 1 *Given the similarity of equilibrium strategy profile in all-pay auction with complete information, and that of the game analyzed in this paper, one may expect a continuum of asymmetric equilibria similar to those in*

[(9)]That is,

$$G^F(p) = \begin{cases} 1-\pi & \text{if } p \in [0,1) \\ 1 & \text{if } p = 1 \end{cases}.$$

[(10)]The convergence result in their setting requires the tail of the preference distribution to be not too fat. In particular, it holds with a finite support, the case we examine here.

Baye et al. (1996). We find that it is indeed the case when $n \in \left[3, n^F\left(\underline{p}\right)\right)$ and $\underline{p} > 0$: for every $\tilde{p} \in \left(\underline{p}, \hat{p}\right)$, there exists an asymmetric equilibrium in which at least two senders induce all $p \in \left(\underline{p}, \hat{p}\right)$, while at least one sender does not induce any $p \in \left(\underline{p}, \tilde{p}\right)$.

5 Arbitrarily Finite State Space and Two Senders

In this section, we extend the analysis to a more general state space. There are two competing senders, and the common state space is denoted by $\Omega \equiv \{u_0, u_1, ..., u_{m-1}\}$. Note that a belief on Ω is represented by a non-negative vector $(p_0, p_1, \cdots, p_{m-1})$ satisfying $\sum_{k=0}^{m-1} p_k = 1$. The increase in the number of dimensions makes the analysis challenging, as $\Pi_i\left(p; G_j\right)$ is now a mapping from a $(m-1)$-dimensional space. Consequently, a sender's best response cannot be solved by inspecting a two-dimensional graph like Figure 4(b). In the analysis below, we show that some, but not all, of the results in the binary state space can be carried to this setting.

Like Section 2, the priors for the respective senders are denoted by $\pi_i, \pi_j \in int\left(\triangle\Omega\right)$. Let $\Lambda_i \subset \triangle\left(\triangle\Omega\right)$ be the space of Bayes-plausible distributions of posterior beliefs for sender i. The buyer's outside option is $\underline{u} < u_{m-1}$. As the buyer selects the proposal that gives him the highest expected utility (provided that it is no less than $\underline{u}$), sender i's payoff depends on his realized posterior p only through the induced value of expected utility $E_p\left[U_i\right]$. Specifically, for each distribution over posteriors $G_i \in \Lambda_i$, denote by $F_{G_i}(v) \equiv \Pr\left(E_p\left[U_i\right] \le v | G_i\right)$ the distribution over expected utility induced by G_i. If sender j uses strategy G_j, then sender i's payoff as a function of posteriors $\Pi_i\left(p; G_j\right)$ is given by

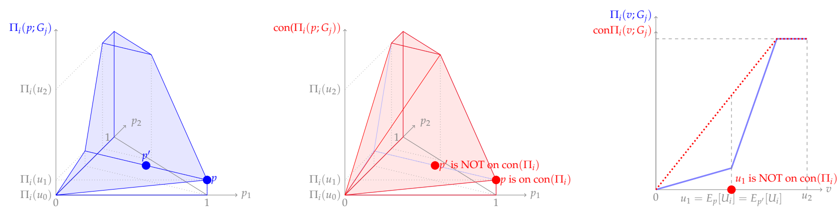
$$\Pi_i\left(p; G_j\right) \equiv \begin{cases} \frac{1}{2}\left(F_{G_j}\left(E_p\left[U_i\right]\right) + \lim_{v' \to E_p[U_i]^-} F_{G_j}\left(v'\right)\right) & E_p\left[U_i\right] \ge \underline{u} \\ 0 & E_p\left[U_i\right] < \underline{u} \end{cases}.$$

With a slight abuse of notation, we sometimes express a sender's payoff as a function of induced expected utility, $\Pi_i\left(v; G_j\right)$, where $v = E_p\left[U\right]$, that is,

$$\Pi_i\left(v; G_j\right) \equiv \begin{cases} \frac{1}{2}\left(F_{G_j}(v) + \lim_{v' \to v^-} F_{G_j}\left(v'\right)\right) & v \ge \underline{u} \\ 0 & v < \underline{u} \end{cases}.$$

Lemma 1 still holds in this setting, as Corollary 2 of Kamenica and Gentzkow (2011) is valid for a general state space. All the results of the binary state space could be carried over if an analogue of Lemma 1 holds for $\Pi_i\left(v; G_j\right)$. Unfortunately, as pointed out in Section 3 of Kamenica and Gentzkow (2011), this is not true whenever the state space has more than two dimensions. For an illustration of this observation, consider Figure 12. In the example, $\Omega = \{u_0, u_1, u_2\}$ and sender i faces a $\Pi_i\left(p; G_j\right)$ as depicted in Figure 12 (a).[11] Note that $p = (0, 1, 0)$, a degenerate distribution at u_1, is on the concave closure of $\Pi_i\left(p; G_j\right)$, as

[11] In Figure 12, the domains of $\Pi_i\left(p; G_j\right)$ and $\text{con}\Pi_i\left(p; G_j\right)$ are 2-simplex.



(a) $\Pi_i(p;G_j)$ (b) $\text{con}(\Pi_i(p;G_j))$ (c) $\Pi_i(v;G_j)$

Figure 12: $\Pi_i\left(p;G_j\right)$, $\text{con}(\Pi_i\left(p;G_j\right))$ and $\Pi_i\left(v;G_j\right)$ $(m=3)$

shown in Figure 12 (b). Now if sender i's prior π_i is at p', then by part (i) of Lemma 1, a best response of sender i is to assign positive weights only to the vertices of $con\Pi_i\left(p;G_j\right)$, including p. However, u_1 is not on the concave closure of $\Pi_i\left(v;G_j\right)$, as shown in Figure 12 (c). Therefore, assigning positive weights only to expected utilities v on the concave closure of $\Pi_i\left(v;G_j\right)$ is NOT a necessary condition for being a best response. The message of this example is that with higher dimensions, when looking for the best response of sender i, it is insufficient to construct the concave closure of $\Pi_i\left(v;G_j\right)$; it is necessary to construct the concave closure of $\Pi_i\left(p;G_j\right)$.

Despite the aforementioned technical difficulty, we show below that the equilibrium strategy must exhibit a linear structure locally. In particular, the equilibrium strategy induces a "piecewise uniform" distribution over expected utilities. Also, the distribution may have upward kinks only at $u\in\Omega$. To illustrate the usefulness of this finding, we provide a full characterization of the symmetric equilibrium in the case of two symmetric senders with $\underline{u}=u_0$ and the state space has three elements.

5.1 Piecewise Linear Structure

We begin with a simple observation that it is without loss of generality to focus on a class of strategy with a simple support. Recall that a sender's payoff depends on his realized posterior only through the induced value of expected utility. Therefore, the space of probability measures over states, $\triangle\Omega$, can be partitioned by level curves $U\left(v\right)\equiv\left\{p\in\triangle\Omega:E_p\left[U_i\right]=v\right\}$ corresponding to different values of expected utilities $v\in\left[u_0,u_{m-1}\right]$. Every posterior in $U\left(v\right)$ thus gives the same probability of winning to a sender. Each level curve $U\left(v\right)$ is a compact and convex set, with extreme points given by $Q\left(v\right)\equiv\left\{q\in\triangle\Omega:E_q\left[U_i\right]=v\text{ and }q_j+q_k=1\text{ for some }j\text{ and }k\right\}$. By the Krein-Milman theorem, every $p\in U\left(v\right)$ can be written as a convex combination of extreme points from $Q\left(v\right)$. Consequently, for any distribution $G\in\Lambda_i$, there exists a $G'\in\Lambda_i$ with support $Q\equiv\cup_{v\in[u_0,u_{m-1}]}Q\left(v\right)$ such that the distribution over induced expected utilities is identical to that of G, i.e., $F_G=F_{G'}$. We, therefore, have the following lemma:

Lemma 7 *If* (G_1, G_2) *is an equilibrium, then there exists an equilibrium* $(G_1', G_2') \in \Lambda_1 \times \Lambda_2$ *with the properties that* $supp\{G_1\}, supp\{G_2\} \subset Q$, $F_{G_1} = F_{G_1'}$, *and* $F_{G_2} = F_{G_2'}$.

Proof. In the Appendix. ■

A consequence of the lemma above is that in every optimal distribution for sender i, positive weights can only assigned to beliefs on the "local concave closure" of $\Pi_i(v; G_j)$. Specifically, denote by $\Pi_i^{[u_{k-1}, u_k]}(v; G_j)$ the restriction of the function $\Pi_i(v; G_j)$ to its subdomain $[u_{k-1}, u_k]$. The local concave closure of $\Pi_i(v; G_j)$ on $[u_{k-1}, u_k]$ is defined as $con\left[\Pi_i^{[u_{k-1}, u_k]}(\cdot; G_j)\right]$. The lemma given below is the analogue of part (ii) of Lemma 1.

Lemma 8 *If* $G_i \in \Lambda_i$ *is a best response to* $G_j \in \Lambda_j$, *then* $v \in supp\{F_{G_i}\} \cap (u_{k-1}, u_k)$ *implies* $\Pi_i(v; G_j) = con\left[\Pi_i^{[u_{k-1}, u_k]}(\cdot; G_j)\right]$, *except possibly for a zero measure under* F_{G_i}.

Proof. In the Appendix. ■

Recall that in the binary state case, the necessary conditions on an equilibrium $(G_i, G_j) \in \Lambda_i \times \Lambda_j$ as stated in Lemma 3 rely only on part (ii) of Lemma 1. With its counterpart, Lemma 8, similar necessary conditions can be shown to hold for equilibrium distributions over expected utilities $\left(F_{G_i}, F_{G_j}\right)$, with the preconditions that they are valid only locally on each interval $[u_{k-1}, u_k]$, for $k = 1, 2, .., m-1$, and provided that both F_{G_i} and F_{G_j} assign positive probabilities on the interval. For a formal statement, define the counterparts of $\bar{p}$ and $\hat{p}$ in the binary-state case as follows:

$$\bar{U}_k(G) = \begin{cases} \max\left(supp\{F_G\} \cap [u_{k-1}, u_k]\right) & \text{if } supp\{F_G\} \cap [u_{k-1}, u_k] \neq \emptyset \\ u_k & \text{otherwise} \end{cases},$$

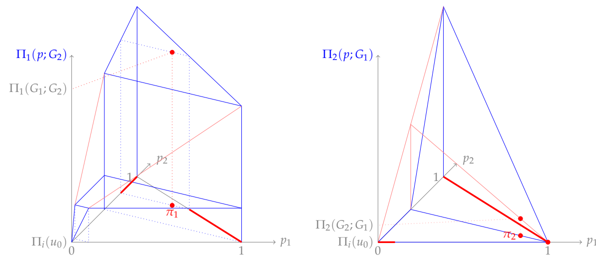
and

$$\hat{U}_k(G) = \begin{cases} \sup\left(supp\{F_G\} \cap [u_{k-1}, u_k)\right) & \text{if } supp\{F_G\} \cap [u_{k-1}, u_k] \neq \emptyset \\ u_k & \text{otherwise} \end{cases}.$$

Also, define $\hat{k} \equiv \min\{k \in \{0, 1, ..., m-1\} : u_k > \underline{u}\}$.

Lemma 9 *Suppose* $i = 1, 2$, $j \neq i$, *and* (G_i, G_j) *is an equilibrium. Let* $k \in \left\{\hat{k}, \hat{k}+1, .., m-1\right\}$ *be such that* F_{G_i} *and* F_{G_j} *both put a positive probability on the interval* (u_{k-1}, u_k), *i.e.,* $\lim_{v \to u_k^-} F_{G_i}(v) - F_{G_i}(u_{k-1}) > 0$ *for* $i = 1, 2$.

1. *If* $k = \hat{k}$, *then* $F_{G_i}(v) = F_{G_i}\left(u_{\hat{k}-1}\right)$ *for all* $v \in [u_{\hat{k}-1}, \underline{u})$.
2. $\bar{U}_k(G_i) = \bar{U}_k(G_j)$.
3. *If* $v \notin [u_0, \underline{u}] \cup \left\{u_{\hat{k}-1}, \cdots, u_{m-1}\right\}$, *then* F_{G_i} *does not put an atom at* v.
4. F_{G_i} *is strictly increasing on* $\left[\max\{\underline{u}, u_{k-1}\}, \hat{U}_k(G_i)\right]$.



(a) $\Pi_1(p;G_2)$ (b) $\Pi_2(p;G_1)$

Figure 13: Example 2

5. $\hat{U}_k(G_i) = \hat{U}_k(G_j)$.

6. F_{G_i} *is linear on* $[\max\{\underline{u}, u_{k-1}\}, \hat{U}_k(G_i)]$.

Proof. In the Appendix. ■

Lemma 9 states that the equilibrium strategy induces a distribution over expected utilities that exhibits a piecewise linear structure. The lemma given below states some global properties of the equilibrium strategies:

Lemma 10 *(i) Suppose* (G_i, G_j) *is an equilibrium. If* F_{G_i} *assigns an atom to some* $u_k \in (\underline{u}, u_{m-1}) \cap \Omega$*, then* F_{G_j} *assigns no atom at* u_k*. (ii) Suppose* $\underline{u} < u_{m-2}$*. Then full disclosure by both senders is not an equilibrium.*

Proof. In the Appendix. ■

Part (i) follows from the simple observation that if both senders assign an atom at some interior utility, then any one of them can profitably deviate by ”improving” the atom marginally to beat the rival's atom. An immediate consequence of this finding is that in every symmetric equilibrium, there is no atom on the interval $(\underline{u}, u_{m-1})$. Part (ii) follows immediately from part (i) as full disclosure implies both senders assign an atom at some interior utility.

Recall that in the binary state case analyzed in Section 3, the equilibrium strategies necessarily feature common support and no atom at interior posteriors. The counterparts of these properties would be that F_{G_i} has a common support for $i = 1, 2$ and that each has no atom at any $v \in (\underline{u}, u_{m-1})$. It turns out that these properties do not hold in a state space with more than two elements, as shown by the example below.

Example 2 *Suppose that* $m = 3$*,* $u_i = i$*, and* $\underline{u} = 0$*. Moreover, the respective priors are* $\pi_1 = \left(\frac{1}{16}, \frac{3}{8}, \frac{9}{16}\right)$*, and*

$\pi_2 = \left(\frac{1}{20}, \frac{17}{20}, \frac{1}{10}\right)$. *Suppose sender 1 uses a strategy G_1 with density function*

$$g_1(p) = \begin{cases} 1 & \text{if } p_1 + p_2 = 1 \text{ and } p_1 \geq \frac{1}{2} \\ 1 & \text{if } p_0 + p_2 = 1 \text{ and } p_0 \leq \frac{1}{4} \\ 0 & \text{otherwise} \end{cases}.$$

The strategy of sender 2, G_2, assigns an atom of size $\frac{71}{95}$ to $u_1 = 1$, and has a density function:

$$g_2(p) = \begin{cases} \frac{1}{5} & \text{if } p_1 + p_2 = 1 \\ \frac{10}{19} & \text{if } p_0 + p_1 = 1 \text{ and } p_1 \leq \frac{1}{10} \\ 0 & \text{otherwise} \end{cases}.$$

The corresponding distributions over expected utilities are given by

$$F_{G_1}(v) = \begin{cases} 0 & v \in [0,1) \\ v-1 & v \in [1,2] \end{cases} \text{; and}$$

$$F_{G_2}(v) = \begin{cases} \frac{10}{19}v & v \in \left[0, \frac{1}{10}\right) \\ \frac{1}{19} & v \in \left[\frac{1}{10}, 1\right) \\ \frac{4}{5} & v = 1 \\ \frac{3+v}{5} & v \in (1,2] \end{cases}.$$

Figure 13 depicts the payoff function that each sender faces given the opponent's strategy. For sender 1, every distribution that assigns a positive measure only on $\{p \in \triangle\Omega : E_p[U_1] > 1\}$ is a best response. For sender 2, every distribution that assigns a positive measure only on the boundary $\{p \in \triangle\Omega : p_0 + p_1 = 1 \text{ or } p_1 + p_2 = 1\}$ is a best response. Therefore, the strategy profile (G_1, G_2) described above is indeed an equilibrium. Note that F_{G_2} has an atom at $u_1 = 1$. Moreover, $\left[0, \frac{1}{10}\right] \in supp\{F_{G_2}\}$ but $\left[0, \frac{1}{10}\right] \notin supp\{F_{G_1}\}$.

5.1.1 Three States and Two Symmetric Senders

In this subsection, we illustrate that Lemmas 9 and 10 are useful in identifying an equilibrium. Specifically, we show that a unique symmetric equilibrium can be fully characterized in a setting in which (i) $\Omega \equiv \{u_0, u_1, u_2\}$, (ii) $\underline{u} = u_0$, and (iii) both senders have a common prior $\pi \in int(\triangle\Omega)$.

Proposition 6 *Suppose the state space consists of three elements, and the two senders have a common prior. Assume $\underline{u} = u_0$. Then a symmetric equilibrium always exists. Moreover, the equilibrium is unique up to the distribution of expected utilities.*

Proof. In the Appendix. ■

Figure 14 depicts the shape of the unique equilibrium distribution of expected utilities, F_G, for different regions of prior $\pi \in int(\triangle\Omega)$. The proof of the proposition is constructive. First, we show, using

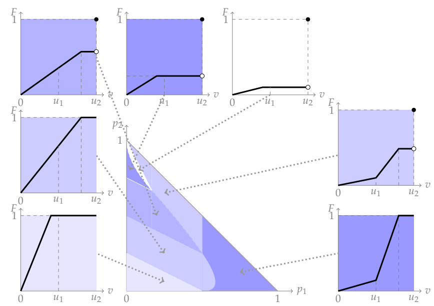
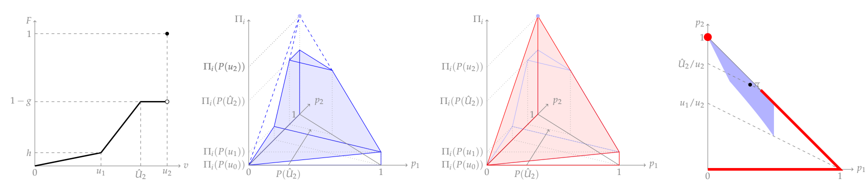


Figure 14: Proposition 6



(a) F (b) Π_i (c) $\text{con}(\Pi_i)$ (d) $\text{supp}F$

Figure 15: A possible form of equilibrium

Lemmas 9 and 10, that the symmetric equilibrium distribution F_G necessarily takes one of the seven forms depicted in Figure 14. Second, we identify necessary and sufficient conditions on the common prior for the existence of a symmetric equilibrium of each particular form. Finally, we show that these conditions cover all possible priors and they do not overlap.

Whereas the equilibrium distribution of expected utility F_G remains linear on each interval of $[u_0, u_1]$ and $[u_1, u_2]$ (except possibly at the top), a novel feature is that it may feature a kink at u_1. Moreover, this kink must be ”upward,” i.e., F_G must be steeper on $[u_1, u_2]$ than $[u_0, u_1]$. Figure 15 is a graphical illustration of a possible form of equilibrium.

Given a payoff function shown in Figure 15(b), it is easy to see that $C(\Pi) = \{p \in \triangle\Omega : p_0 + p_1 = 1\} \cup \{p \in \triangle\Omega : p_1 + p_2 = 1 \text{ and } p_1u_1 + p_2u_2 \leq \hat{U}_2\} \cup (0, 0, 1)$. Moreover, because the concave closure of $\Pi(p; G)$ is linear, as depicted in Figure 15(c), every Bayes-plausible distribution that assigns a positive weight only on $C(\Pi)$ constitutes an optimal response. Note that in an equilibrium, the kink at u_1 for F_G can only be ”upward” (see Figure 15(a)). If it were a downward kink, $\Pi(p; G)$ would be locally concave along the line $\{p \in \triangle\Omega : u_1p_1 + u_2p_2 = u_1\}$, and the optimal response would put a positive weight on either $[u_0, u_1]$ or $[u_1, u_2]$, but not both.[12]

[12] In this case, whether the optimal response puts a positive weight only on $[u_0, u_1]$ or $[u_1, u_2]$ depends on whether the prior π has an expected value above or below u_1.

Finally, note that we do not have equilibrium uniqueness in general. This is because typically, there are multiple distributions over posteriors G that can generate the same distribution over expected utilities F_G. However, it can be shown that, in the current setting, whenever F_G has a kink at u_1, the symmetric equilibrium distribution over posteriors is necessarily unique.

6 Concluding Remarks

We studied competition in Bayesian persuasion in which each sender can disclose only information about his proposal. An appealing feature of our model is that it has a unique equilibrium with a simple structure, which allows us to obtain clean comparative statics results. We believe that the framework can be fruitfully adapted to study other problems that involve competing persuaders. For instance, in persuading a body of voters, multiple receivers are involved, and each may have a different preference. It is interesting to investigate which group of voters each sender would target at. Other avenues for future research include analyzing the case of a general (and asymmetric) state space, as well as the interaction of competition in information disclosure with that in other dimensions such as pricing and product design.

7 Appendix

Proof of Proposition 1

(i) We invoke Corollary 3.3 of Reny (1999) to establish equilibrium existence. To this end, we need to show that the game is compact, quasiconcave, reciprocally upper semicontinuous, and payoff secure.

Compactness: Endow each $\triangle(\triangle\Omega)$ with the weak* topology and $\triangle(\triangle\Omega)^n$ by the corresponding product topology. To see the strategy set of sender i, Λ_i, is compact, it suffices to show that it is closed. Take a sequence $\left\{G_i^k\right\}_k \subset \Lambda_i$ that converges to some $G_i^* \in \triangle(\triangle\Omega)$. As each G_i^k satisfies Bayes-plausibility, $\int p dG_i^k(p) = \pi_i$, we have $\int p dG_i^*(p) = \pi_i$. Thus, $G_i^* \in \Lambda_i$.

Quasi-concavity: Define $\Pi_i : [u_0, u_{m-1}]^n \to [0,1]$ by

$$\Pi_i(v_i, v_{-i}) = \begin{cases} 1/k & v_i \geq \max v_{-i} \text{ and } v_i \geq \underline{u} \\ 0 & \text{otherwise} \end{cases}$$

where k is the number of senders (including sender i) who induced (receiver's) expected utility of v_i. Thus, $\Pi_i(v_i, v_{-i})$ is sender i's expected payoff in the event that the realized profile of expected utility is (v_i, v_{-i}). Define $V_i(G_i, G_{-i})$ as sender i's payoff if the strategy profile is $(G_i, G_{-i}) \in \Lambda_i \times \Lambda_{-i}$:

$$V_i(G_i, G_{-i}) = \int\int \Pi_i\left(E_{p_i}[U_i], E_{p_{-i}}[U_{-i}]\right) dG_{-i}(p_{-i})\, dG_i(p_i).$$

Since $V_i(G_i, G_{-i})$ is linear in G_i, it is quasi-concave.

Reciprocal upper semicontinuity: A sufficient condition for reciprocally upper semicontinuity is that the sum of sender's payoffs

$$\sum_{i=1}^{n} V_i(G) = \int \sum_{i=1}^{n} \Pi_i\left(E_{p_i}[U_i], E_{p_{-i}}[U_{-i}]\right) dG(p)$$

is upper semicontinuous in $G \equiv (G_i, G_{-i})$. To this end, it suffices to note that $\sum_{i=1}^{n} \Pi_i\left(E_{p_i}[U_i], E_{p_{-i}}[U_{-i}]\right) = 1_{\left[\max_{i=1,\dots,n} E_{p_i}[U_i] \geq \underline{u}\right]}$ is a upper semicontinuous function in p.

Payoff security: Each sender's payoff depends on posterior realization only through the distribution $F_G(v) \equiv \Pr\left(E_p[U] \leq v | G\right) = \int_{E_p[U] \leq v} dG(p)$ over the induced value of expected receiver's utility $E_p[U]$. Also, let $F_{G_{-i}}$ be the distribution of $\max_{j \neq i} E_{p_j}\left[U_j\right]$. Fix an arbitrary strategy profile (G_i, G_{-i}) and $\varepsilon > 0$. To show the payoff security at (G_i, G_{-i}), it suffices to show that there exists a $\tilde{G}_i \in \Lambda_i$ such that $F_{\tilde{G}_i}$ is continuous on $[u_0, u_{m-1})$ and $V_i\left(\tilde{G}_i, G_{-i}\right) > V_i(G_i, G_{-i}) - \varepsilon/2$. This is because for $\tilde{G}_i$ that is continuous on $[u_0, u_{m-1})$, $V_i\left(\tilde{G}_i, G'_{-i}\right)$ is lower semicontinuous with respect to G'_{-i}.(13) Consequently, there exists a neighborhood $O(G_{-i})$ of G_{-i} such that for all $G'_{-i} \in O(G_{-i})$, $V_i\left(\tilde{G}_i, G'_{-i}\right) > V_i\left(\tilde{G}_i, G_{-i}\right) - \varepsilon/2$.

Below we thus show the existence of a $\tilde{G}_i \in \Lambda_i$ such that F_{G_i} is continuous on $[u_0, u_{m-1})$ and $V_i\left(\tilde{G}_i, G_{-i}\right) > V_i(G_i, G_{-i}) - \varepsilon/2$. First, we show that it is without loss of generality to assume that for any G_i, there exists a $p \in supp\{G_i\}$ such that $p_{m-1} > 0$ and $E_p[U] = \max supp\left\{F_{G_i}\right\}$. To this end, for each $k \in \{1, 2, ..., m-1\}$, let $V_k(G_i)$ be the supremum of the set of v such that $v = E_p[U]$ for some $p \in supp\{G_i\}$ such that $p_k > 0$. (14)

Lemma 11 *For each $G_i \in \Lambda_i$, there exists a $G'_i \in \Lambda_i$ such that $F_{G_i} = F_{G'_i}$, and $V_k\left(G'_i\right)$ is weakly increasing in k.*

Proof. Suppose that $V_k(G_i) < V_{k'}(G_i)$ for some $k' < k$. Then there exists an $\tilde{\varepsilon} \geq 0$ such that $p_k = 0$, $p_{k'} > 0$ for all $p \in A \equiv \left\{p \in supp\{G_i\} : E_p[U_i] \geq V_{k'}(G_i) - \tilde{\varepsilon}\right\}$ and $\mu_A \equiv \int_A 1 dG_i(p) > 0$. Recall that $\pi_{i\kappa} > 0$ for all κ. Thus there exists an $\varepsilon' > 0$ such that the set $B \equiv \left\{p \in supp\{G_i\} : p_k > \varepsilon' \text{ and } E_p[U_i] < V_{k'}(G_i) - \tilde{\varepsilon}\right\}$ has a positive measure under G_i, i.e., $\mu_B \equiv \int_B 1 dG_i(p) > 0$.

We show that one can "transfer" the prior probability π_k that was assigned by G_i to B to posteriors that induce expected utilities weakly higher than $V_{k'}(G_i) - \tilde{\varepsilon}$. Specifically, for any pair $p \in B$ and $p' \in A$, there exists $\alpha_{(p,p')} \in (0,1)$, $\underline{p}_{(p,p')}$ and $\bar{p}_{(p,p')}$ such that $(1-\alpha)\underline{p} + \alpha\bar{p} = p$, $\bar{p}_k > 0$, and $E_{\bar{p}}[U_i] = E_{p'}[U_i]$. Suppose that $\mu_B \leq \mu_A$ and define $q : \triangle\Omega \to \triangle\Omega$ as follows: For $p \in L$,

$$q(p) \equiv \frac{1}{\mu_A} \int_{p' \in A} \left(\left(1 - \alpha_{(p,p')}\right) \times \underline{p}_{(p,p')} + \alpha_{(p,p')} \times p'\right) dG_i(p').$$

(13)To see this, define $W(p_{-i})$ as the probability that sender i wins by using strategy $\tilde{G}_i$, conditional on the posterior realization of all other senders being $p_{-i} \in (\triangle\Omega)^{n-1}$. By definition, $V_i\left(\tilde{G}_i, G_{-i}\right) = \int W(p_{-i}) dG_{-i}(p_{-i})$. As $F_{\tilde{G}_i}$ is continuous on $[u_0, u_{m-1})$, $W(p_{-i})$ is lower semicontinuous in p_{-i}. Therefore, by the Portmanteau theorem, for every sequence $\left\{G^k_{-i}\right\}_{k \in \mathbb{N}} \subset \Lambda_{-i}$ that converges in weak* topology to G_{-i}, we have $\liminf V_i\left(\tilde{G}_i, G^k_{-i}\right) \geq V_i\left(\tilde{G}_i, G_{-i}\right)$. That is, $V_i\left(\tilde{G}_i, G'_{-i}\right)$ is lower semicontiuous with respect to G'_{-i}.

(14)That is, $V_k(G_i) \equiv \sup\left\{v \in [u_0, u_{m-1}] : v = E_p[U], \exists p \in supp\{G_i\} \text{ such that } p_k > 0\right\}$.

For $p' \in A$,

$$q\left(p'\right) \equiv \frac{1}{\mu_B}\int_{p\in B}\left(\left(1-\frac{\mu_L}{\mu_A}\alpha_{(p,p')}\right)\times p' + \left(\frac{\mu_L}{\mu_A}\alpha_{(p,p')}\right)\times \bar{p}_{(p,p')}\right)dG_i\left(p\right)$$

Otherwise, $q\left(p\right) \equiv p$.

Define $G_i'\left(q\left(p\right)\right) \equiv G_i\left(p\right)$. As $E_{q(p)}\left[U_i\right] = E_p\left[U_i\right]$ for all p by construction, we have $F_{G_i}\left(v\right) = F_{G_i'}\left(v\right)$. To see $G_i'\left(q\left(p\right)\right) \in \Lambda_i$,

$$\begin{aligned}\int_{B\cup A} q\left(p\right)dG_i &= \frac{1}{\mu_A}\int_{p\in B}\left[\int_{p'\in A}\left(\alpha_{(p,p')}\times\left(\bar{p}_{(p,p')}-p'\right)\right)dG_i\left(p'\right)\right]dG_i\left(p\right)\\ &= \int_{B\cup A} p dG_i.\end{aligned}$$

By construction, under G_i', expected utilities weakly higher than $V_{k'}\left(G_i\right) - \tilde{\varepsilon}$ are induced by posteriors $p \in \triangle\Omega$ such that $p_k > 0$. Therefore, $V_k\left(G_i'\right) \geq V_{k'}\left(G_i'\right)$.

For the case $\mu_B > \mu_A$, we can define $q\left(p\right)$ as follows:

$$q\left(p\right) = \begin{cases} \frac{1}{\mu_A}\int_{p'\in A}\left(\left(1-\frac{\mu_A}{\mu_L}\alpha_{(p,p')}\right)\times \underline{p}_{(p,p')} + \frac{\mu_A}{\mu_B}\alpha_{(p,p')}\times p'\right)dG_i\left(p'\right) & p\in B\\ \frac{1}{\mu_B}\int_{p'\in B}\left(\left(1-\alpha_{(p',p)}\right)\times p + \left(\alpha_{(p',p)}\right)\times\bar{p}_{(p',p)}\right)dG_i\left(p'\right) & p\in A\\ p & \text{otherwise}\end{cases}.$$

Q.E.D. ■

Lemma 12 *Take an arbitrarily small $\varepsilon > 0$. Then for any $\left(G_i, G_{-i}\right) \in \Lambda_i \times \Lambda_{-i}$, there exists a $\tilde{G}_i \in \Lambda_i$ such that F_{G_i} is continuous on $\left[u_0, u_{m-1}\right)$ and $V_i\left(\tilde{G}_i, G_{-i}\right) > V_i\left(G_i, G_{-i}\right) - \varepsilon/2$.*

Proof. Notice that the set of discontinuity points D of F_{G_i} is countable. We, thus, denote D by $D = \{d_1, d_2, \cdots, d_l, \cdots\}$, where $d_l < d_{l+1}$. Denote the size of atom at d_l by $t_l \equiv \int 1_{\{d_l\}} dF_{G_i}\left(d_l\right)$. By Lebesgue's decomposition theorem, we can express $G_i = \bar{G}_i + \hat{G}_i$, where $\bar{G}_i$ is absolutely continuous and $\hat{G}_i$ is singular (both with respect to the Lebesgue measure on $\mathbb{R}^{m-1}$). Note that it is without loss of generality to assume that there is a unique $p_i \in \triangle\Omega$ such that $E_{p_i}\left[U_i\right] = d_l$ and $\hat{G}_i\left(p_i\right) > 0$. Let $\bar{u} \equiv \max_{p\in supp\{G_i\}} E_p\left[U_i\right]$. We consider three cases: (a) $\bar{u} = u_{m-1} \in D$, (b) $\bar{u} \in D \backslash u_{m-1}$, and (c) $\bar{u} \notin D$.

We start with case (a). For each $d_l < \bar{u}$, by mixing the masses t_l at d_l, and $m_l < \left(\frac{\varepsilon}{4+\varepsilon}\right)^l$ at u_{m-1}, we can construct a $G_i' \in \Lambda_i$ with the following properties: (i) $F_{G_i'}\left(v\right)$ is continuous at d_l; and (ii) $F_{G_i'}\left(v | v < u_{m-1}\right) \leq F_{G_i}\left(v | v < u_{m-1}\right)$. One possible construction is as follows. Take a sequence $\{m_l\}_{l\in\mathbb{N}}$ such that $m_l \in \left(0, \left(\frac{\varepsilon}{4+\varepsilon}\right)^l\right)$. Define $\hat{G}_i' : \triangle\Omega \to [0,1]$ as follows:

$$\hat{G}_i'\left(p_i\right) \equiv \begin{cases} \hat{G}_i\left(e_{m-1}\right) - m_l & \text{if } p_i = e_{m-1}\\ 0 & \text{if } E_{p_i}\left[U_i\right] = d_l\\ \hat{G}_i\left(p_i\right) & \text{otherwise}\end{cases},$$

where e_{m-1} is the unit vector which is positive only in the last entry. Denote by $\bar{g}_i$ the Radon–Nikodym derivative of $\bar{G}_i$. Define also $\bar{g}'_i : \triangle\Omega \to \mathbb{R}_+$ by

$$\bar{g}'_i(p_i) \equiv \begin{cases} \bar{g}_i(p_i) + \frac{(m_l+t_l)^2}{2m_l} & \text{if } p_i = (1-\beta)\, p'_i + \beta e_{m-1} \text{ for some } \beta \in \left[0, \frac{2m_l}{m_l+t_l}\right] \text{ and } E_{p'_i}[U_i] = d_l \\ \bar{g}_i(p_i) & \text{otherwise} \end{cases}.$$

Intuitively, we mix the masses at d_l and u_{m-1} to form a uniform density over the set

$$\left\{(1-\beta)\, p'_i + \beta e_{m-1} \colon \beta \in \left[0, \frac{2m_l}{m_l+t_l}\right] \text{ and } E_{p'_i}[U_i] = d_l\right\}.$$

The measure $G'_i \equiv \hat{G}'_i + \int \bar{g}'_i dp_i$ satisfies properties (i) and (ii) above. Because of property (ii), we have $V_i(G'_i, G_{-i}) > V_i(G_i, G_{-i}) - \left(\frac{\varepsilon}{2+\varepsilon}\right)^l$. Applying this procedure inductively arrives at a $\tilde{G}_i \in \Lambda_i$ such that $F_{\tilde{G}_i}(v)$ is continuous on $[u_0, u_{m-1})$ and $V_i(\tilde{G}_i, G_{-i}) > V_i(G_i, G_{-i}) - \sum_{l=1}^{\infty}\left(\frac{\varepsilon}{4+\varepsilon}\right)^l = V_i(G_i, G_{-i}) - \frac{\varepsilon}{4}$.

Next, consider case (b). Applying the procedure in case (a) with u_{m-1} replaced by $\bar{u}$, one can construct a $G'_i \in \Lambda_i$ such that $F_{G'_i}(v)$ is continuous on $[u_0, \bar{u})$ and $V_i(G'_i, G_{-i}) > V_i(G_i, G_{-i}) - \frac{\varepsilon}{4}$. By Lemma 11, it is without loss of generality to assume that $\bar{u} = \sup V_{m-1}(G'_i)$, i.e., there exists a (unique) $p \in supp\{G'_i\}$ such that $E_p[U_i] = \bar{u}$ and $p_{m-1} > 0$. As $\bar{u} < u_{m-1}$, there exists a $u_k \in \Omega$ such that $p_k > 0$ and $u_k < \bar{u}$. Denote the size of the atom that G'_i assigns to p by γ. Denote $x \equiv \frac{\delta'}{\delta'+\varepsilon'}e_k + \frac{\varepsilon'}{\delta'+\varepsilon'}e_{m-1} \in \triangle\Omega$ and $y \equiv \frac{p-\delta' e_k - \varepsilon' e_{m-1}}{1-(\delta'+\varepsilon')} \in \triangle\Omega$.(15) Clearly, one can choose $\varepsilon' > 0$ sufficiently small relative to $\delta' > 0$ such that $E_y[U_i] > \bar{u}$, $E_x[U_i] < \bar{u}$, and $\delta' + \varepsilon' < \frac{\varepsilon}{4}$. As $p = (\delta'+\varepsilon')\, x + (1-(\delta'+\varepsilon'))\, y$, we can replace the atom at p with a convex combination of an atom of size $\gamma(\delta'+\varepsilon')$ at x, and an atom of size $\gamma(1-(\delta'+\varepsilon'))$ at y.(16) Finally, as x is nondegenerate, one can replace the atom at x with a continuous density such that the induced expected utility admits a continuous density. Likewise, one can replace the atom at y with a continuous density such that the induced expected utility admits a continuous density with support exceeding $\bar{u}$. The resulting strategy $\tilde{G}_i$ induces a continuous distribution $F_{\tilde{G}_i}$ over $[u_0, u_{m-1}]$. Moreover, $V_i(\tilde{G}_i, G_{-i}) > V_i(G'_i, G_{-i}) - \frac{\varepsilon}{4} > V_i(G_i, G_{-i}) - \frac{\varepsilon}{2}$.

Finally, consider case (c). Notice that there exists $\varepsilon'' > 0$ such that $\int 1_{[\bar{u}-\varepsilon'', \bar{u}]} dF_{G_i}(v) \in (0, \varepsilon/2)$. Then, the positive measure on $[\bar{u}-\varepsilon'', \bar{u}]$ can be used to eliminate the jumps at D using the same argument as (a). ■

(ii) Suppose $\pi_i = \pi_j$, so that $\Lambda_i = \Lambda_j$ for all i, j. First, as each sender i's payoff is linear in G_i, the game is diagonally quasiconcave. Moreover, the argument in part (i) shows that the game is diagonally payoff

(15)Here e_k is the unit vector that is positive only at the k-th entry.

(16)That is, one can replace $\hat{G}'_i$, the singular part of G'_i, with

$$\hat{G}''_i(p_i) = \begin{cases} 0 & \text{if } p_i = p \\ \gamma(\delta'+\varepsilon') & \text{if } p_i = x \\ \gamma(1-(\delta'+\varepsilon')) & \text{if } p_i = y \\ \hat{G}'_i(p_i) & \text{otherwise} \end{cases}.$$

secure. Finally,

$$V_i(G,...,G) = \frac{1}{n}\int 1_{[E_p[U_i]\geq \underline{u}]} d(G(p))^n$$

is upper semicontinuous as a function of G, as $1_{[E_p[U_i]\geq \underline{u}]}$ is a upper semicontinuous function in p.[17] Therefore, by Corollary 4.3 of Reny (1999), a symmetric equilibrium exists. Q.E.D.

Proof of Lemma 2

(i) It follows from the discussion in the text.

(ii) Suppose, $\pi_j < 2\frac{1-\underline{p}}{2-\underline{p}}$. We identify conditions under which it is an equilibrium for sender j to fully disclose, and sender i to partially disclose. Suppose that sender j adopts full disclosure. Sender i's payoff is then given by (3), and it is apparent that $supp\{G_i\} \subset \left\{0, \underline{p}, 1\right\}$.[18] Notice that there is no equilibrium in which sender j discloses fully if $1 \notin supp\{G_i\}$. This is because if $1 \notin supp\{G_i\}$, then the sender j can increase his payoff by inducing posteriors 0 and $1-\varepsilon$ for a very small $\varepsilon > 0$.

Now, suppose, $supp\{G_i\} = \left\{\underline{p}, 1\right\}$. It is, therefore, necessary that $\pi_i > \underline{p}$. By Bayes-plausibility, sender i induces posterior $\underline{p}$ with probability $\frac{1-\pi_i}{1-\underline{p}}$ and posterior 1 with the complementary probability. Consequently, sender j's payoff is given by

$$\Pi_j(p;G_i) = \begin{cases} 0 & \text{if } p \in \left[0,\underline{p}\right) \\ \frac{1-\pi_i}{2\left(1-\underline{p}\right)} & \text{if } p = \underline{p} \\ \frac{1-\pi_i}{1-\underline{p}} & \text{if } p \in \left(\underline{p},1\right) \\ \frac{2-\left(\pi_i+\underline{p}\right)}{2\left(1-\underline{p}\right)} & \text{if } p = 1 \end{cases}.$$

Full disclosure is optimal for sender j, i.e., $\{0,1\} = supp\left\{G_j\right\}$ if and only if

$$\Pi_j(1;G_i) \geq \frac{\lim_{p'\to\underline{p}^+}\Pi_j(p';G_i)}{\underline{p}} \Leftrightarrow \pi_i \geq \frac{\underline{p}^2 + 2\left(1-\underline{p}\right)}{2-\underline{p}}$$

Since $\frac{\underline{p}^2+2\left(1-\underline{p}\right)}{2-\underline{p}} > \underline{p}$, there exists an equilibrium in which sender j discloses fully if $\pi_i \geq \frac{\underline{p}^2+2\left(1-\underline{p}\right)}{2-\underline{p}}$.

Next, suppose $\pi_j = 2\frac{1-\underline{p}}{2-\underline{p}}$, so that $C(\Pi_i) = \left\{0,\underline{p},1\right\}$ and any Bayes-plausible distribution with support $\left\{0,\underline{p},1\right\}$ is a best response for sender i. Suppose that sender i puts positive probability $q < \pi_i$ on posterior 1, $\frac{\pi_i - q}{\underline{p}}$ on posterior $\underline{p}$, and the complementary probability on posterior 0.[19] Then sender j's

[17] More specifically, take a sequence $\{G_k\}$ that converges to G in weak* topology. Then G_k^n converges to G^n in weak* as well. As $\limsup \int f dG_k^n \leq \int f dG^n$ for all bounded, upper semi-continuous function f, $V(G,...,G)$ is upper semicontinuous as a function of G.

[18] Also see Figure 3-(b).

[19] It is immediate that the distribution satisfies Bayes-plausibility for all $q < \pi_i$.

payoff is given by

$$\Pi_j(p;G_i) = \begin{cases} 0 & \text{if } p \in \left[0, \underline{p}\right) \\ 1 - q - \frac{\pi_i - q}{2\underline{p}} & \text{if } p = \underline{p} \\ 1 - q & \text{if } p \in \left(\underline{p}, 1\right) \\ 1 - \frac{q}{2} & \text{if } p = 1 \end{cases}.$$

Full disclosure is optimal for sender j if and only if

$$\Pi_j(1;G_i) \geq \frac{\lim_{p' \to \underline{p}^+} \Pi_j(p';G_i)}{\underline{p}} \Leftrightarrow q \geq \frac{2\left(1 - \underline{p}\right)}{2 - \underline{p}}.$$

Since Bayes-plausibility implies $q < \pi_i$, there exists an equilibrium in which sender j discloses fully if $\pi_i > \frac{2\left(1-\underline{p}\right)}{2-\underline{p}}$. Q.E.D.

Proof to Lemma 3

1. $G_i(p) = G_i(0)$ for all $p \in \left(0, \underline{p}\right)$: For all $p \in \left(0, \underline{p}\right)$ and $G_j \in \Lambda_j$, we have $0 = \Pi_i\left(p;G_j\right) < \Pi_i\left(1;G_j\right)$. Consequently, $\left(0, \underline{p}\right) \cap C(\Pi_i) = \emptyset$, and G_i puts a zero measure on $\left(0, \underline{p}\right)$.

2. $\bar{p}(G_i) = \bar{p}\left(G_j\right)$: Suppose, $\bar{p}(G_i) > \bar{p}\left(G_j\right)$. Observe that $\Pi_i\left(p;G_j\right) = \Pi_i\left(\bar{p}(G_i);G_j\right) = 1$ for all $p \in \left(\bar{p}\left(G_j\right), \bar{p}(G_i)\right)$. First, suppose that $\pi_i > \bar{p}\left(G_j\right)$. If such an equilibrium exists, then sender i attains a payoff of 1 without any disclosure, and sender j attains a zero payoff. Sender j, however, can always attain a strictly positive payoff by full disclosure, a contradiction. Second, suppose that $\pi_i \leq \bar{p}\left(G_j\right)$. Choose a positive $\varepsilon < \frac{1}{2}\left(\bar{p}(G_i) - \bar{p}\left(G_j\right)\right)$ such that $G_i\left(\bar{p}(G_i) - \varepsilon\right) < 1$.[20] Sender i can improve his payoff by "transferring" probability weights from the interval $[\bar{p}(G_i) - \varepsilon, \bar{p}(G_i)]$ to a point mass at $\bar{p}\left(G_j\right) + \varepsilon$. Specifically, define $G_i' \in \Lambda_i$ by

$$G_i'(p) = \begin{cases} G_i(p) & \text{if } p < \bar{p}\left(G_j\right) + \varepsilon \\ G_i\left(\bar{p}\left(G_j\right) + \varepsilon\right) + \frac{\int_{\bar{p}(G_i)-\varepsilon}^{1} p dG_i(p)}{\bar{p}\left(G_j\right)+\varepsilon} & \text{if } p = \bar{p}\left(G_j\right) + \varepsilon \\ G_i(p) + \frac{\int_{\bar{p}(G_i)-\varepsilon}^{1} p dG_i(p)}{\bar{p}\left(G_j\right)+\varepsilon} & \text{if } p \in \left(\bar{p}\left(G_j\right) + \varepsilon, \bar{p}(G_i) - \varepsilon\right) \\ 1 & \text{if } p \geq \bar{p}(G_i) - \varepsilon \end{cases}.$$

   By adopting G_i' instead of G_i, sender i's probability of winning increases by $\frac{\int_{\bar{p}(G_i)-\varepsilon}^{1} p dG_i(p)}{\bar{p}\left(G_j\right)+\varepsilon} - \int_{\bar{p}(G_i)-\varepsilon}^{1} dG_i(p) > 0$, a contradiction.

3. No atom at $p \neq 0, \underline{p}$, or 1: Suppose that G_i contains an atom at some $\tilde{p} \in \left(\underline{p}, 1\right)$. This implies $\tilde{p} \in C(\Pi_i)$. Moreover, $\Pi_j(p;G_i)$ exhibits an upward jump at $\tilde{p}$. Therefore, there exists some $\varepsilon > 0$

[20] For notational simplicity, if G_i has no atom at $\bar{p}(G_i)$, choose an ε such that G_i is continuous at $\bar{p}(G_i) - \varepsilon$.

such that $[\tilde{p}-\varepsilon,\tilde{p}]\cap C\left(\Pi_j\right)=\emptyset$. Consequently, $G_j\left(p\right)=G_j\left(\tilde{p}-\varepsilon\right)$ for all $p\in[\tilde{p}-\varepsilon,\tilde{p}]$, which in turn implies $\Pi_i\left(p,G_j\right)=\Pi_i\left(\tilde{p},G_j\right)$ for all $p\in[\tilde{p}-\varepsilon,\tilde{p}]$. If $\Pi_i\left(p,G_j\right)>\Pi_i\left(\tilde{p},G_j\right)$ for some $p>\tilde{p}$, then it contradicts that $\tilde{p}\in C\left(\Pi_i\right)$. On ther other hand, if $\Pi_i\left(p,G_j\right)=\Pi_i\left(\tilde{p},G_j\right)$ for all $p>\tilde{p}$, then we have $\bar{p}\left(G_j\right)\leq\tilde{p}-\varepsilon<\tilde{p}\leq\bar{p}\left(G_i\right)$, contradicting part 2 of the lemma.

4. G_i is strictly increasing on $\left[\underline{p},\hat{p}\left(G_i\right)\right]$: The claim is trivial if $\left(\underline{p},\hat{p}\left(G_i\right)\right)=\emptyset$, so assume $\underline{p}<\hat{p}\left(G_i\right)$. Suppose, the claim does not hold, i.e., $supp\left(G_i\right)^c\cap\left[\underline{p},\hat{p}\left(G_i\right)\right]\neq\emptyset$. Let I be a maximal open interval in $supp\left(G_i\right)^c\cap\left[\underline{p},\hat{p}\left(G_i\right)\right]$. First, consider the case where $\sup I=\hat{p}\left(G_i\right)$. By the definition of $\hat{p}\left(G_i\right)$, we have $\hat{p}\left(G_i\right)<\bar{p}\left(G_i\right)=1$. Then G_i must have an atom at $\hat{p}\left(G_i\right)$, contradicting part 3 of the lemma. Next, consider the case where $\sup I<\hat{p}\left(G_i\right)$. By the definition of $\hat{p}\left(G_i\right)$, there exists a $\tilde{p}>\sup I$ such that $G_i\left(\tilde{p}\right)>G_i\left(\inf I\right)$. Thus, $I\cap C\left(\Pi_j\right)=\emptyset$, and $G_j\left(p\right)=G_j\left(\inf I\right)$ for all $p\in I$. This in turn implies that $\Pi_i\left(p,G_j\right)$ is also constant on the interval I. Moreover, G_j does not contain any atom at any $p\in\left(\underline{p},\hat{p}\left(G_i\right)\right)$, so $\Pi_i\left(p,G_j\right)$ is continuous at $p=\sup I$. This implies that there exists an $\varepsilon>0$ such that $\left(\sup I,\sup I+\varepsilon\right)\cap C\left(\Pi_i\right)=\emptyset$, contradicting the maximality of I.

5. $\hat{p}\left(G_i\right)=\hat{p}\left(G_j\right)$: Suppose $\hat{p}\left(G_i\right)>\hat{p}\left(G_j\right)$. By part 2, $\bar{p}\left(G_i\right)=\bar{p}\left(G_j\right)$, so $\hat{p}\left(G_j\right)<\bar{p}\left(G_j\right)$. By definitions, G_j has an atom at $\bar{p}\left(G_j\right)$, and by part 3, this requires $\bar{p}\left(G_i\right)=\bar{p}\left(G_j\right)=1$. Consequently, for all $p\in\left[\hat{p}\left(G_j\right),1\right)$, $G_j\left(p\right)=G_j\left(\hat{p}\left(G_j\right)\right)$ and $\Pi_i\left(p,G_j\right)=\Pi_i\left(\hat{p}\left(G_j\right),G_j\right)$. As $\Pi_i\left(1,G_j\right)>\Pi_i\left(\hat{p}\left(G_j\right),G_j\right)$, we have $\left(\hat{p}\left(G_j\right),1\right)\cap C\left(\Pi_i\right)=\emptyset$, and G_i must assign no measure on $\left(\hat{p}\left(G_j\right),1\right)$. This contradicts the definition of $\hat{p}\left(G_i\right)$.

6. $G_i\left(p\right)$ is linear on $\left(\underline{p},\hat{p}\left(G_i\right)\right]$: By Caratheodory theorem, and the existence of a best response of sender j, $\left(\pi_j,con\left[\Pi_j\right]\left(\pi_j\right)\right)$ can be written as a convex combination of two points on the graph of Π_j.[21] Denote these pair of points by $\left(p',\Pi_j\left(p'\right)\right)$ and $\left(p'',\Pi_j\left(p''\right)\right)$. Take any $\tilde{p}\in\left(\underline{p},\hat{p}\left(G_i\right)\right]$, it is necessary that $\left(\tilde{p},\Pi_j\left(\tilde{p}\right)\right)$ lies on the straight line passing through $\left(p',\Pi_j\left(p'\right)\right)$ and $\left(p'',\Pi_j\left(p''\right)\right)$. Suppose that it lies strictly below. Then putting positive weight on the neighborhood of $\tilde{p}$ would yield a strictly lower payoff than $con\left[\Pi_j\right]\left(\pi_j\right)$, contradicting that $\tilde{p}\in supp\left\{G_j\right\}$. Next, suppose that it lies strictly above. Then a convex combination of either $\{p',\tilde{p}\}$ or $\{\tilde{p},p''\}$ would yield a payoff strictly exceeding that of $\{p',p''\}$, contradicting that $con\left[\Pi_j\right]\left(\pi_j\right)$ is the optimal payoff for sender j. Q.E.D.

Proof of Lemma 4

(i) Suppose, $\underline{p}>0$ and $b_i>0$. Then $\Pi_j\left(p;G_i\right)$ exhibits a jump at $\underline{p}$, so $\underline{p}\notin C\left(\Pi_j\right)$. By Lemma 1, G_j must assign zero probability to $\underline{p}$, so $b_j=0$. An analogous argument applies to the second part of the statement. (ii) Suppose $\underline{p}>0$ and $a_i+b_i=0$. Then there exists a neighborhood of $\underline{p}$ that is not in $C\left(\Pi_j\right)$.

[21]See also Proposition 4 in the appendix of Kamenica and Gentzkow (2011).

Consequently, $\underline{p} \notin supp\left(G_j\right)$, which is a contradiction. (iii) Suppose $g_i > 0$. Then by part 6 of Lemma 3, we have $\hat{p} < 1$. Now by part 2 of Lemma 3, $\bar{p}\left(G_j\right) = \bar{p}\left(G_i\right) = 1$, so $g_j > 0$. Q.E.D.

Proof of Proposition 2

As discussed in Section 3.3, it is without loss of generality to impose $b_j = 0$. Also, there are four possible forms of equilibria if $\underline{p} > 0$ and two possible forms if $\underline{p} = 0$. Below, we study each case in turn. Specifically, for each form of equilibria, we identify the exact conditions on the prior pair $\left(\pi_i, \pi_j\right)$ under which the particular form of equilibria exists, and completely solve the equilibria for these cases.

Using Theorem 1, the Bayes-plausibility conditions are as follows:

$$\underline{p} b_i + \frac{1}{2}\left(1 - g_i - (a_i + b_i)\right)\left(\hat{p} + \underline{p}\right) + g_i = \pi_i; \tag{14}$$

$$\frac{1}{2}\left(1 - g_j - a_j\right)\left(\hat{p} + \underline{p}\right) + g_j = \pi_j. \tag{15}$$

We first consider $\underline{p} > 0$. By part (ii) of Lemma 4, we have $a_j > 0$.

In cases (1) and (2), the support of G_j is $\{0\} \cup \left[\underline{p}, \hat{p}\right]$, with no atom in the interval $\left[\underline{p}, \hat{p}\right]$. By Lemma 1, the necessary condition for G_j being a best response is, thus, $\{0, 1\} \cup \left(\underline{p}, \hat{p}\right) \subset C\left(\Pi_j\right)$. By Theorem 1, this is equivalent to requiring the following slope condition:

$$\frac{1}{\hat{p}} = \frac{a_i + b_i}{\underline{p}}. \tag{16}$$

In a similar vein, the support of G_i includes $[\underline{p}, \hat{p}]$, possibly with an atom at $\underline{p}$. By Lemma 1, the necessary condition for G_i being a best response is thus $\left(\underline{p}, \hat{p}\right) \subset C\left(\Pi_i\right)$. By Theorem 1, this is equivalent to requiring the following slope condition:

$$\frac{1}{\hat{p}} \leq \frac{a_j}{\underline{p}}. \tag{17}$$

Case (1) : $g_i = g_j = 0$ **and** $a_i > 0$**:** In case (1), $a_i > 0$ (i.e., $0 \in supp\left\{G_i\right\}$), so inequality (17) holds with equality. Substituting $g_i = g_j = 0$ into (14) and (15), and setting (17) at equality, we have a system of four equations with four unknowns (a_i, b_i, a_j, and $\hat{p}$). Straightforward calculation shows that the unique solution is given by

$$\hat{p} = \pi_j + \sqrt{\pi_j^2 + \underline{p}^2};$$
$$a_i = \frac{\underline{p}^2 + \hat{p}^2 - 2\hat{p}\pi_i}{2\underline{p}\hat{p}}; b_i = \frac{\underline{p}^2 - \hat{p}^2 + 2\pi_i\hat{p}}{2\underline{p}\hat{p}}; \text{ and } a_j = \frac{\underline{p}}{\hat{p}}.$$

Notice that the necessary conditions for the existence of equilibrium in this class are that $\hat{p} \in \left(\underline{p}, 1\right]$, $b_i \geq 0$, and $a_i > 0$. That $\underline{p} < \hat{p}$ follows from $\underline{p} < \pi_j + \sqrt{\pi_j^2 + \underline{p}^2}$. That $\hat{p} \leq 1$ follows if and only if $\pi_j \leq \frac{1-\underline{p}^2}{2}$. Since $b_i = \frac{\left(\pi_i - \pi_j\right)\left(\pi_j + \sqrt{\pi_j^2 + \sqrt{\pi_j^2 + \underline{p}^2}}\right)}{\underline{p}\left(\pi_j + \sqrt{\pi_j^2 + \underline{p}^2}\right)}$, $b_i \geq 0$ if and only if $\pi_j \leq \pi_i$. Also $a_i > 0$ if and only if either $\pi_i < \underline{p}$

or $\hat{p} > \pi_i + \sqrt{\pi_i^2 - \underline{p}^2}$. To sum up, the following are the necessary conditions for the existence of equilibrium in this class (a) $\pi_j \leq \min\left\{\frac{1-\underline{p}^2}{2}, \pi_i\right\}$ and (b) either $\pi_i < \underline{p}$ or $\pi_j + \sqrt{\pi_j^2 + \underline{p}^2} > \pi_i + \sqrt{\pi_i^2 - \underline{p}^2}$.

Now we check sufficiency using Lemma 1. Because of slope condition (16), con $\left[\Pi_j\right](\cdot)$ is linear on $[0, \hat{p}]$ and flat on $[\hat{p}, 1]$. As $\pi_j < \hat{p}$, every Bayes-plausible posterior distribution that assigns positive weights only on $\{0\} \cup (\underline{p}, \hat{p}] \subset C\left(\Pi_j\right)$ is optimal. Therefore, G_j is indeed a best response. Likewise, because of the slope condition (17), con $[\Pi_i](\cdot)$ is linear on $[0, \hat{p}]$ and flat on $[\hat{p}, 1]$. As $\pi_i < \hat{p}$, every Bayes-plausible posterior distribution that assigns positive weights only on $\{0\} \cup [\underline{p}, \hat{p}] \subset C(\Pi_i)$ is optimal. Therefore, G_i is indeed a best response.

Case (2): $g_i = g_j = 0$ and $a_i = 0$: Substituting $g_i = g_j = a_i = 0$ in (14), (15), and (16), we have a system of three equations in three unknowns (b_i, a_j, and $\hat{p}$). Straightforward calculation shows that the unique solution is given by

$$\hat{p} = \pi_i + \sqrt{\pi_i^2 - \underline{p}^2}; a_j = 1 - \frac{2\pi_j}{\hat{p} + \underline{p}}; \text{ and } b_i = \frac{\underline{p}}{\hat{p}}.$$

The condition (17) holds if and only if $\hat{p} \geq \pi_j + \sqrt{\pi_j^2 + \underline{p}^2}$. Notice that $\hat{p} \geq \pi_j + \sqrt{\pi_j^2 + \underline{p}^2}$ implies $a_j \geq 0$. Furthermore, $\hat{p} \in \left(\underline{p}, 1\right]$ if and only if $\underline{p} \leq \pi_i \leq \frac{1}{2}\left(1 + \underline{p}^2\right)$. The following are the necessary conditions for the existence of equilibrium in this class are thus (a) $\underline{p} \leq \pi_i \leq \frac{1}{2}\left(1 + \underline{p}^2\right)$ and (b) $\pi_i + \sqrt{\pi_i^2 - \underline{p}^2} \geq \pi_j + \sqrt{\pi_j^2 + \underline{p}^2}$.

Now we check sufficiency using Lemma 1. Because of slope condition (16), con $\left[\Pi_j\right](\cdot)$ is linear on $[0, \hat{p}]$ and flat on $[\hat{p}, 1]$. As $\pi_j < \hat{p}$, every Bayes-plausible posterior distribution that assigns positive weights only on $\{0\} \cup (\underline{p}, \hat{p}] \subset C\left(\Pi_j\right)$ is optimal. Therefore, G_j is indeed a best response. Likewise, because of slope condition (17), con $[\Pi_i](\cdot)$ is linear on $\left[\underline{p}, \hat{p}\right]$ and flat on $[\hat{p}, 1]$. As $\underline{p} \leq \pi_i < \hat{p}$, every Bayes-plausible posterior distribution that assigns positive weights only on $[\underline{p}, \hat{p}] \subset C(\Pi_i)$ is optimal. Therefore, G_i is indeed a best response.

In cases (3) and (4), there is an atom at the top: $g_i, g_j > 0$. The slope conditions now have to take into account the atoms at the top.

In these case, the support of G_j is $\{0, 1\} \cup \left[\underline{p}, \hat{p}\right]$, with no atom in the interval $\left[\underline{p}, \hat{p}\right]$. By Lemma 1, a necessary condition for G_j being a best response is thus $\{0, 1\} \cup \left(\underline{p}, \hat{p}\right) \subset C\left(\Pi_j\right)$. By Theorem 1, this is equivalent to requiring the following slope conditions

$$1 - \frac{g_i}{2} = \frac{1 - g_i}{\hat{p}} = \frac{a_i + b_i}{\underline{p}}. \tag{18}$$

If these slope conditions are satisfied, then con $\left[\Pi_j\right](\cdot)$ is linear on $\triangle\Omega$, so every Bayes-plausible posterior distribution that assigns positive weights only on $C\left(\Pi_j\right) = \{0, 1\} \cup (\underline{p}, \hat{p}]$ is optimal. Therefore, (18) is also sufficient for the optimality of G_j.

In a similar vein, the support of G_i includes $[\underline{p},\hat{p}]\cup\{1\}$, possibly with an atom at $\underline{p}$. By Lemma 1, the necessary condition for G_i being a best response is thus $\{0,1\}\cup\left(\underline{p},\hat{p}\right)\subset C\left(\Pi_i\right)$. This is equivalent to requiring the following slope conditions:

$$\frac{1-\frac{g_j}{2}-a_j}{1-\underline{p}}=\frac{1-g_j-a_j}{\hat{p}-\underline{p}}\le\frac{a_j}{\underline{p}}. \tag{19}$$

Case (3): $g_i,g_j>0$ **and** $a_i>0$: Notice that $a_i>0$ (i.e., $0\in supp\{G_i\}$) implies that inequality (19) holds with equality. Rearranging the slope conditions (18) and (19), we get

$$g_i=g_j=2\frac{1-\hat{p}}{2-\hat{p}};a_j=\frac{\underline{p}}{2-\hat{p}};a_i+b_i=\frac{\underline{p}}{2-\hat{p}}.$$

Together with Bayes-plausibility conditions (14) and (15), we have a system of 6 equations in 6 unknowns. The unique solution is given by

$$\begin{aligned}
\hat{p}&=\left(2-\pi_j\right)-\sqrt{\pi_j^2+\underline{p}^2}\\
g_i&=g_j=2\frac{1-\hat{p}}{2-\hat{p}};a_j=\frac{\underline{p}}{2-\hat{p}}\\
a_i&=\frac{1}{\underline{p}}\left(\frac{\left(2-\hat{p}\right)^2+\underline{p}^2}{2\left(2-\hat{p}\right)}-\pi_i\right);b_i=\frac{1}{\underline{p}}\left(\pi_i-\frac{\left(2-\hat{p}\right)^2-\underline{p}^2}{2\left(2-\hat{p}\right)}\right).
\end{aligned}$$

First, $\hat{p}\in\left(\underline{p},1\right)$ if and only if $\frac{1-\underline{p}^2}{2}<\pi_j<\frac{2\left(1-\underline{p}\right)}{2-\underline{p}}$. Second, $a_i>0$ if and only if $\pi_j+\sqrt{\pi_j^2+\underline{p}^2}>\pi_i+\sqrt{\pi_i^2-\underline{p}^2}$, or $\pi_i<\underline{p}$. Also, $b_i\ge 0$, if and only if $\pi_i\ge\pi_j$. Thus the necessary conditions for the existence of equilibrium in this class is (a) $\frac{1-\underline{p}^2}{2}<\pi_j<\frac{2\left(1-\underline{p}\right)}{2-\underline{p}}$; $\pi_i\ge\pi_j$; and (b) either $\pi_j+\sqrt{\pi_j^2+\left(\underline{p}\right)^2}>\pi_i+\sqrt{\pi_i^2-\underline{p}^2}$, or $\pi_i<\underline{p}$.

Now, we check sufficiency. Slope condition (18) guarantee the optimality of G_j. Likewise, slope condition (19) holding with equality implies that con $[\Pi_i](\cdot)$ is linear on $\triangle\Omega$, so every Bayes-plausible posterior distribution that assigns positive weights only on $C\left(\Pi_i\right)=\{0,1\}\cup[\underline{p},\hat{p}]$ is optimal. Therefore, G_i is also optimal.

Case (4): $g_i,g_j>0$ **and** $a_i=0$: Substituting $a_i=0$, and rearranging the slope conditions (18) and (19), we get

$$g_i=2\frac{1-\hat{p}}{2-\hat{p}};b_i=\frac{\underline{p}}{2-\hat{p}}.$$

Together with $\frac{1-\frac{g_j}{2}-a_j}{1-\underline{p}}=\frac{1-g_j-a_j}{\hat{p}-\underline{p}}$, and Bayes-plausibility conditions (14) and (15), we have a system of five equations in five unknowns. The unique solution is given by

$$\hat{p} = 2 - \pi_i - \sqrt{(\pi_i)^2 - \underline{p}^2};$$
$$g_i = 2\frac{1-\hat{p}}{2-\hat{p}}; b_i = \frac{\underline{p}}{2-\hat{p}};$$
$$g_j = \frac{4(1-\hat{p})\pi_j}{(2-\hat{p})^2 - \underline{p}^2}; a_j = \frac{2(1-\pi_j) - \left(\hat{p} - \underline{p}\right)}{2 - \left(\hat{p} - \underline{p}\right)}$$

First, $\hat{p} \in \left(\underline{p}, 1\right)$ if and only if $\frac{1+\underline{p}^2}{2} < \pi_i < \frac{2\left(1-\underline{p}\right)+\underline{p}^2}{2-\underline{p}}$. The inequality requirement in (19) translates into $\pi_i + \sqrt{\pi_i^2 - \underline{p}^2} \geq \pi_j + \sqrt{\underline{p}^2 + \pi_j^2}$. Thus the necessary condition for the existence of equilibrium in this class is: (a) $\frac{1+\underline{p}^2}{2} < \pi_i < \frac{2\left(1-\underline{p}\right)+\underline{p}^2}{2-\underline{p}}$; and (b) $\pi_i + \sqrt{\pi_i^2 - \underline{p}^2} \geq \pi_j + \sqrt{\underline{p}^2 + \pi_j^2}$.

Now we check sufficiency. Slope condition (18) guarantees the optimality of G_j. Likewise, slope condition (19) implies that con $[\Pi_i](\cdot)$ is piecewise linear with a single kink at $\underline{p}$. As $\underline{p} < \pi_i$, every Bayes-plausible posterior distribution that assigns positive weights only on $[\underline{p}, \hat{p}] \cup \{1\} \subset C(\Pi_i)$ is optimal. Therefore, G_i is indeed a best response.

Finally it is straightforward to verify that the parameter regions identified in each of the four cases above, together with those identified in Lemma 2 covers all pairs of priors with $\pi_i \geq \pi_j$. Moreover, the regions of parameters for each case do not overlap, except along the boundary covered in Lemma 2.

The scenario of $\underline{p} = 0$ is a special case of the above analysis. Here, according to Lemma 2, equilibrium in which some sender fully discloses does not exist. Moreover, according to part (i) of Lemma 4, only cases 2 and 4 are relevant because of the specification $a_j \geq 0 = a_i$. Furthermore, with $\underline{p} = 0$, we only need the conditions concerning g_i and g_j for case 4 in slope condition (19).

Case 2: $g_i = g_j = 0$ **and** $a_i = 0$: Substituting $g_i = g_j = a_i = 0$ into (14), and (15), we have a system of two equations in two unknowns (a_j and $\hat{p}$). Straightforward calculation shows that the unique solution is given by

$$\hat{p} = 2\pi_i, \text{ and } a_j = \frac{\pi_i - \pi_j}{\pi_i}.$$

Note that $\hat{p} \leq 1$ if and only if $\pi_i \leq \frac{1}{2}$. Moreover, $a_j \geq 0$ if and only if $\pi_i \geq \pi_j$. Therefore, the necessary conditions for the existence of equilibrium in this class are thus $\pi_j \leq \pi_i \leq \frac{1}{2}$.

Now we check sufficiency. It is clear that both con $[\Pi_i](\cdot)$ and con $\left[\Pi_j\right](\cdot)$ are linear on $[0, \hat{p}]$ and flat on $[\hat{p}, 1]$. As $\pi_i \leq \hat{p}$, every Bayes-plausible posterior distribution that assigns positive weights only on $(0, \hat{p}] \subset C(\Pi_i)$ is optimal for sender i. Likewise, as $\pi_j \leq \hat{p}$, every Bayes-plausible posterior distribution that assigns positive weights only on $[0, \hat{p}] \subset C\left(\Pi_j\right)$ is optimal for sender j.

Case 4: $g_i, g_j > 0$ **and** $a_i = 0$: Substituting $a_i = 0$, the slope conditions (18) and (19) give $g_i = 2\frac{1-\hat{p}}{2-\hat{p}}$ and $a_j = \frac{1-g_j-\hat{p}\left(1-\frac{g_j}{2}\right)}{1-\hat{p}}$. Together with the Bayes plausibility conditions (14) and (15), we have a system of four equations in four unknowns. The unique solution is given by

$$\hat{p} = 2(1-\pi_i), g_i = \frac{2\pi_i - 1}{\pi_i};$$
$$g_j = \frac{\pi_j}{\pi_i^2}(2\pi_i - 1); a_j = \frac{\pi_i - \pi_j}{\pi_i}.$$

First, $g_i, g_j > 0$ if and only if $\pi_i > \frac{1}{2}$. Second, $a_j \geq 0$ if and only if $\pi_i \geq \pi_j$. The necessary conditions for the existence of equilibrium in this class is thus $\pi_i > \frac{1}{2}$ and $\pi_i \geq \pi_j$.

Now we check sufficiency. By slope conditions (18) and (19), both con $[\Pi_i](\cdot)$ and con $[\Pi_j](\cdot)$ are linear on $[0,1]$, so every Bayes-plausible posterior distribution that assigns positive weights only on $C(\Pi_i)$ and $C(\Pi_j)$, respectively, is optimal. Therefore, G_i is optimal as $C(\Pi_i) \subset (0,\hat{p}] \cup \{1\}$, $supp\{G_i\} = [0,\hat{p}] \cup \{1\}$, and G_i has no atom at 0. Likewise, G_j is optimal as $supp\{G_j\} = C(\Pi_j) = [0,\hat{p}] \cup \{1\}$. Q.E.D.

Proof of Proposition 3

It is straightforward though tedious to check that the equilibrium strategies (G_i, G_j) are continuous in (π_i, π_j). Therefore, it suffices to check the proposition holds for changes in interior of the each of the following parameter regions:

1. $\pi_j \leq \frac{1-\underline{p}^2}{2}$; $\pi_j \leq \pi_i$; and either $\pi_i < \underline{p}$ or $\pi_j + \sqrt{\pi_j{}^2 + \underline{p}^2} > \pi_i + \sqrt{\pi_i{}^2 - \underline{p}^2}$;

1'. $\pi_i \leq \frac{1-\underline{p}^2}{2}$; $\pi_i \leq \pi_j$; and either $\pi_j < \underline{p}$ or $\pi_i + \sqrt{\pi_i^2 + \underline{p}^2} > \pi_j + \sqrt{\pi_j^2 - \underline{p}^2}$;

2. $\underline{p} \leq \pi_i \leq \frac{1}{2}\left(1 + \underline{p}^2\right)$ and $\pi_i + \sqrt{\pi_i^2 - \underline{p}^2} \geq \pi_j + \sqrt{\pi_j^2 + \underline{p}^2}$;

2'. $\underline{p} \leq \pi_j \leq \frac{1}{2}\left(1 + \underline{p}^2\right)$ and $\pi_j + \sqrt{\pi_j^2 - \underline{p}^2} \geq \pi_i + \sqrt{\pi_i^2 + \underline{p}^2}$;

3. $\frac{1-\underline{p}^2}{2} < \pi_j < \frac{2(1-\underline{p})}{2-\underline{p}}$; $\pi_i \geq \pi_j$; and either $\pi_j + \sqrt{\pi_j^2 + \underline{p}^2} > \pi_i + \sqrt{\pi_i^2 - \underline{p}^2}$, or $\pi_i < \underline{p}$;

3'. $\frac{1-\underline{p}^2}{2} < \pi_i < \frac{2(1-\underline{p})}{2-\underline{p}}$; $\pi_j \geq \pi_i$; and either $\pi_i + \sqrt{\pi_i^2 + \underline{p}^2} > \pi_j + \sqrt{\pi_j^2 - \underline{p}^2}$, or $\pi_j < \underline{p}$;

4. $\frac{1+\underline{p}^2}{2} < \pi_i < \frac{2(1-\underline{p})+\underline{p}^2}{2-\underline{p}}$ and $\pi_i + \sqrt{\pi_i^2 - \underline{p}^2} \geq \pi_j + \sqrt{\underline{p}^2 + \pi_j^2}$;

4'. $\frac{1+\underline{p}^2}{2} < \pi_j < \frac{2(1-\underline{p})+\underline{p}^2}{2-\underline{p}}$ and $\pi_j + \sqrt{\pi_j^2 - \underline{p}^2} \geq \pi_i + \sqrt{\underline{p}^2 + \pi_i^2}$;

5. $\pi_i, \pi_j > 2\frac{1-\underline{p}}{2-\underline{p}}$;

6. $\pi_j < 2\frac{1-\underline{p}}{2-\underline{p}}$ and $\pi_i \geq \frac{\underline{p}^2+2(1-\underline{p})}{2-\underline{p}}$; and

6'. $\pi_i < 2\frac{1-\underline{p}}{2-\underline{p}}$ and $\pi_j \geq \frac{\underline{p}^2+2(1-\underline{p})}{2-\underline{p}}$.

We start with the proof that G_i' first-order stochastically dominates G_i. For (1)-(4′), we are done if we show that $a_i, a_i + b_i, 1 - g_i$ are weakly decreasing in π_i, and $\hat{p}$ is weakly increasing in π_i, with at least one of the relations above being strict.

1. $a_i = \frac{\underline{p}^2+\hat{p}^2-2\hat{p}\pi_i}{2\underline{p}\hat{p}}$ is decreasing in π_i while $\hat{p} = \pi_j + \sqrt{\pi_j^2 + \underline{p}^2}$ and $a_i + b_i = \frac{\underline{p}}{\hat{p}}$ do not depend on π_i.

1′. $a_i = \frac{\underline{p}}{\pi_i+\sqrt{\pi_i^2+\underline{p}^2}}$ is decreasing in π_i and $b_i = 0$ is constant. Thus, $a_i + b_i$ is decreasing in π_i. Moreover, $\hat{p} = \pi_i + \sqrt{\pi_i^2 + \underline{p}^2}$ is increasing in π_i.

2. $\hat{p} = \pi_i + \sqrt{\pi_i^2 - \underline{p}^2}$ is increasing in π_i. $a_i = 0$ and $b_i = \frac{\underline{p}}{\hat{p}}$ is decreasing in π_i. Thus, $a_i + b_i$ is decreasing in π_i.

2′. $\hat{p} = \pi_j + \sqrt{\pi_j^2 - \underline{p}^2}$ is constant in π_i. Therefore, $a_i = 1 - \frac{2\pi_i}{\hat{p}+\underline{p}}$ is decreasing in π_i, and $b_i = \frac{\underline{p}}{\hat{p}}$ is constant. Thus $a_i + b_i$ is decreasing in π_i.

3. $\hat{p} = \left(2 - \pi_j\right) - \sqrt{\pi_j^2 + \underline{p}^2}$ is constant in π_i. Therefore, $g_i = 2\frac{1-\hat{p}}{2-\hat{p}}$ stays constant. Moreover, $a_i = \frac{1}{\underline{p}}\left(\frac{(2-\hat{p})^2+\underline{p}^2}{2(2-\hat{p})} - \pi_i\right)$ is decreasing in π_i, and $a_i + b_i = \frac{\underline{p}}{2-\hat{p}}$ is constant in π_i.

3′. $\hat{p} = (2 - \pi_i) - \sqrt{\pi_i^2 + \underline{p}^2}$ is decreasing in π_i. All of $1 - g_i = \frac{\hat{p}}{2-\hat{p}}$, $a_i = \frac{\underline{p}}{2-\hat{p}}$, and $a_i + b_i = a_i$ are decreasing in π_i.

4. $\hat{p} = 2 - \pi_i - \sqrt{\pi_i^2 - \underline{p}^2}$ is decreasing in π_i. Moreover, $a_i = 0$. Also, both $1 - g_i = \frac{\hat{p}}{2-\hat{p}}$ and $a_i + b_i = \frac{\underline{p}}{2-\hat{p}}$ are decreasing in π_i.

4′. $\hat{p}$ is constant in π_i. Both $1 - g_i = \frac{\hat{p}}{2-\hat{p}}$ and $b_i = 0$ are constant. Moreover, $a_i = 1 - \frac{2\pi_i}{2-\left(\hat{p}-\underline{p}\right)}$ is decreasing in π_i.

5. Since sender i discloses fully, it is straightforward that G_i' first-order stochastically dominates G_i.

6. Here, the support of G_i consists only of $\underline{p}$ and 1. The atom at $\underline{p}$, given by $\frac{1-\pi_i}{1-\underline{p}}$, is decreasing in π_i.

6′. Since i discloses fully, it is straightforward that G_i' first-order stochastically dominates G_i.

Now we show that $G_j' = G_j$ or G_j' is a mean-preserving spread of G_j. We are done if we show that $\int_{\underline{p}}^{p} G_i\left(x\right) dx$ is weakly increasing in π_i for all p.

1. $\hat{p} = \pi_j + \sqrt{\pi_j^2 + \underline{p}^2}$ is constant in π_i. $a_j = \frac{\underline{p}}{\hat{p}}$ and $b_j = 0$. Therefore G_j does not change.

1′. $\hat{p} = \pi_i + \sqrt{\pi_i^2 + \underline{p}^2}$ is increasing π_i, and hence

$$\int_{\underline{p}}^{p} G_j\left(x\right) dx = \begin{cases} \left(\frac{\underline{p}^2+\hat{p}^2-2\hat{p}\pi_j}{2\underline{p}\hat{p}}\right) p & \text{if } p \in [0, \underline{p}) \\ \frac{1}{2\hat{p}}\left(p^2 + \hat{p}^2 - 2\pi_j\hat{p}\right) & \text{if } p \in (\underline{p}, \hat{p}] \\ p - \pi_j & \text{if } p \in (\hat{p}, 1] \end{cases}$$

is increasing in π_i.

2. $\hat{p} = \pi_i + \sqrt{\pi_i^2 - \underline{p}^2}$ is increasing in π_i, and hence

$$\int_{\underline{p}}^{p} G_j(x)\,dx = \begin{cases} \left(1 - \frac{2\pi_j}{\hat{p}+\underline{p}}\right) p & \text{if } p \in [0, \underline{p}) \\ \left(1 - \frac{2\pi_j}{\hat{p}+\underline{p}}\right) p + \pi_j \frac{1}{\hat{p}^2 - \underline{p}^2} \left(p - \underline{p}\right)^2 & \text{if } p \in (\underline{p}, \hat{p}] \\ p - \pi_j & \text{if } p \in (\hat{p}, 1] \end{cases}$$

is increasing in π_i.

2'. $\hat{p} = \pi_j + \sqrt{\pi_j^2 - \underline{p}^2}$ is constant in π_i. $a_j = b_j = 0$. Thus G_j stays constant.

3. $\hat{p} = (2 - \pi_j) - \sqrt{\pi_j^2 + \underline{p}^2}$ is constant in π_i, and hence so are $a_j = \frac{\underline{p}}{2-\hat{p}}$ and $g_j = 2\frac{1-\hat{p}}{2-\hat{p}}$. Thus G_j does not change.

3'. $\hat{p} = (2 - \pi_i) - \sqrt{\pi_i^2 + \underline{p}^2}$ is decreasing in π_i, and hence

$$\int_{\underline{p}}^{p} G_i(x)\,dx = \begin{cases} \frac{1}{\underline{p}} \left(\frac{(2-\hat{p})^2 + \underline{p}^2}{2(2-\hat{p})} - \pi_i \right) p & \text{if } p \in [0, \underline{p}) \\ \frac{(2-\hat{p})^2 - 2\pi_i(2-\hat{p}) + p^2}{2(2-\hat{p})} & \text{if } p \in (\underline{p}, \hat{p}] \\ \frac{2 - 2\pi_i + \hat{p}\pi_i - 2\hat{p} + p\hat{p}}{2-\hat{p}} & \text{if } p \in (\hat{p}, 1] \end{cases}.$$

is increasing in π_i.

4. $\hat{p} = 2 - \pi_i - \sqrt{\pi_i^2 - \underline{p}^2}$ is decreasing in π_i. Thus,

$$\int_{\underline{p}}^{p} G_j(x)\,dx = \begin{cases} \frac{2\left(1-\pi_j\right) - \left(\hat{p} - \underline{p}\right)}{2 - \left(\hat{p} - \underline{p}\right)} p & \text{if } p \in [0, \underline{p}) \\ \frac{p(2-\hat{p})^2 - p\pi_j(4 - 2\hat{p} - p) + \left(\pi_j - p\right)\underline{p}^2}{(2-\hat{p})^2 - \underline{p}^2} & \text{if } p \in (\underline{p}, \hat{p}] \\ \frac{4p\left(1-\pi_j\right)(1-\hat{p}) - \left(\hat{p}^2 - \underline{p}^2\right)\left(\pi_j - p\right)}{(2-\hat{p})^2 - \underline{p}^2} & \text{if } p \in (\hat{p}, 1] \end{cases}.$$

is increasing in π_i.

4'. $a_j = 0$. $\hat{p} = 2 - \pi_j - \sqrt{\pi_j^2 - \underline{p}^2}$ is constant π_i, and so are $g_j = 2\frac{1-\hat{p}}{2-\hat{p}}; b_j = \frac{\underline{p}}{2-\hat{p}}$.

5. Since sender j discloses fully, G_j does not change.

6. Since sender j discloses fully, G_j does not change.

6'. $G_i\left(\underline{p}\right) = G(p) = \frac{1-\pi_j}{1-\underline{p}}$ for all $p \in \left(\underline{p}, 1\right)$ and does not depend on π_i. Q.E.D.

Proof of Lemma 5

Suppose that all senders except i disclose fully, i.e., they adopt

$$G^F(p) \equiv \begin{cases} 1-\pi & \text{if } p \in [0,1) \\ 1 & \text{if } p = 1 \end{cases}.$$

Then sender i's payoff of inducing posterior p is

$$\Pi\left(p; G^F\right) = \begin{cases} 0 & \text{if } p < \underline{p} \\ (1-\pi)^{n-1} & \text{if } p \in \left[\underline{p}, 1\right) \\ \frac{1-(1-\pi)^n}{n\pi} & \text{if } p = 1 \end{cases}.$$

By constructing the concave closure of Π, full disclosure by sender i is a best response to G^F if and only if $\Pi\left(1; G^F\right) \geq \frac{\Pi\left(\underline{p}; G^F\right)}{\underline{p}}$, or equivalently,

$$\frac{1-(1-\pi)^n}{n\pi} \geq \frac{(1-\pi)^{n-1}}{\underline{p}} \Leftrightarrow \underline{p} \geq \frac{n\pi(1-\pi)^{n-1}}{1-(1-\pi)^n} \equiv p^F(n).$$

It is immediate that $p^F(n)$ is strictly decreasing in n and $\lim_{n\to\infty} p^F(n) = 0$. Therefore, the condition $\underline{p} \geq p^F(n)$ is equivalent to $n \geq n^F\left(\underline{p}\right)$, where $n^F\left(\underline{p}\right)$ is the inverse of $p^F(\cdot)$. Q.E.D.

Proof of Lemma 6

It suffices to show that a symmetric equilibrium strategy $G \in \Lambda$ has the following properties. (1) $supp(G) \cap \left(0, \underline{p}\right) = \emptyset$; (2) G does not contain any atom except possibly at $p = 0$ or 1; (3) G is strictly increasing on $\left[\underline{p}, \hat{p}(G)\right]$; (4) $(G(p))^{n-1}$ is linear on $\left[\underline{p}, \hat{p}(G)\right]$; and (5) If $\underline{p} = 0$, then G has no atom at 0. If $\underline{p} > 0$, then G has an atom at 0.

(1) For any $G \in \Lambda$, $0 = \Pi(p; G) < \Pi(1; G)$ for all $p \in \left(0, \underline{p}\right)$. Therefore, $\left(0, \underline{p}\right) \cap C(\Pi) = \emptyset$.

(2) Suppose G has an atom at some $\tilde{p} \in [\underline{p}, 1)$. Then $\Pi(p; G)$ exhibits an upward jump at $p = \tilde{p}$, so $\tilde{p} \notin C(\Pi)$. By part (ii) of Lemma 1, there cannot be an atom at $\tilde{p}$, a contradiction.

(3) The claim is trivial if $\left(\underline{p}, \hat{p}(G)\right) = \emptyset$, so assume $\underline{p} < \hat{p}(G)$. Suppose that the claim does not hold, i.e., $supp\{G\}^c \cap \left[\underline{p}, \hat{p}(G)\right] \neq \emptyset$. Let I be a maximal open interval in $supp\{G\}^c \cap \left[\underline{p}, \hat{p}(G_i)\right]$. First, consider the case where $\sup I = \hat{p}(G)$. By the definition of $\hat{p}(G)$, we have $\hat{p}(G) < \bar{p}(G) = 1$. Second, G must have an atom at $\hat{p}(G)$, contradicting property (2) above. Finally, consider the case where $\sup I < \hat{p}(G)$. By the definition of $\hat{p}(G)$, there exists a $\tilde{p} > \sup I$ such that $G(\tilde{p}) > G(\inf I)$. By property (2), $\Pi(p, G)$ is continuous on the interval $[\sup I, \tilde{p}]$. Consequently, there exists an $\varepsilon > 0$ such that $(\sup I, \sup I + \varepsilon) \cap C(\Pi) = \emptyset$, contradicting the maximality of I.

(4) First, we establish that $\Pi(p; G)$ is linear on the interval $\left[\underline{p}, \hat{p}(G)\right]$. By Caratheodory theorem, and the existence of a best response of every sender, $(\pi, con[\Pi](\pi))$ can be written as a convex combination

of two points on the graph of Π. Denote these pair of points by $(p', \Pi(p'))$ and $(p'', \Pi(p''))$. Take any $\tilde{p} \in (\underline{p}, \hat{p}(G)]$, it is necessary that $(\tilde{p}, \Pi(\tilde{p}))$ lies on the straight line passing through $(p', \Pi(p'))$ and $(p'', \Pi(p''))$. Suppose, it lies strictly below, then putting positive weight on the neighborhood of $\tilde{p}$ would yield a strictly lower payoff than $con[\Pi](\pi)$, contradicting that $\tilde{p} \in supp\{G\}$. Second, suppose it lies strictly above. Then a convex combination of either $\{p', \tilde{p}\}$ or $\{\tilde{p}, p''\}$ would yield a payoff strictly exceeding that of $\{p', p''\}$, contradicting that $con[\Pi](\pi)$ is the optimal payoff for a sender.

The linearity of $\Pi(p;G)$ is linear on the interval $\left[\underline{p}, \hat{p}(G)\right]$ implying that G has no atom at $\hat{p}(G)$. Together with property (2) above, G has no atom on the whole interval $[\underline{p}, \hat{p}(G)]$. We, thus, have $\Pi(\cdot;G) = (G_i(\cdot))^{n-1}$ on $[\underline{p}, \hat{p}(G)]$.

(5) Suppose $\underline{p} = 0$ and that G has an atom at $\underline{p} = 0$. Then $\Pi(p;G)$ exhibits an upward jump at $p = 0$, so $0 \notin C(\Pi)$. By part (ii) of Lemma 1, there cannot be an atom at 0, a contradiction.

Suppose $\underline{p} > 0$. By properties (2) to (4) above, $\Pi(p;G)$ is zero on the interval $[0, \underline{p})$, and increases linearly on the interval $\left[\underline{p}, \hat{p}(G)\right]$. If G has no atom at $p = 0$, then $\Pi(p;G)$ has an upward kink at $\underline{p}$, and $[\underline{p}, \hat{p}(G)) \cap C(\Pi) = \emptyset$. This contradicts property (3) above.

Proof of Proposition 4 Suppose all senders choose the specified in Lemma 6. By Lemma 6, there are two cases to consider: (1) $g = 0$ and (2) $g > 0$ and $\hat{p} < 1$.

First consider case (1) with $\underline{p} > 0$. Suppose that every other sender adopts strategy (9). A sender's payoff of inducing posterior p is given by

$$\Pi(p;G) = \begin{cases} 0 & \text{if } p \in [0, \underline{p}) \\ a^{n-1} + \frac{1-a^{n-1}}{\hat{p}-\underline{p}}\left(p - \underline{p}\right) & \text{if } p \in (\underline{p}, \hat{p}] \\ 1 & \text{if } p \in (\hat{p}, 1] \end{cases}.$$

In order that the sender finds it optimal to adopt strategy (9), it is necessary and sufficient to ensure $\{0\} \cup \left[\underline{p}, \hat{p}\right] \subset C(\Pi)$. This translates into the following slope condition:

$$\frac{\Pi(\hat{p})}{\hat{p}} = \frac{\Pi\left(\underline{p}\right)}{\underline{p}} \Leftrightarrow a = \left(\frac{\underline{p}}{\hat{p}}\right)^{\frac{1}{n-1}}.$$

Substitute this into Bayes-plausibility condition (10), we get

$$H(\hat{p}) \equiv \left(\frac{1}{\hat{p}}\right)^{\frac{1}{n-1}} \frac{1}{n} \times \left(\hat{p}^{\frac{n}{n-1}} - \underline{p}^{\frac{n}{n-1}}\right) = \pi. \tag{20}$$

Observe that $H(\hat{p})$ is strictly increasing, $H\left(\underline{p}\right) < \pi$, and continuous. Therefore, there exists a unique $\hat{p} \in \left(\underline{p}, 1\right]$ if and only if $H(1) \geq \pi$, that is, $\underline{p}^{\frac{n}{n-1}} \leq 1 - n\pi$. Therefore, if we define $p^T(n)$ by

$$p^T(n) \equiv \begin{cases} (1 - n\pi)^{\frac{n-1}{n}} & \text{if } n \leq \frac{1}{\pi} \\ -1 & \text{if } n > \frac{1}{\pi} \end{cases},$$

an equilibrium with partial disclosure of this form exists if and only if $p \leq p^T(n)$.

Next, consider case (1) with $\underline{p} = 0$. Since $a = 0$, the necessary condition collapses to

$$\int_0^{\hat{p}} pd\left(\frac{p}{\hat{p}}\right)^{\frac{1}{n-1}} = \pi \Leftrightarrow \hat{p} = n\pi.$$

Thus, an equilibrium exists in this case if and only if $n \leq \frac{1}{\pi}$.

Case (2) with $\underline{p} > 0$ has been discussed in the text. It is shown that an equilibrium exists in this case if and only if the solution to equation (13) lies in the interval $[0, \pi]$. Denote the left hand side of equation (13) by $L_1(g)$. As $L_1(g)$ is strictly increasing in g, it has at most one root. Moreover, as $L_1(g)$ is continuous, a root exists in the interval $(0, \pi]$ if and only if $L_1(0) < 0$ and $L_1(\pi) \geq 0$. Observe that

$$\begin{aligned} & L_1(0) < 0 \\ & \Leftrightarrow \frac{1}{n} - \frac{1}{n}\left(\underline{p}\right)^{\frac{n}{n-1}} < \pi \\ & \Leftrightarrow \underline{p} > p^T(n), \end{aligned}$$

and

$$\begin{aligned} & L_1(\pi) \geq 0 \\ & \Leftrightarrow \frac{\pi(1-\pi)^n}{1-(1-\pi)^n} - \left(\frac{\underline{p}}{n}\right)^{\frac{n}{n-1}}\left(\frac{1-(1-\pi)^n}{\pi}\right)^{\frac{1}{n-1}} \geq 0 \\ & \Leftrightarrow \underline{p} \leq \frac{n\pi(1-\pi)^{n-1}}{1-(1-\pi)^n} = p^F(n). \end{aligned}$$

It can be shown that $p^F(n) > p^T(n)$ for all $n \geq 2$.[(22)]

Finally, consider case (2) with $\underline{p} = 0$. Suppose that every other sender adopts strategy (9). A sender's payoff of inducing posterior p is given by

$$\Pi(p; G) = \begin{cases} (1-g)^{n-1}\frac{p-\underline{p}}{\hat{p}-\underline{p}} & \text{if } p \in [0, \hat{p}] \\ (1-g)^{n-1} & \text{if } p \in (\hat{p}, 1) \\ \frac{1-(1-g)^n}{ng} & \text{if } p = 1 \end{cases}.$$

In order that the sender finds it optimal to adopt strategy (9), it is necessary and sufficient to ensure $[0, \hat{p}] \cup \{1\} \subset C(\Pi)$. This translates into the following slope condition:

$$\frac{\Pi(\hat{p})}{\hat{p}} = \Pi(1) \Leftrightarrow \hat{p} = \frac{ng(1-g)^{n-1}}{1-(1-g)^n}.$$

(22) To see this, suppose $p^T(n) > 0$, as otherwise, it holds trivially. Then

$$p^F(n) > p^T(n) \Leftrightarrow \frac{n\pi(1-\pi)^{n-1}}{1-(1-\pi)^n} > (1-n\pi)^{\frac{n-1}{n}} \Leftrightarrow \left(\frac{n\pi}{1-(1-\pi)^n}\right)^{\frac{n}{n-1}} > \frac{1-n\pi}{(1-\pi)^n}.$$

The final inequality holds, as the left-hand side exceeds 1, whereas the right-hand side is less than 1.

Substituting this into Bayes plausibility condition (10), we get

$$L_2(g) \equiv \frac{g}{1-(1-g)^n} - \pi = 0. \tag{21}$$

As $L_2(g)$ is strictly increasing and continuous in g, and $L_2(\pi) > 0$, there is at most one root, which lies in the interval $(0, \pi]$ if and only if $L_2(0) < 0$.

$$L_2(0) < 0 \Leftrightarrow n > \frac{1}{\pi}.$$

To summarize, in the case $\underline{p} = 0$, a symmetric equilibrium with $g = 0$ exists if and only if $n \le \frac{1}{\pi}$. A symmetric equilibrium with $g > 0$ exists if and only if $n > \frac{1}{\pi}$.

In the case of $\underline{p} > 0$, an equilibrium with $g = 0$ exists if and only if $\underline{p} \le p^T(n)$. A partial-disclosure equilibrium with $g > 0$ exists if and only if $\underline{p} \in \left(p^T(n), p^F(n)\right)$. By Lemma 5, a full-disclosure equilibrium exists if and only if $\underline{p} \ge p^F(n)$. We have thus established the existence and uniqueness of the symmetric equilibrium. Q.E.D.

Proof of Proposition 5

Define $S_n : \mathbb{N} \times [0,1] \to [0,1]$ by $S_n(x) \equiv \int_0^x G_n(p)\,dp$.

(i) We begin with the case $\underline{p} = 0$. Using the formula from the proof of Proposition 4, if $n \le \frac{1}{\pi}$, then $S_n(x)$ is given by

$$S_n(x) = \begin{cases} \frac{n-1}{n}\left(\frac{x^n}{n\pi}\right)^{\frac{1}{n-1}} & \text{if } x \le n\pi \\ x - \pi & \text{if } x > n\pi \end{cases}.$$

On the other hand, if $n > \frac{1}{\pi}$, then $S_n(x)$ is given by

$$S_n(x) = \begin{cases} \frac{n-1}{n}\left(\frac{x^n}{n\pi}\right)^{\frac{1}{n-1}} & \text{if } x \le \hat{p} \\ 1 - \pi - (1-g)(1-x) & \text{if } x > \hat{p} \end{cases},$$

where $(\hat{p}, g)$ are the solution to the system of equations

$$\hat{p} = \frac{ng(1-g)^{n-1}}{1-(1-g)^n}; \frac{g}{1-(1-g)^n} - \pi = 0. \tag{22}$$

We show that $S_n(x)$ is weakly increasing in n for all x, and strictly so for some x. Take a pair of natural numbers $n_1 < n_2$. For $k = 1, 2$, denote by $(\hat{p}_k, g_k)$ the solution to (22) if $n_k > \frac{1}{\pi}$.

First, suppose that $n_2 \le \frac{1}{\pi}$. In this case, it suffices to note that for all $x < n\pi$, $\left(\frac{n-1}{n}\right)\left(\frac{x^n}{n\pi}\right)^{\frac{1}{n-1}}$ is strictly increasing in n. Thus, $S_{n_1}(x) < S_{n_2}(x)$ for $x \le n_2\pi$, and $S_{n_1}(x) = S_{n_2}(x)$ for $x > n_2\pi$.

Second, consider the case $n_2 > \frac{1}{\pi} \geq n_1$ and $n_1\pi < \hat{p}_2$. For $x \leq n_1\pi$, $S_{n_2}(x) > S_{n_1}(x)$ follows from that $\left(\frac{n-1}{n}\right)\left(\frac{x^n}{n\pi}\right)^{\frac{1}{n-1}}$ is strictly increasing in n. For $x \in (n_1\pi, \hat{p}_2)$, since $x < n_2\pi$, we have

$$\begin{aligned} S_{n_2}(x) &= \frac{n_2-1}{n_2}\left(\frac{x^{n_2}}{n_2\pi}\right)^{\frac{1}{n_2-1}} \\ &> \frac{\frac{x}{\pi}-1}{\frac{x}{\pi}}\left(\frac{x^{\frac{x}{\pi}}}{\frac{x}{\pi}\pi}\right)^{\frac{1}{\frac{x}{\pi}-1}} \\ &= x-\pi = S_{n_1}(x). \end{aligned}$$

For $x \in [\hat{p}_2, 1)$, $S_{n_2}(x) = 1-\pi-(1-g)(1-x) > x-\pi = S_{n_1}(x)$ follows as $g > 0$.

Now consider the case $n_2 > \frac{1}{\pi} \geq n_1$ and $n_1\pi > \hat{p}_2$. We only need to show $S_{n_2}(x) \geq S_{n_1}(x)$ for $x \in (\hat{p}_2, n_1\pi)$, as other regions are covered above. For $x \in (\hat{p}_2, n_1\pi)$,

$$S_{n_2}(x) - S_{n_1}(x) = 1-\pi-(1-g_2)(1-x) - \frac{n-1}{n}\left(\frac{x^n}{n\pi}\right)^{\frac{1}{n-1}}$$

is concave in x. Moreover, $S_{n_2}(\hat{p}_2) - S_{n_1}(\hat{p}_2) > 0$, and $S_{n_2}(n_1\pi) - S_{n_1}(n_1\pi) > 0$. Therefore, $S_{n_2}(x) > S_{n_1}(x)$ for all $x \in (\hat{p}_2, n_1\pi)$.

Finally consider the case $n_2 \geq n_1 > \frac{1}{\pi}$. There are two possibilities: (i) $\hat{p}_1 \leq \hat{p}_2$, and (ii) $\hat{p}_1 > \hat{p}_2$. Suppose $\hat{p}_1 \leq \hat{p}_2$. For $x \in (0, \hat{p}_2]$, $S_{n_2}(x) > S_{n_1}(x)$ follows from $\frac{\partial}{\partial n}S_n(x) > 0$. Next, for $x > \hat{p}_2$, we have $S'_{n_1}(x) = 1-g_1 > 1-g_2 \geq S'_{n_2}(x)$. Combined with the fact that $S_{n_2}(1) = S_{n_1}(1) = 1-\pi$, we have $S_{n_2}(x) > S_{n_1}(x)$ for all $x \in (\hat{p}_1, 1)$.

Next suppose $\hat{p}_1 > \hat{p}_2$. For $x \in (0, \hat{p}_2]$, $S_{n_2}(x) > S_{n_1}(x)$ follows from $\frac{\partial}{\partial n}S_n(x) > 0$. For $x \in (\hat{p}_1, 1)$, $S_{n_2}(x) > S_{n_1}(x)$ follows from $S'_{n_2}(x) = 1-g_2 < 1-g_1 = S'_{n_1}(x)$ and $S_{n_2}(1) = S_{n_1}(1) = 1$. For $x \in (\hat{p}_2, \hat{p}_1)$, it follows from that $S_{n_2}(x) - S_{n_1}(x)$ is concave, $S_{n_2}(\hat{p}_2) - S_{n_1}(\hat{p}_2) > 0$ and $S_{n_2}(\hat{p}_1) - S_{n_1}(\hat{p}_1) > 0$.

Below we consider the case $\underline{p} > 0$. Define $n^F\left(\underline{p}\right)$ as the inverse of $p^F(n)$, and $n^T\left(\underline{p}\right)$ as the inverse of $p^T(n)$. Note both n^F and n^T are well-defined because p^F and p^T are decreasing function with range $[0,1]$. Moreover, $n^T\left(\underline{p}\right) < n^F\left(\underline{p}\right)$ for all $\underline{p} < 1$, $n^T(1) = n^F(1) = 1$, and $n^T(0) = \frac{1}{\pi}$.

Suppose $n \leq n^T\left(\underline{p}\right)$. Using formula from the proof of Proposition 4, $S_n(x)$ is given by

$$S_n(x) = \begin{cases} ax & \text{if } x \in \left[0, \underline{p}\right] \\ a\underline{p} + \frac{n-1}{n}a\left[\left(\frac{x^n}{\underline{p}}\right)^{\frac{1}{n-1}} - \underline{p}\right] & \text{if } x \in \left(\underline{p}, \hat{p}\right) \\ x-\pi & \text{if } x \in [\hat{p}, 1] \end{cases},$$

where $(a, \hat{p})$ is solution to the system of equations

$$\frac{1}{n}\left(\frac{1}{a^{n-1}} - a\right) = \frac{\pi}{\underline{p}}, \text{ and } \hat{p} = \frac{p}{a^{n-1}}. \tag{23}$$

However, if $n \in \left(n^T\left(\underline{p}\right), n^F\left(\underline{p}\right)\right)$, $S_n(x)$ is given by

$$S_n(x) = \begin{cases} ax & \text{if } x \in \left[0, \underline{p}\right] \\ a\underline{p} + \frac{n-1}{n} a \left[\left(\frac{x^n}{\underline{p}}\right)^{\frac{1}{n-1}} - \underline{p}\right] & \text{if } x \in \left(\underline{p}, \hat{p}\right) \\ 1 - \pi - (1-g)(1-x) & \text{if } x \in [\hat{p}, 1] \end{cases},$$

where $(a, \hat{p}, g)$ is solution to the system of equations

$$\frac{1}{n}\left(\frac{1}{a^{n-1}} - a\right) = \frac{\pi}{\underline{p}}, a = \left(\underline{p}\frac{1-(1-g)^n}{ng}\right)^{\frac{1}{n-1}}, \text{ and } \hat{p} = \frac{ng(1-g)^{n-1}}{1-(1-g)^n}. \tag{24}$$

Finally, if $n \geq n^F\left(\underline{p}\right)$, then $S_n(x) = (1-\pi)x$.

We show that $S_n(x)$ is weakly increasing in n for all x and strictly so for some x. Take a pair of natural numbers $n_1 < n_2$. Denote by $(a_i, \hat{p}_i, g_i)$ the solution to the system (23) if $n_i \in \left(n^T\left(\underline{p}\right), n^F\left(\underline{p}\right)\right)$; $(a_i, \hat{p}_i)$ the solution to the system (24) if $n_i \leq n^T\left(\underline{p}\right)$.

First suppose $n_2 \leq n^T\left(\underline{p}\right)$. Note that

$$\frac{\partial}{\partial n}\left(\frac{1}{n}\left(\frac{1}{a^{n-1}} - a\right)\right) = -\frac{1}{a^{n-1}n^2}\left(\ln a^n - a^n + 1\right) > 0$$

and

$$\frac{\partial}{\partial a}\left(\frac{1}{n}\left(\frac{1}{a^{n-1}} - a\right)\right) = -\frac{n-1+a^n}{na^n} < 0.$$

Thus, by the first equation in (23), we have $a_1 < a_2$. Moreover, (20) implies $\hat{p}_2 > \hat{p}_1$. Consequently, $S_{n_2}(x) > S_{n_1}(x)$ for $x \in \left(0, \underline{p}\right) \cup (\hat{p}_1, \hat{p}_2)$. Clearly, $S_{n_2}(x) = S_{n_1}(x)$ for $x \geq \hat{p}_2$. It remains to consider the interval $\left[\underline{p}, \hat{p}_1\right]$. On this interval, $\frac{d}{dx}\left(S_{n_2}(x) - S_{n_1}(x)\right)$ switches sign from positive to negative only once. Together with $S_{n_2}\left(\underline{p}\right) - S_{n_1}\left(\underline{p}\right) > 0$ and $S_{n_2}(\hat{p}_1) - S_{n_1}(\hat{p}_1) > 0$, we have $S_{n_2}(x) > S_{n_1}(x)$ for all $x \in \left[\underline{p}, \hat{p}_1\right]$.

Next suppose $n_2 \in (n^T\left(\underline{p}\right), n^F\left(\underline{p}\right)]$ and $n_1 \leq n^T\left(\underline{p}\right)$. Similar to above, we have $a_2 > a_1$. Moreover, $g_2 > 0$. Therefore, $S_{n_2}(x) > S_{n_1}(x)$ for $x \in (0, \underline{p}] \cup [\max\{\hat{p}_1, \hat{p}_2\}, 1)$. Furthermore, on the interval $(\underline{p}, \max\{\hat{p}_1, \hat{p}_2\})$, $\frac{d}{dx}\left(S_{n_2}(x) - S_{n_1}(x)\right)$ changes sign from positive to negative only once. Therefore, $S_{n_2}(x) > S_{n_1}(x)$ for all $x \in (\underline{p}, \max\{\hat{p}_1, \hat{p}_2\})$.

Finally, consider the case $n_1, n_2 \in (n^T\left(\underline{p}\right), n^F\left(\underline{p}\right)]$. Similar to above, we have $a_2 > a_1$. The key step is to establish that $g_1 < g_2$. To this end, define $g : \mathbb{R} \to [0,1]$ and $a : \mathbb{R} \to [0,1]$ as the solution of (24), both as a function of n. Differentiating both sides of the first equation of (24) and rearranging, we get

$$\frac{\partial a}{\partial n} = \frac{-\frac{\pi}{\underline{p}} - a\left(\frac{n\pi}{a\underline{p}} + 1\right)\ln(a)}{(n-1)\left(\frac{n\pi}{a\underline{p}} + 1\right) + 1}.$$

Next, differentiating the second equation of (24) and rearranging, we get

$$\frac{\partial a}{\partial n}=\frac{1}{n-1}\left(-a\ln a+\frac{\underline{p}}{nga^{n-2}}\left(-\ln(1-g)(1-g)^{n}-g\frac{a^{n-1}}{\underline{p}}+n\left[(1-g)^{n-1}-\frac{a^{n-1}}{\underline{p}}\right]\frac{\partial g}{\partial n}\right)\right).$$

Equating the two equations above and rearranging, we get

$$\frac{\partial g}{\partial n}=\frac{\left(\frac{\ln a+1}{\pi(n-1)+a\underline{p}}\right)a^{n}+\frac{(1-g)^{n}}{g}\ln(1-g)}{ng\left[(1-g)^{n-1}-\frac{a^{n-1}}{\underline{p}}\right]}. \tag{25}$$

The denominator of $\frac{\partial g}{\partial n}$ in (25) is negative, so it suffices to show the numerator is negative. The term $\frac{\ln a+1}{\pi(n-1)+a\underline{p}}$ is increasing in a, and $a\leq 1-\pi$. Moreover, the term $\frac{(1-g)^{n}}{g}\ln(1-g)$ is increasing in g and $g\leq\pi$. Therefore, an upper bound for the numerator of $\frac{\partial g}{\partial n}$ in (25) is given by

$$\left[\left(\frac{\ln(1-\pi)+1}{\pi(n-1)+(1-\pi)\underline{p}}\right)+\frac{1}{\pi}\ln(1-\pi)\right](1-\pi)^{n}. \tag{26}$$

Note that the expression in (26) is clearly negative if $\ln(1-\pi)<-1$. Below, suppose $\ln(1-\pi)\geq-1$.

Suppose $n\geq\frac{1}{\pi}$. The bracketed term in the expression in (26) is bounded from above by

$$\frac{\ln(1-\pi)+1}{1-\pi}+\frac{1}{\pi}\ln(1-\pi). \tag{27}$$

To see the upper bound above is negative, note that[23]

$$\begin{aligned}&\frac{\partial}{\partial\pi}\left(\frac{\ln(1-\pi)+1}{1-\pi}+\frac{1}{\pi}\ln(1-\pi)\right)\\&=-\frac{1}{\pi^{2}(1-\pi)^{2}}\left(\pi+(1-\pi)\ln(1-\pi)-\pi(\pi+\ln(1-\pi))\right)<0.\end{aligned}$$

Consequently,

$$\frac{\ln(1-\pi)+1}{1-\pi}+\frac{1}{\pi}\ln(1-\pi)\leq\lim_{\pi\to0}\left(\frac{\ln(1-\pi)+1}{1-\pi}+\frac{1}{\pi}\ln(1-\pi)\right)=0.$$

Finally, consider the case $n<\frac{1}{\pi}$. Recall $n>n^{T}\left(\underline{p}\right)$ is equivalent to $\underline{p}>p^{T}(n)$. The bracketed term in (26) is bounded from above by

$$\left(\frac{\ln(1-\pi)+1}{\pi(n-1)+(1-\pi)p^{T}(n)}\right)+\frac{1}{\pi}\ln(1-\pi).$$

We claim that $\pi(n-1)+(1-\pi)p^{T}(n)\geq 1-\pi$. To see this,

$$\begin{aligned}&\pi(n-1)+(1-\pi)p^{T}(n)-(1-\pi)\\&=\pi(n-1)+(1-\pi)\left((1-n\pi)^{\frac{n-1}{n}}\right)-(1-\pi)\\&=(1-n\pi)^{\frac{n-1}{n}}\left((1-\pi)-(1-n\pi)^{\frac{1}{n}}\right)\\&>0.\end{aligned}$$

[23] We have used the fact that for all $\pi\in(0,1)$, $\pi+(1-\pi)\ln(1-\pi)>0$ and $\pi+\ln(1-\pi)<0$.

Using this claim, the bracketed term in the expression in (26) is bounded from above by (27), which has been shown to be nonpositive above. We have thus established that $\frac{\partial g}{\partial n} > 0$, so $g_1 < g_2$.

The findings that $a_2 > a_1$ and $g_2 > g_1$ imply that $S_{n_2}(x) > S_{n_1}(x)$ for $x \in (0, \underline{p}] \cup [\max\{\hat{p}_1, \hat{p}_2\}, 1)$. Furthermore, on the interval $(\underline{p}, \max\{\hat{p}_1, \hat{p}_2\})$, $\frac{d}{dx}(S_{n_2}(x) - S_{n_1}(x))$ changes sign from positive to negative only once. Therefore, $S_{n_2}(x) > S_{n_1}(x)$ for all $x \in (\underline{p}, \max\{\hat{p}_1, \hat{p}_2\})$.

(ii) First, consider the case of $\underline{p} = 0$. It suffices to show that the root of L_2 (defined in (21)) approaches π as $n \to \infty$. To see this, observe that $L_2(g)$ is decreasing in n for all $g \in [0, \pi]$. Moreover, for all $g \in [0, \pi)$, there exists a sufficiently large $n' \in \mathbb{N}$ such that $L_2(g) < 0$.

Second, consider the case of $\underline{p} > 0$. It suffices to show that $p^F(n) \to 0$ as $n \to \infty$. By L'Hosiptal rule,

$$\lim_{n\to\infty} p^F(n) = \frac{\pi}{1-\pi} \lim_{n\to\infty} \frac{n}{(1-\pi)^{-n} - 1} = \frac{\pi}{1-\pi} \frac{1}{-\ln(1-\pi)} \lim_{n\to\infty} (1-\pi)^n = 0.$$

Q.E.D.

Proof of Lemma 8 By Lemma 7, it is without loss to assume the support of G_i is in Q. Suppose F_{G_i} assigns a positive measure to $\left\{v \in (u_{k-1}, u_k) : \Pi_i(v; G_j) < con\left[\Pi_i^{[u_{k-1}, u_k]}(v; G_j)\right]\right\}$. Then there exists a distinct pair $j, k \in \{0, 1, 2, ..., m-1\}$ such that G_i assigns positive weight on

$$\left\{q \in \triangle\Omega : q_j + q_k = 1, E_q[U_i] \in (u_{k-1}, u_k), \text{ and } \Pi_i(E_q[U_i]; G_j) < con\left[\Pi_i^{[u_{k-1}, u_k]}(E_q[U_i]; G_j)\right]\right\}.$$

Now as $\{q \in \triangle\Omega : q_j + q_k = 1\}$ is a two-dimensional space, part (ii) of Lemma 1 applies. Consequently, G_i is not a best response, a contradiction. Q.E.D.

Proof of Lemma 9 In part 1, it is without loss to assume $\underline{u} > u_{\hat{k}-1}$, as the claim holds trivially otherwise. As $F_{G_i}\left(u_{\hat{k}+1}\right) - F_{G_i}(u_{\hat{k}}) > 0$ for both $i = 1, 2$, we have $\Pi_i(u_{\hat{k}}; G_j) > \Pi_i(v; G_j) = 0$ for all $v \in [u_{\hat{k}-1}, \underline{u})$. Consequently, $[u_{\hat{k}-1}, \underline{u}) \cap C(\Pi_i) = \emptyset$, and by Lemma 8, F_{G_i} assigns zero measure to the interval $[u_{\hat{k}-1}, \underline{u})$. The proof for the rest of lemma follows from the same arguments as in the proof of Lemma 3, by replacing π_i with $E[U_i | U_i \in [u_{k-1}, u_k]]$, and noting that Lemma 8 holds. Q.E.D.

Proof of Lemma 10 (i) Suppose that $u_k \in (\underline{u}, u_{m-1}) \cap \Omega$ and both F_{G_i} and F_{G_j} assign an atom to u_k with respective sizes of α_i and α_j. As $\pi_j \in int(\triangle\Omega)$, the prior probability that $U_j = u_{m-1}$, denoted by $\pi_{j,m-1}$, is positive. Choose an $\varepsilon < \pi_{j,m-1}$ and mix the atom at u_k with a mass ε of u_{m-1}, sender j's payoff would increase by at least $\alpha_i(\alpha_j + \varepsilon) - \frac{1}{2}\alpha_i\alpha_j - \varepsilon > 0$ for ε sufficiently small. Thus, a profitable deviation exists for sender j.

(ii) Full disclosure by both senders implies that both assigns an atom at u_{m-2}. By part (i), this cannot be an equilibrium. Q.E.D.

Proof of Proposition 6 It is without loss to normalize $u_0 = 0$. Moreover, denote by π^k the prior probability that the utility being u_k, i.e., $\pi^k \equiv \Pr(U_i = u_k)$, for $k = 0, 1, 2$. Observe also that the Bayes-plausibility condition holds in expected utility, i.e., for all $G_i \in \Lambda$,

$$E_\pi [U_i] = \int_{[u_0, u_{m-1}]} v dF_{G_i}(v). \tag{28}$$

The following lemma derives further necessary conditions for the symmetric equilibrium on top of those in Lemmas 9 and 10.

Lemma 13 *Suppose that G is a symmetric equilibrium strategy.*

(i) $F_G(u_1) > 0$.

(ii) Suppose, $\lim_{v \to u_2^-} F_G(v) - F_G(u_1) > 0$. *Then* $\hat{U}_1(G) = u_1$. *Also,* $\frac{F_G(u_1)}{u_1} \le \frac{F_G(\hat{U}_2(G)) - F_G(u_1)}{\hat{U}_2(G) - u_1}$.

Proof. (i) If $F_G(u_1) = 0$, then $F_G(v)$ puts all its weights on $(u_1, u_2]$. Recall that by part (iii) of Lemma 9 and part (i) of Lemma 10, F_G has no atom on (u_1, u_2). Moreover, by part (vi) of Lemma 9, F_G is linear on $[u_1, \hat{U}]$, for some $\hat{U} \in (u_1, u_2]$. It is apparent that $C(\Pi) \subset \{p \in \triangle\Omega : p_0 + p_1 = 1 \text{ or } p_1 + p_2 = 1\}$. Moreover, as $\pi^0 > 0$, the best response to F_G necessarily puts a positive weight on posteriors with $p_0 + p_1 = 1$. This contradicts that $F_G(u_1) = 0$.

(ii) Suppose that F_G assigns positive weights on (u_1, u_2). Suppose further that $\hat{U}_1(G) < u_1$. By part (iv) of Lemma 9, F_G is increasing and linear on $(u_1, \hat{U}_2(G))$, as well as $(0, \hat{U}_1(G))$. Consequently, $\Pi(p; G)$ has a kink along the line

$$\left\{p \in \triangle\Omega : p = \alpha\left(1 - \frac{\hat{U}_1}{u_1}, \frac{\hat{U}_1}{u_1}, 0\right) + (1-\alpha)\left(0, \frac{\hat{U}_2(G) - u_1}{u_2 - u_1}, \frac{u_2 - \hat{U}_2(G)}{u_2 - u_1}\right) \text{ for some } \alpha \in [0,1]\right\}.$$

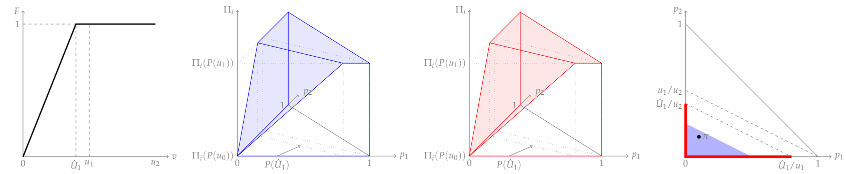
Therefore, the best response assigns no weight on either $(0, \hat{U}_1(G))$ or $(u_1, \hat{U}_2(G))$, which is a contradiction.

Now, suppose that F_G assigns positive weights on (u_1, u_2) and $\frac{F_G(u_1)}{u_1} > \frac{F_G(\hat{U}_2(G)) - F_G(u_1)}{\hat{U}_2(G) - u_1}$. In this case, $\Pi(p; G)$ has a kink along the line $\{p \in \triangle\Omega : p_1 u_1 + p_2 u_2 = u_1\}$. Therefore, the best response assigns no weight on either $(0, u_1)$ or $(u_1, \hat{U}_2(G))$, which is a contradiction. ■

With this lemma, the symmetric equilibrium G necessarily takes one of the following seven forms.

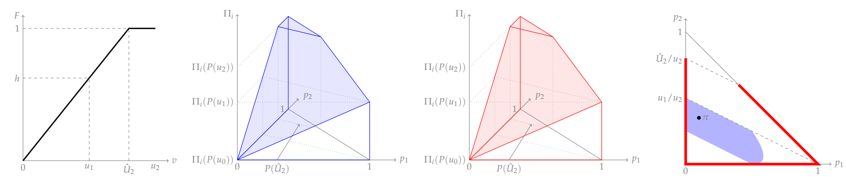
1.
$$F_G(v) = \begin{cases} \frac{1}{\hat{U}_1} v & \text{if } v \in [0, \hat{U}_1) \\ 1 & \text{if } v \in [\hat{U}_1, u_2] \end{cases},$$
for some $\hat{U}_1 \in (0, u_1)$.

2.
$$F_G(v) = \begin{cases} \frac{h}{u_1} v & \text{if } v \in [0, u_1) \\ h + \frac{1-h}{\hat{U}_2 - u_1}(v - u_1) & \text{if } v \in [u_1, \hat{U}_2) \\ 1 & \text{if } v \in [\hat{U}_2, u_2] \end{cases},$$



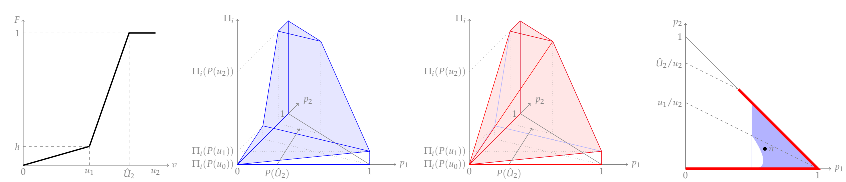
(a) F (b) Π_i (c) $\text{con}(\Pi_i)$ (d) $\text{supp}F$

Figure 16: Form 1



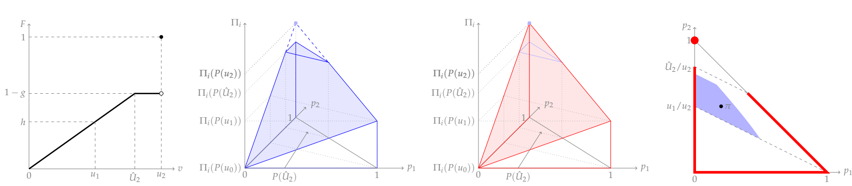
(a) F (b) Π_i (c) $\text{con}(\Pi_i)$ (d) $\text{supp}F$

Figure 17: Form 2



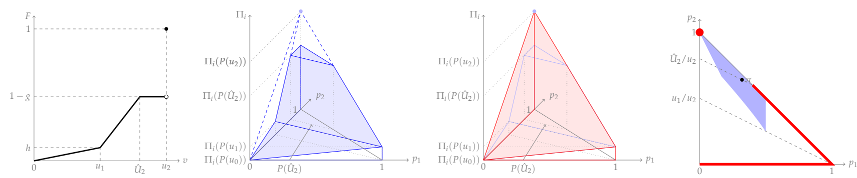
(a) F (b) Π_i (c) $\text{con}(\Pi_i)$ (d) $\text{supp}F$

Figure 18: Form 3



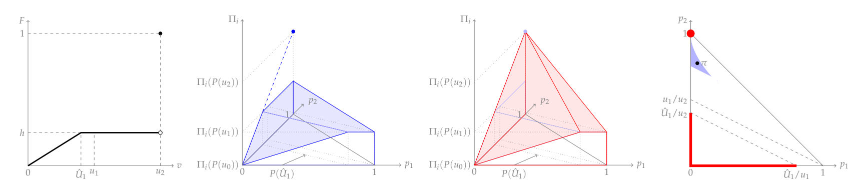
(a) F (b) Π_i (c) $\text{con}(\Pi_i)$ (d) $\text{supp}F$

Figure 19: Form 4



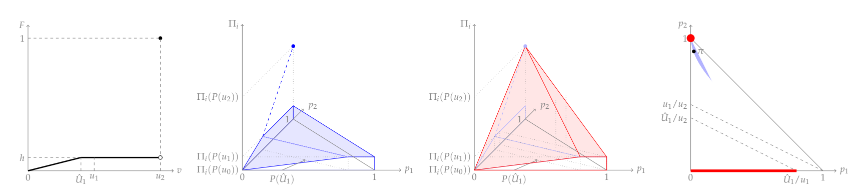
(a) F (b) Π_i (c) $\text{con}(\Pi_i)$ (d) $\text{supp}F$

Figure 20: Form 5



(a) F (b) Π_i (c) $\text{con}(\Pi_i)$ (d) $\text{supp}F$

Figure 21: Form 6



(a) F (b) Π_i (c) $\text{con}(\Pi_i)$ (d) $\text{supp}F$

Figure 22: Form 7

for some $h \in (0,1)$ and $\hat{U}_2 \in (u_1, u_2]$ such that $\frac{h}{u_1} = \frac{1-h}{\hat{U}_2 - u_1}$.

3.

$$F_G(v) = \begin{cases} \frac{h}{u_1} v & \text{if } v \in [0, u_1) \\ h + \frac{1-h}{\hat{U}_2 - u_1}(v - u_1) & \text{if } v \in [u_1, \hat{U}_2) \\ 1 & \text{if } v \in [\hat{U}_2, u_2] \end{cases},$$

for some $h \in (0,1)$ and $\hat{U}_2 \in (u_1, u_2]$ such that $\frac{h}{u_1} < \frac{1-h}{\hat{U}_2 - u_1}$.

4.

$$F_G(v) = \begin{cases} \frac{h}{u_1} v & \text{if } v \in [0, u_1) \\ h + \frac{1-\frac{g}{2}-h}{u_2 - u_1}(v - u_1) & \text{if } v \in [u_1, \hat{U}_2) \\ 1-g & \text{if } v \in [\hat{U}_2, u_2] \\ 1 & \text{if } v = u_2 \end{cases},$$

for some $h \in (0,1)$, $g \in (0, \pi^2)$, and $\hat{U}_2 \in (u_1, u_2]$ such that $\frac{h}{u_1} = \frac{1-\frac{g}{2}-h}{u_2 - u_1}$.

5.

$$F_G(v) = \begin{cases} \frac{h}{u_1} v & \text{if } v \in [0, u_1) \\ h + \frac{1-\frac{g}{2}-h}{u_2 - u_1}(v - u_1) & \text{if } v \in [u_1, \hat{U}_2) \\ 1-g & \text{if } v \in [\hat{U}_2, u_2] \\ 1 & \text{if } v = u_2 \end{cases},$$

for some $h \in (0,1)$, $g \in (0, \pi^2)$, and $\hat{U}_2 \in (u_1, u_2]$ such that $\frac{h}{u_1} < \frac{1-\frac{g}{2}-h}{u_2 - u_1}$.

6.

$$F_G(v) = \begin{cases} \frac{h}{\hat{U}_1} v & \text{if } v \in [0, \hat{U}_1) \\ h & \text{if } v \in [\hat{U}_1, u_2) \\ 1 & \text{if } v = u_2 \end{cases},$$

for some $h \in [1 - \pi_2, 1)$ and $\hat{U}_1 \in (0, u_1)$ such that $\frac{h}{\hat{U}_1} = \frac{\frac{1}{2}(1+h)}{u_2}$.

7.

$$F_G(v) = \begin{cases} \frac{h}{\hat{U}_1} v & \text{if } v \in [0, \hat{U}_1) \\ h & \text{if } v \in [\hat{U}_1, u_2) \\ 1 & \text{if } v = u_2 \end{cases},$$

for some $h \in [1 - \pi_2, 1)$ and $\hat{U}_1 \in (0, u_1)$ such that $\frac{h}{\hat{U}_1} < \frac{\frac{1}{2}(1+h)}{u_2}$.

Below we consider each form in turn, and derive necessary conditions on the common prior π for the existence of each particular form of equilibrium.

Form 1: Bayes-plausibility in expected utility (28) requires that

$$u_2\pi^2 + u_1\pi^1 = \int_0^{\hat{U}_1} \frac{1}{\hat{U}_1} x dx \Leftrightarrow \hat{U}_1 = 2\left(u_2\pi^2 + u_1\pi^1\right).$$

Requirement $\hat{U}_1 \in (0, u_1)$ implies

$$u_2\pi^2 + u_1\pi^1 \leq \frac{u_1}{2} \Leftrightarrow \pi^2 \leq \left(\frac{1}{2} - \pi^1\right)\frac{u_1}{u_2}. \tag{29}$$

We claim that for every π that satisfies inequality (29), there exists a symmetric equilibrium G that takes Form 1 with $\hat{U}_1 = 2\left(u_2\pi^2 + u_1\pi^1\right)$. An explicit density over posteriors can be constructed as follows:

$$\tilde{g}(p) = \begin{cases} \frac{\pi^1 u_1^2}{2\left(u_2\pi^2 + u_1\pi^1\right)^2} & \text{if } p_2 = 0 \text{ and } p_1 \leq \frac{2\left(u_2\pi^2 + u_1\pi^1\right)}{u_1} \\ \frac{\pi^2 u_2^2}{2\left(u_2\pi^2 + u_1\pi^1\right)^2} & \text{if } p_1 = 0 \text{ and } p_2 \leq \frac{2\left(u_2\pi^2 + u_1\pi^1\right)}{u_2} \\ 0 & \text{otherwise} \end{cases}.$$

If the sender's opponent uses this density function, then the sender faces a payoff function given in Figure 16(b). It is apparent that whenever $E_\pi[U_i] < \hat{U}_1$, every Bayes-plausible distribution that assigns weights only to $\left\{p \in \triangle\Omega : E_\pi[U_i] \leq \hat{U}_1\right\}$ is a best response. The density function $\tilde{g}$ above is therefore a best response.

Form 2: Bayes-plausibility in expected value (28) requires that

$$\pi^1 u_1 + \pi^2 u_2 = \int_0^{u_1} \frac{h}{u_1} x dx + \int_{u_1}^{\hat{U}_2} \frac{1-h}{\hat{U}_2 - u_1} x dx \Leftrightarrow \hat{U}_2 = \frac{2\left(\pi^1 u_1 + \pi^2 u_2\right) - u_1}{1-h}. \tag{30}$$

Since $\frac{h}{u_1} = \frac{1-h}{\hat{U}_2 - u_1}$, we have

$$h = \frac{1}{2\left(u_2\pi^2 + u_1\pi^1\right)} u_1, \text{ and } \hat{U}_2 = 2\left(u_2\pi^2 + u_1\pi^1\right). \tag{31}$$

Requirements $h < 1$ and $u_1 < \hat{U}_2 \leq u_2$ translate into

$$\frac{u_1}{2u_2}\left(1 - 2\pi^1\right) < \pi^2 \leq \frac{u_2 - 2u_1\pi^1}{2u_2}. \tag{32}$$

Furthermore, Bayes-plausibility imposes constraints on feasible priors that can generate such F_G, specifically; it imposes the following upper bound on π^1:

$$\begin{aligned} \pi^1 &\leq \int_0^{u_1} \frac{h}{u_1}\frac{x}{u_1} dx + \int_{u_1}^{\hat{U}_2} \frac{1-h}{\hat{U}_2 - u_1}\frac{u_2 - x}{u_2 - u_1} dx \\ &= u_2 \frac{4\left(1 - \pi^1 - \pi^2\right)\left(u_1\pi^1 + u_2\pi^2\right) - u_1}{16\left(u_2\pi^2 + u_1\pi^1\right)\left(u_2 - u_1\right)} + \pi^1 \end{aligned}$$

If π^1 exceeds the upper bound above, then for any distribution of posterior G that induces F_G, Bayes-plausibility is violated in the dimension of u_1.[24] The inequality can be simplified into

$$4\left(1 - \pi^1 - \pi^2\right)\left(\pi^1 u_1 + \pi^2 u_2\right) - u_1 \geq 0. \tag{33}$$

[24]Loosely speaking, there is an "excess supply" of π^1.

We claim that every π that satisfies the inequalities (32) and (33), there exists a symmetric equilibrium G that takes Form 2 with h and $\hat{U}_2$ given by (31). An explicit density over posteriors can be constructed as follows. Define $\alpha \equiv \frac{4\pi^1(u_2-u_1)(\pi^1u_1+\pi^2u_2)}{4u_2(\pi^1u_1+\pi^2u_2)-4(\pi^1u_1+\pi^2u_2)^2-u_1u_2}$. Note $\alpha > 0$ under (33).

$$\tilde{g}(p) = \begin{cases} \frac{\alpha u_1}{2(u_2\pi^2+u_1\pi^1)} & \text{if } p_2 = 0 \\ \frac{\alpha(u_2-u_1)}{2(u_2\pi^2+u_1\pi^1)} & \text{if } p_0 = 0 \text{ and } p_2 \leq \frac{2(u_2\pi^2+u_1\pi^1)-u_1}{u_2-u_1} \\ (1-\alpha)\frac{u_2}{2(u_2\pi^2+u_1\pi^1)} & \text{if } p_1 = 0 \text{ and } p_2 \leq \frac{2(u_2\pi^2+u_1\pi^1)}{u_2} \\ 0 & \text{otherwise} \end{cases}.$$

The density function $\tilde{g}$ over posteriors defined above put positive weight only on the boundary of $\triangle\Omega$. Note that other distributions over posteriors may also induce the same distribution over expected utilities.

If the sender's opponent uses this density function, then the sender faces a payoff function given in Figure 17(b). It is apparent that whenever $E_\pi[U_i] < \hat{U}_2$, every Bayes-plausible distribution that assigns weights only to $\{p \in \triangle\Omega : E_\pi[U_i] \leq \hat{U}_2\}$ is a best response. The density function g above is, therefore, a best response.

Form 3: Like the analysis of Form 2 above, (30) has to hold. Moreover, for those posteriors with expected utilities less than $\hat{U}_2$, only those with $p_2 = 0$ or $p_0 = 0$ are on $\operatorname{con}(\Pi_i)$ (see Figure 18(b)). Therefore, weights can only be assigned to $\{p \in \triangle\Omega : p_2 = 0\}$ and $\{p \in \triangle\Omega : p_0 = 0 \text{ and } E_p[U_i] \leq \hat{U}_2\}$. Bayes-plausibility now requires:

$$\pi^0 = \int_0^{u_1} \left(\frac{h}{u_1}\frac{u_1-x}{u_1}\right)dx; \text{ and } \pi^2 = \int_{u_1}^{\hat{U}_2}\left(\frac{1-h}{\hat{U}_2-u_1}\frac{x-u_1}{u_2-u_1}\right)dx.$$

Upon solving, we have

$$h = 2\pi^0; \text{ and } \hat{U}_2 = \frac{2\pi^2(u_2-u_1)}{1-2\pi^0} + u_1.$$

Requirements $h < 1$ and $\hat{U}_2 \in (u_1, u_2]$ translate into

$$\pi^1 \geq \frac{1}{2}.$$

Finally, condition $\frac{h}{u_1} < \frac{1-h}{\hat{U}_2-u_1}$ translates into

$$4\left(1-\pi^1-\pi^2\right)\left(\pi^1u_1+\pi^2u_2\right) - u_1 < 0.$$

If the sender's opponent uses this distribution with h and $\hat{U}_2$ as determined above, then the sender faces a payoff function given in Figure 18(b). It is apparent that whenever the conditions on π above holds, every Bayes-plausible distribution that assigns weights only to

$$\{p \in \triangle\Omega : p_2 = 0\} \text{ and } \{p \in \triangle\Omega : p_0 = 0 \text{ and } E_p[U_i] \leq \hat{U}_2\}$$

is a best response.

Form 4: First note that the atom at u_2, denoted by g, must be such that the slopes on $[u_1, u_2]$ are aligned:

$$\frac{1-\frac{g}{2}-h}{u_2-u_1}=\frac{\frac{g}{2}}{u_2-\hat{U}_2}\Leftrightarrow g=2\left(1-h\right)\frac{u_2-\hat{U}_2}{2u_2-\hat{U}_2-u_1}. \tag{34}$$

Second, Bayes-plausibility in expected value requires that

$$\pi^1 u_1+\pi^2 u_2=\int_0^{u_1}\frac{h}{u_1}xdx+\int_{u_1}^{\hat{U}_2}\frac{1-\frac{g}{2}-h}{u_2-u_1}xdx+gu_2. \tag{35}$$

Since $\frac{h}{u_1}=\frac{1-\frac{g}{2}-h}{u_2-u_1}$, we have

$$g=\frac{2\left(\pi^1 u_1+\pi^2 u_2\right)-u_2}{\pi^1 u_1+\pi^2 u_2}, h=\frac{u_1}{2\left(\pi^1 u_1+\pi^2 u_2\right)}, \text{and } \hat{U}_2=2\left(u_2-\left(\pi^1 u_1+\pi^2 u_2\right)\right). \tag{36}$$

Requirements $h<1$, $\hat{U}_2\in(u_1, U_2]$, and $g\leq\pi^2$ translate into

$$\frac{u_2}{2}<\pi^1 u_1+\pi^2 u_2<\frac{2u_2-u_1}{2}, \text{and } \pi^1\leq\frac{1-\left(2-\pi^2\right)\pi^2}{2-\pi^2}\frac{u_2}{u_1}. \tag{37}$$

Furthermore, Bayes-plausibility imposes the following upper bound on π^1:

$$\pi^1\leq\int_0^{u_1}\frac{h}{u_1}\frac{x}{u_1}dx+\int_{u_1}^{\hat{U}_2}\frac{1-\frac{g}{2}-h}{u_2-u_1}\frac{u_2-x}{u_2-u_1}dx.$$

Upon simplification,

$$4\left(1-\left(\pi^1+\pi^2\right)\right)\left(\pi^1 u_1+\pi^2 u_2\right)-u_1\geq 0. \tag{38}$$

Note that the constraint $\pi^1\leq\frac{1-\left(2-\pi^2\right)\pi^2}{2-\pi^2}\frac{u_2}{u_1}$ in (37) is guaranteed by (38), so can be dropped.

We claim that any π that satisfies the inequalities (37) and (38), there exists a symmetric equilibrium G that takes Form 4 with g, h, and $\hat{U}_2$ given by (36). An explicit density of G over posteriors can be constructed as follows. Define $\alpha\equiv\frac{4\pi^1(u_2-u_1)\left(\pi^1 u_1+\pi^2 u_2\right)}{4u_2\left(\pi^1 u_1+\pi^2 u_2\right)-4\left(\pi^1 u_1+\pi^2 u_2\right)^2-u_1 u_2}$.

$$g\left(p\right)=\begin{cases}\alpha\frac{u_1}{2\left(\pi^1 u_1+\pi^2 u_2\right)} & \text{if } p_2=0\\ \alpha\frac{u_2-u_1}{2\left(\pi^1 u_1+\pi^2 u_2\right)} & \text{if } p_0=0 \text{ and } p_2\leq\frac{2\left(u_2-\left(\pi^1 u_1+\pi^2 u_2\right)\right)-u_1}{u_2-u_1}\\ \left(1-\alpha\right)\frac{u_2}{2\left(\pi^1 u_1+\pi^2 u_2\right)} & \text{if } p_1=0 \text{ and } p_2\leq\frac{2\left(u_2-\left(\pi^1 u_1+\pi^2 u_2\right)\right)}{u_2}\\ 0 & \text{otherwise}\end{cases}.$$

In addition, distribution G puts an atom of size $\frac{2\left(\pi^1 u_1+\pi^2 u_2\right)-u_2}{\pi^1 u_1+\pi^2 u_2}$ at $p=(0,0,1)$.

If sender i's opponent uses strategy G above, then $con\left[\Pi_i\right]$ is linear (see Figure 19(c)). Consequently, every Bayes-plausible distribution that assigns positive weights only on $C\left(\Pi_i\right)=\left\{p\in\triangle\Omega: E_p\left[U_i\right]\notin\left(\hat{U}_2, u_2\right)\right\}$ is a best response. Therefore, G defined above is indeed a best response.

Form 5: Similar to the analysis in Form 4 above, (35) has to hold. Moreover, only posteriors with $p_2 = 0$ and those with $p_0 = 0$ are on $\text{con}(\Pi_i)$. Therefore, weights can only be assigned on $(0, 0, 1)$, $\{p \in \triangle\Omega : p_2 = 0\}$ and $\{p \in \triangle\Omega : p_0 = 0 \text{ and } E_p[U_i] \leq \hat{U}_2\}$. Consequently, Bayes-plausibility condition requires:

$$\pi^0 = \int_0^{u_1} \frac{h}{u_1} \frac{u_1 - x}{u_1} dx \text{ and } \pi^2 = \int_{u_1}^{\hat{U}_2} \left(\frac{1 - \frac{g}{2} - h}{u_2 - u_1} \frac{x - u_1}{u_2 - u_1} \right) dx + g.$$

Upon solving,

$$h = 2\pi^0, \text{ and } \hat{U}_2 = 2u_2 - u_1 - \frac{2(u_2 - u_1)\pi^2}{1 - 2\pi^0}.$$

This gives

$$g = \frac{1}{\pi^2} \left(1 - 2\pi^0\right) \left(2\pi^0 + 2\pi^2 - 1\right) < \pi^2.$$

Requirement $\hat{U}_2 \in (u_1, u_2]$ translate into

$$\frac{1}{2} \left(1 - \pi^2\right) < \pi^1 < \frac{1}{2}.$$

Finally, inequality $\frac{h}{u_1} < \frac{1 - \frac{g}{2} - h}{u_2 - u_1}$ simplifies into

$$4\left(1 - \left(\pi^1 + \pi^2\right)\right) \left(\pi^1 u_1 + \pi^2 u_2\right) - u_1 < 0.$$

If the sender's opponent uses this distribution with g, h and $\hat{U}_2$ as determined above, then the sender faces a payoff function given in Figure 20(b). As $con[\Pi_i]$ is linear (see Figure 20(c)), every Bayes-plausible distribution that assigns positive weights only on

$$C(\Pi_i) = \{p \in \triangle\Omega : p_2 = 0\} \cup \{p \in \triangle\Omega : p_0 = 0 \text{ and } E_p[U_i] \leq \hat{U}_2\} \cup (0, 0, 1)$$

is a best response.

Form 6: Bayes-plausibility in expected value in (28) requires that

$$u_2\pi^2 + u_1\pi^1 = \int_0^{\hat{U}_1} \frac{h}{\hat{U}_1} x dx + (1 - h) u_2 \Leftrightarrow \hat{U}_1 = \frac{2}{h} \left(u_2\pi^2 + u_1\pi^1 - (1 - h) u_2\right). \tag{39}$$

Since $\frac{h}{\hat{U}_1} = \frac{\frac{1}{2}(1+h)}{u_2}$, we have

$$h = \frac{u_2}{u_2\pi^2 + u_1\pi^1} - 1, \text{ and } \hat{U}_1 = 2\left(u_2 - \left(u_2\pi^2 + u_1\pi^1\right)\right). \tag{40}$$

Requirement $\hat{U}_1 \leq u_1$ and $h \in [1 - \pi^2, 1)$ translates into

$$\left(1 - \pi^2\right) \frac{u_2}{u_1} - \frac{1}{2} \leq \pi^1 \leq \frac{u_2}{u_1} \frac{\left(1 - \pi^2\right)^2}{2 - \pi^2}. \tag{41}$$

Furthermore, Bayes-plausibility imposes the following upper bound on π^1:

$$\pi^1 \leq \int_0^{\hat{U}_1} \frac{h}{\hat{U}_1} \frac{x}{u_1} dx.$$

The inequality can be simplified into $\pi^1 \le \frac{u_2}{u_1}\frac{\left(1-\pi^2\right)^2}{2-\pi^2}$, which coincides with the upper bound in (41).

We claim that any π that satisfies inequalities in (41), there exists a symmetric equilibrium G that takes Form 6 with h and $\hat{U}_1$ given by (40). An explicit density over posteriors can be constructed as follows. Define $\alpha \equiv \frac{u_1\pi^1\left(\pi^1 u_1+\pi^2 u_2\right)}{\left(u_2-\left(\pi^1 u_1+\pi^2 u_2\right)\right)^2}$. For all $p \neq (0,0,1)$,

$$g(p) = \begin{cases} \alpha\frac{u_1}{2\left(\pi^1 u_1+\pi^2 u_2\right)} & \text{if } p_2 = 0 \text{ and } p_1 \le \frac{2\left(u_2-\left(u_2\pi^2+u_1\pi^1\right)\right)}{u_1} \\ (1-\alpha)\frac{u_2}{2\left(\pi^1 u_1+\pi^2 u_2\right)} & \text{if } p_1 = 0 \text{ and } p_2 \le \frac{2\left(u_2\pi^2+u_1\pi^1\right)}{u_2} \\ 0 & \text{otherwise} \end{cases}.$$

In addition, distribution G puts an atom of size $\frac{u_2}{u_2\pi^2+u_1\pi^1}$ at $p=(0,0,1)$.

If the sender's opponent uses this strategy G, then the sender faces a payoff function given in Figure 21(b). It is apparent that whenever the conditions on π above holds, every Bayes-plausible distribution that assigns weights only to $(0,0,1)$ and $\left\{p \in \triangle\Omega : E_p\left[U_i\right] \le \hat{U}_1\right\}$ is a best response. Therefore, G is indeed a best response.

Form 7: As in Form 6, (39) has to hold. In this case, only $(0,0,1)$ and posteriors with $p_2 = 0$ are on $\text{con}\left(\Pi_i\right)$. Therefore, weights can only be assigned on $(0,0,1)$ and $\left\{p \in \triangle\Omega : p_2 = 0 \text{ and } E_p\left[U_i\right] \le \hat{U}_1\right\}$. Consequently, Bayes-plausibility condition requires that:

$$\pi^1 = \int_0^{\hat{U}_1} \frac{h}{\hat{U}_1}\frac{x}{u_1}dx \text{ and } \pi^2 = 1-h.$$

Upon solving, we have

$$h = 1-\pi^2, \text{and } \hat{U}_1 = \frac{2\pi^1}{1-\pi^2}u_1.$$

Requirement $\hat{U}_1 \in (0, u_1]$ translates into

$$\pi^2 \le 1-2\pi^1.$$

Finally, condition $\frac{h}{\hat{U}_1} < \frac{\frac{1}{2}(1+h)}{u_2}$ translates into

$$\pi^1 > \frac{u_2\left(1-\pi^2\right)^2}{u_1\left(2-\pi^2\right)}.$$

If the sender's opponent uses this form of strategy with h and $\hat{U}_1$ as determined above, the sender faces a payoff function depicted by Figure 22(b). If the prior satisfies the conditions above, then every Bayes-plausible distribution that assigns weights only to $(0,0,1)$ and $\left\{p \in \triangle\Omega : p_2 = 0 \text{ and } E_p\left[U_i\right] \le \hat{U}_1\right\}$ is a best response.

The region of priors for each form of equilibrium is summarized as follows.

Form 1: $\pi^2 \le \frac{1}{2}\left(1-2\pi^1\right)\frac{u_1}{u_2}$.

Form 2: $\frac{u_1}{2u_2}\left(1-2\pi^1\right) < \pi^2 \le \frac{1}{2} - \frac{u_1}{u_2}\pi^1$ and $4\left(1-\pi^1-\pi^2\right)\left(\pi^1 u_1+\pi^2 u_2\right) - u_1 \ge 0$.

Form 3: $\pi^1 \geq \frac{1}{2}$ and $4\left(1-\pi^1-\pi^2\right)\left(\pi^1 u_1+\pi^2 u_2\right)-u_1<0$.

Form 4: $\frac{1}{2}-\frac{u_1}{u_2}\pi^1<\pi^2<1-\frac{u_1}{u_2}\left(\frac{1}{2}+\pi^1\right)$ and $4\left(1-\left(\pi^1+\pi^2\right)\right)\left(\pi^1 u_1+\pi^2 u_2\right)-u_1 \geq 0$.

Form 5: $\pi^2>1-2\pi^1$ and $\pi^1<\frac{1}{2}$ and $4\left(1-\left(\pi^1+\pi^2\right)\right)\left(\pi^1 u_1+\pi^2 u_2\right)-u_1<0$.

Form 6: $\pi^2 \geq 1-\frac{u_1}{u_2}\left(\frac{1}{2}+\pi^1\right)$ and $\pi^1 \leq \frac{u_2}{u_1}\frac{\left(1-\pi^2\right)^2}{2-\pi^2}$.

Form 7: $\pi^2 \leq 1-2\pi^1$ and $\pi^1>\frac{u_2}{u_1}\frac{\left(1-\pi^2\right)^2}{2-\pi^2}$.

These regions are graphed in Figure 14. We have verified that these conditions on the common prior are necessary and sufficient for the existence of a symmetric equilibrium of each form. Finally, it is straightforward to verify that they partition the space of all possible priors. Q.E.D.

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